Estimating methane emission source and rate with continuous monitoring systems: Model updates and active research directions

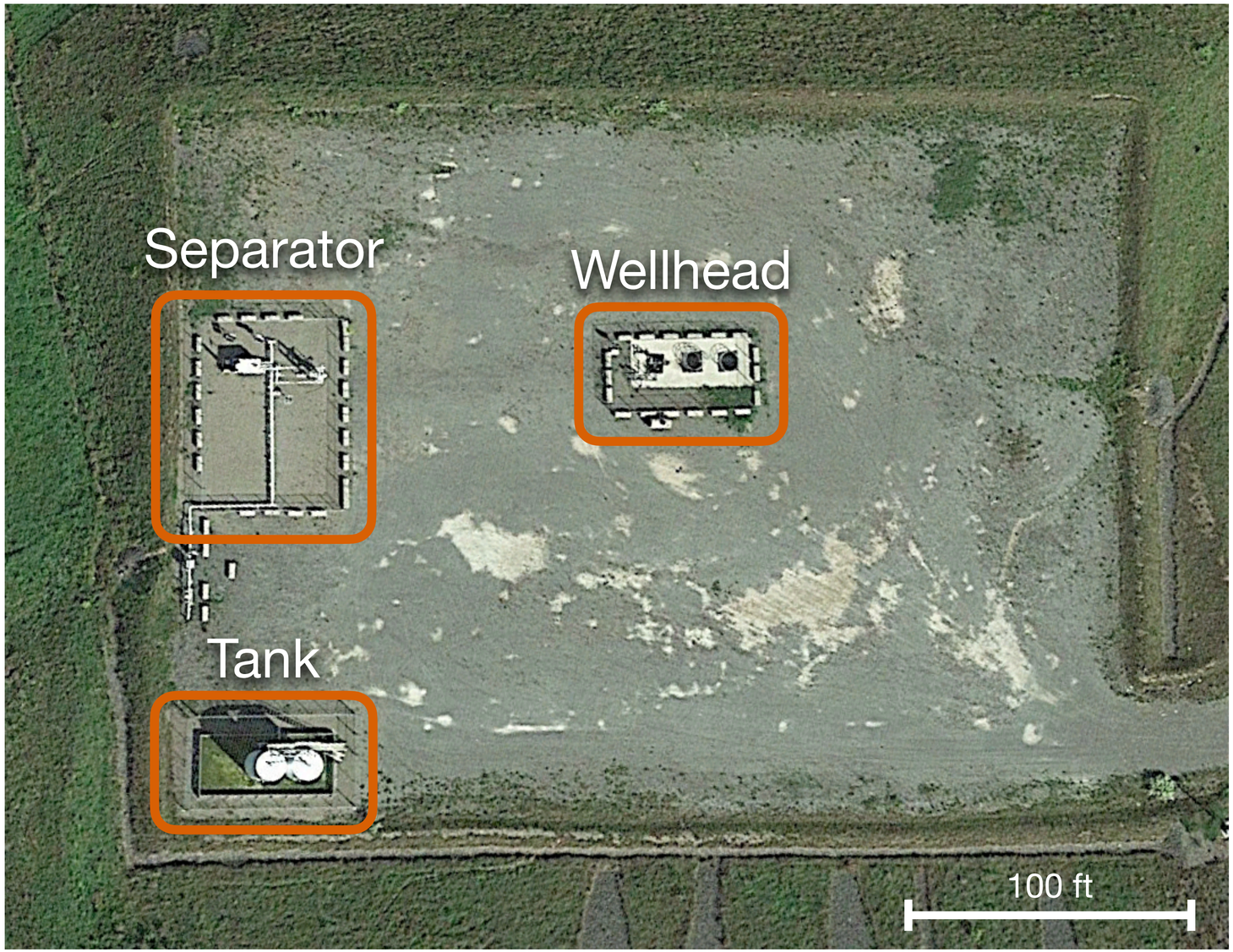
William Daniels (wdanie16@jh.edu)

with many contributions from the Colorado School of Mines team!

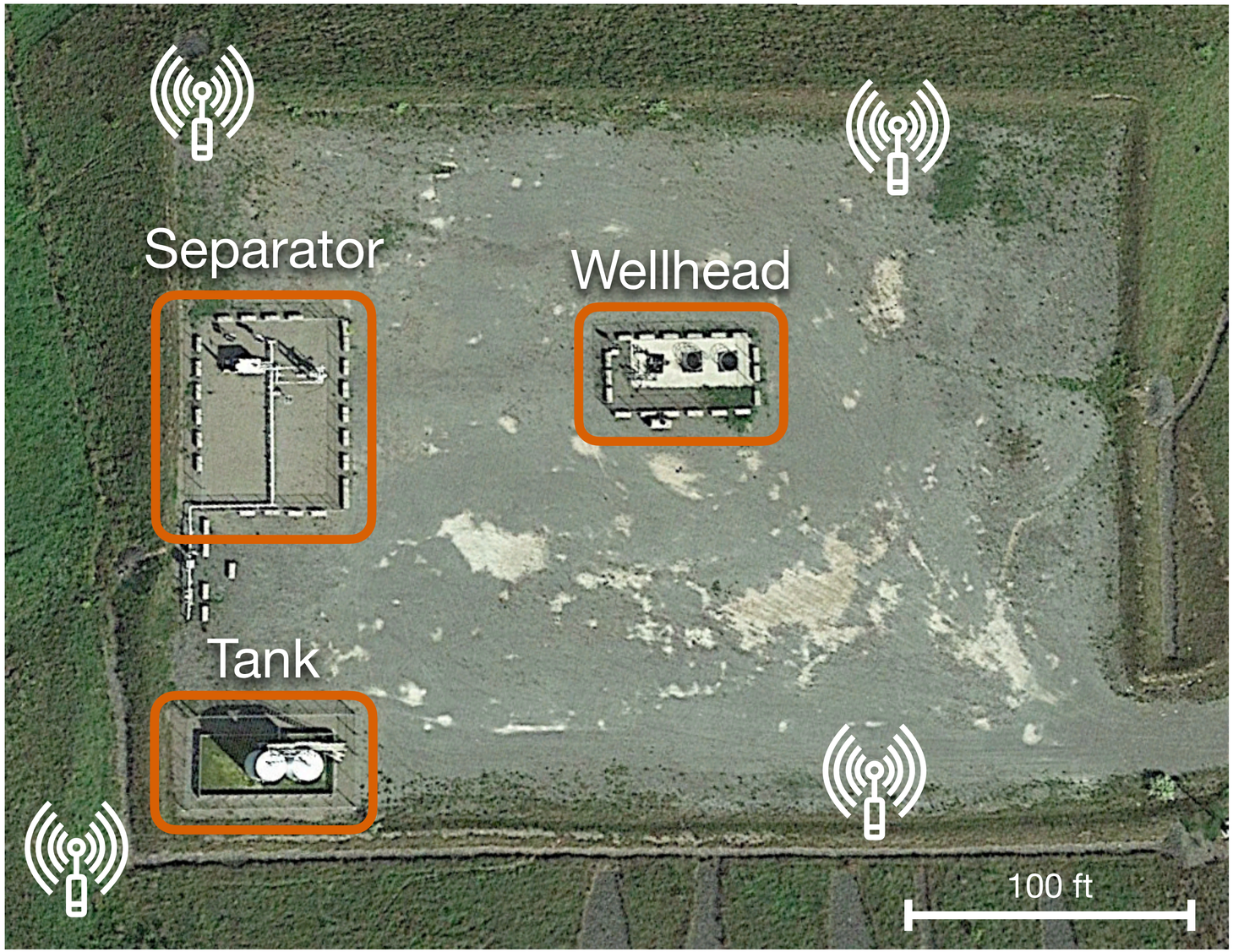


Energy Emissions Modeling and Data Lab (EEMDL) Annual Meeting
Continuous Monitoring System (CMS) Workshop
October 13, 2026

Example production oil and gas site



Example production oil and gas site



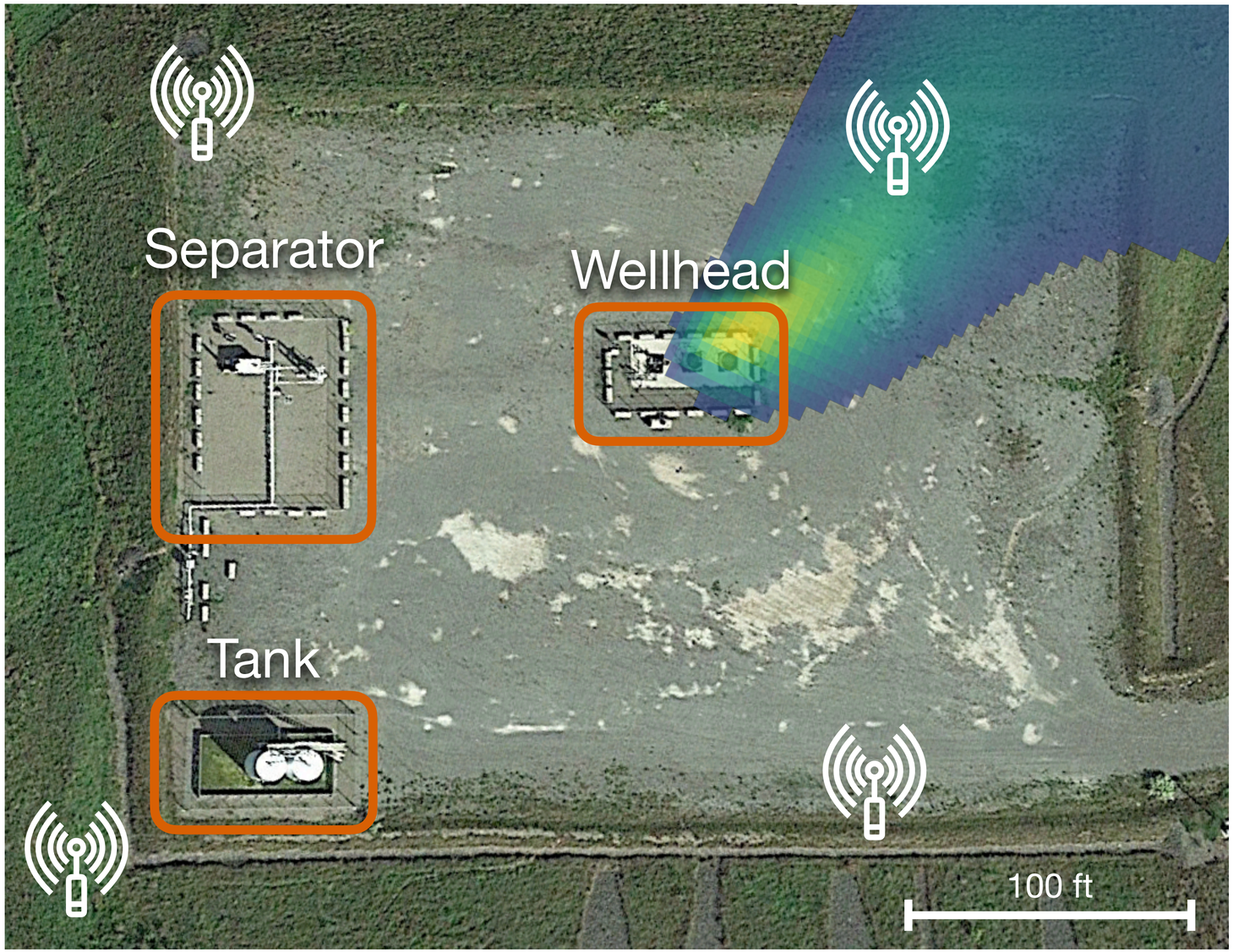
continuous monitoring system (CMS)

..or..

point sensor network



Example production oil and gas site



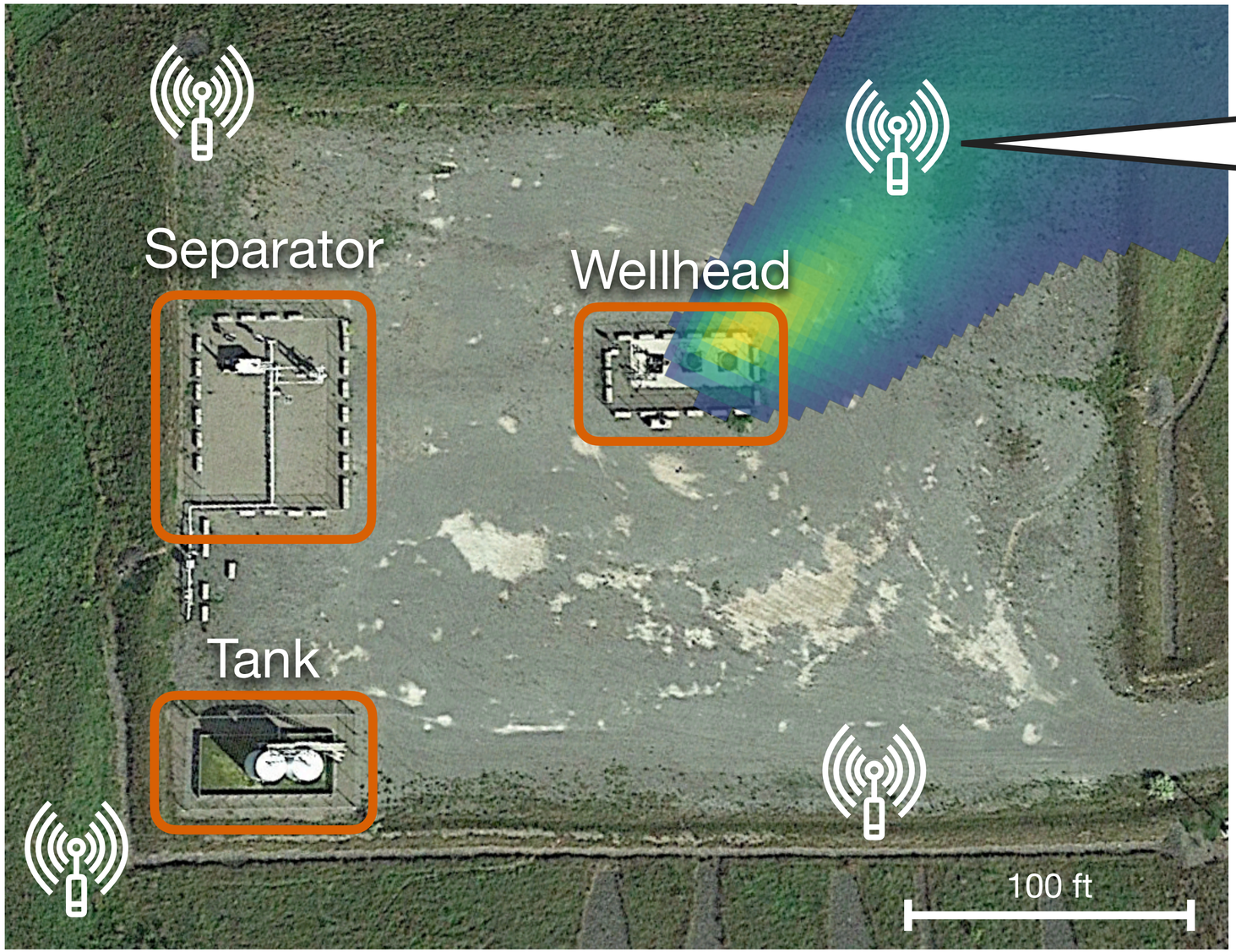
continuous monitoring system (CMS)

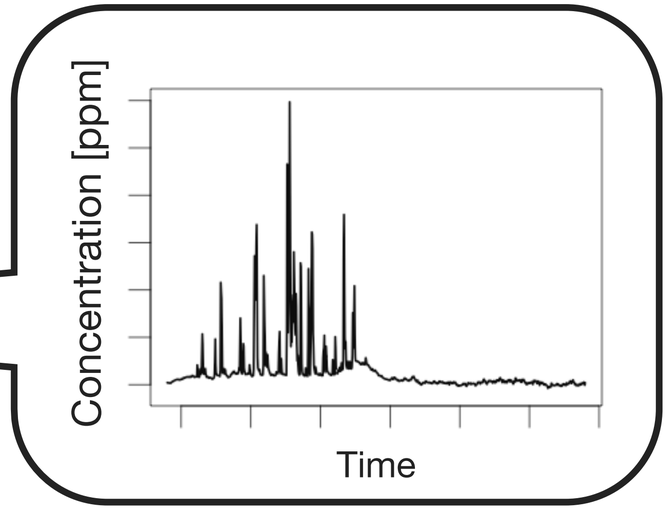
..or..

point sensor network



Example production oil and gas site



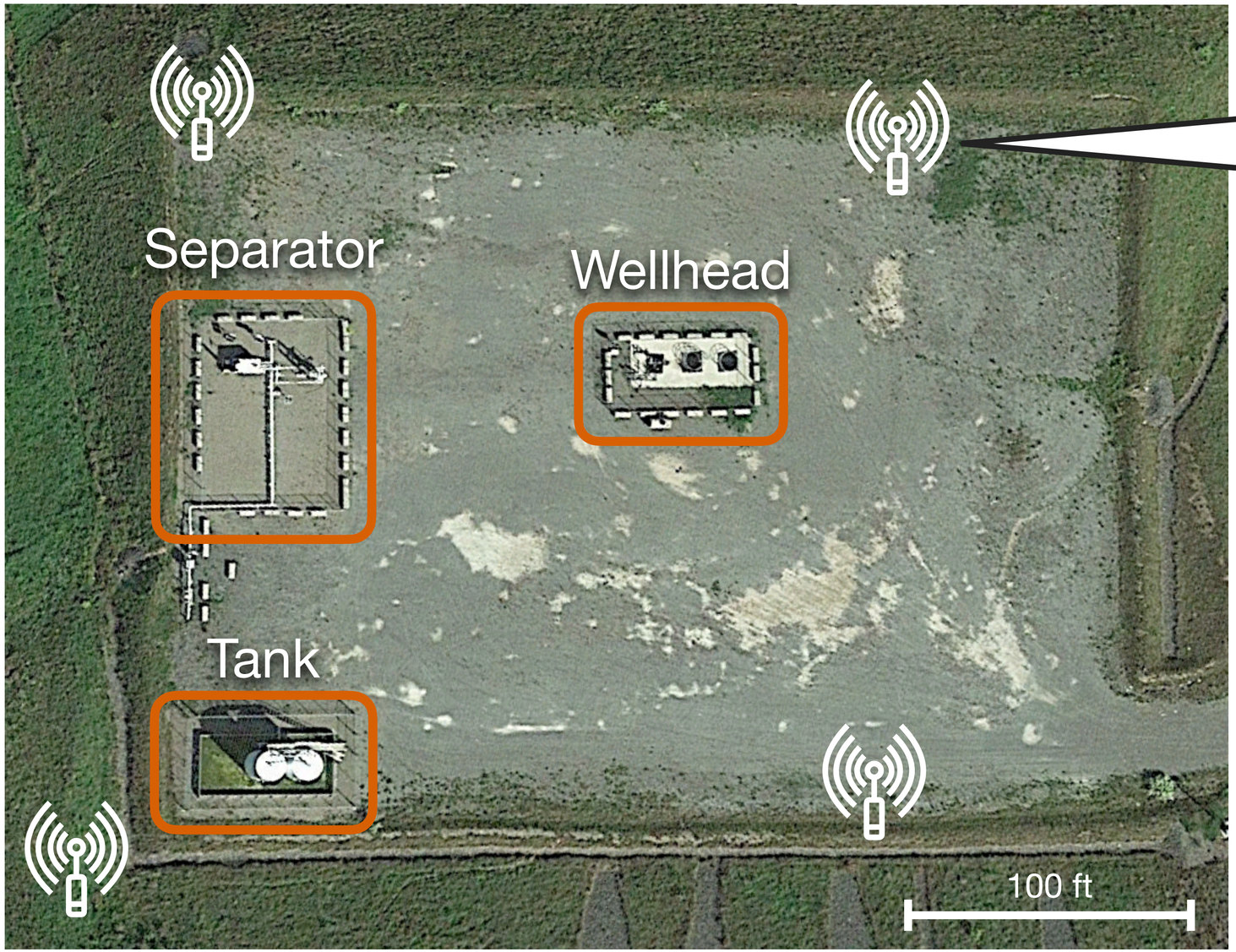


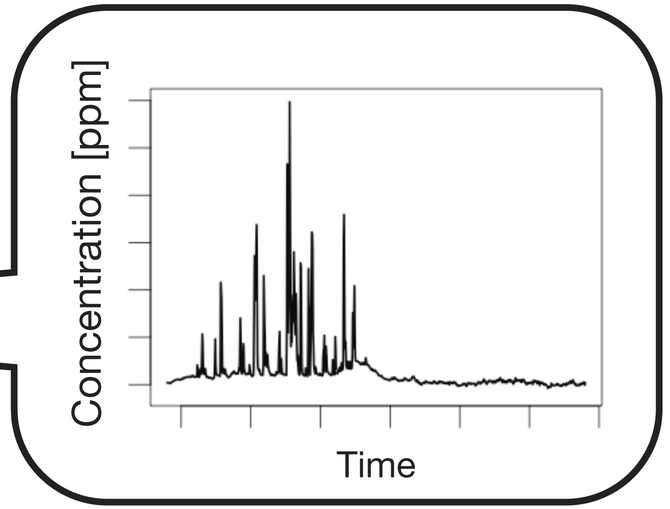
Event detection:
When is an emission happening?

Localization:
Where is the emission coming from?

Quantification:
How much is being emitted?

Example production oil and gas site

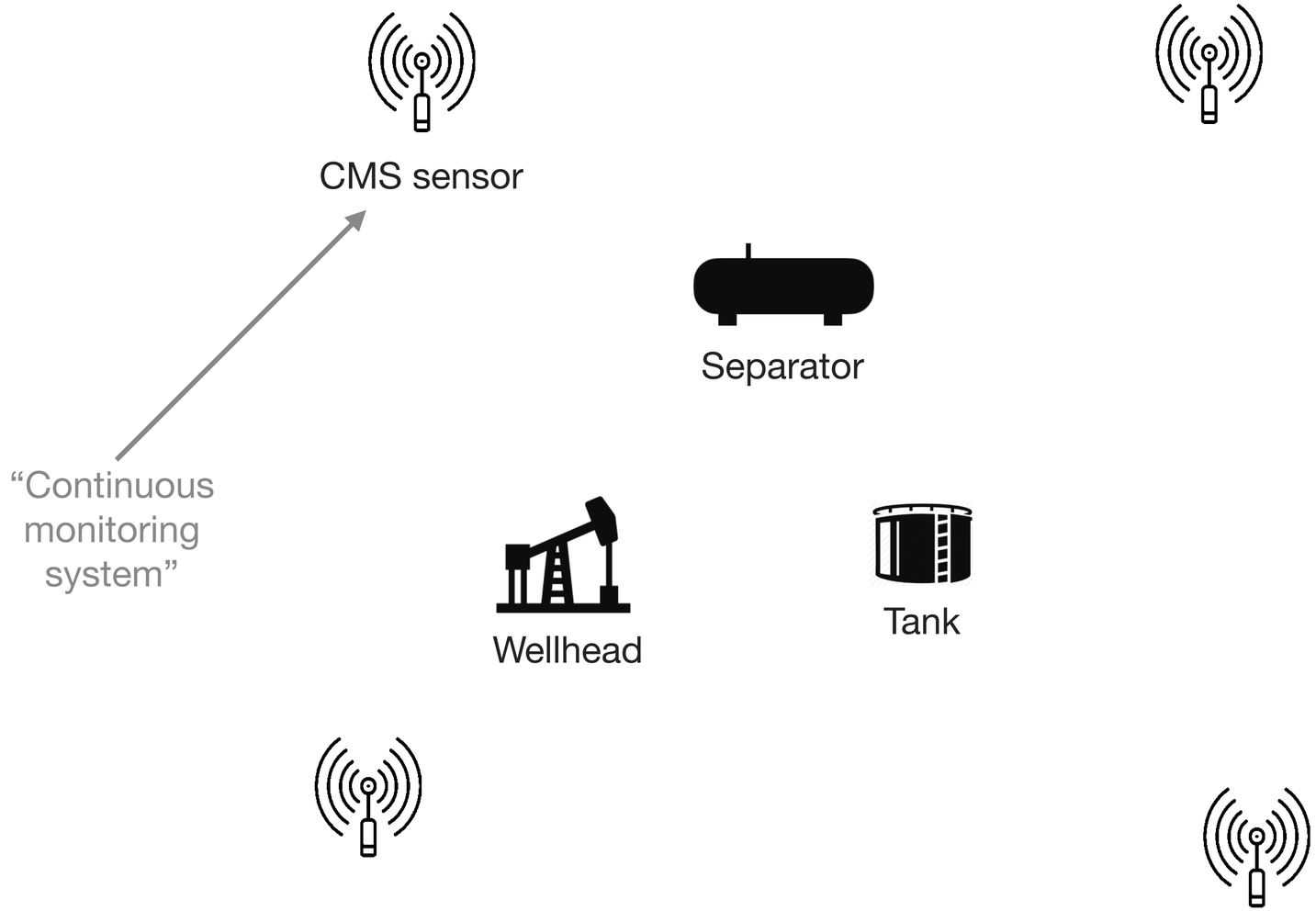




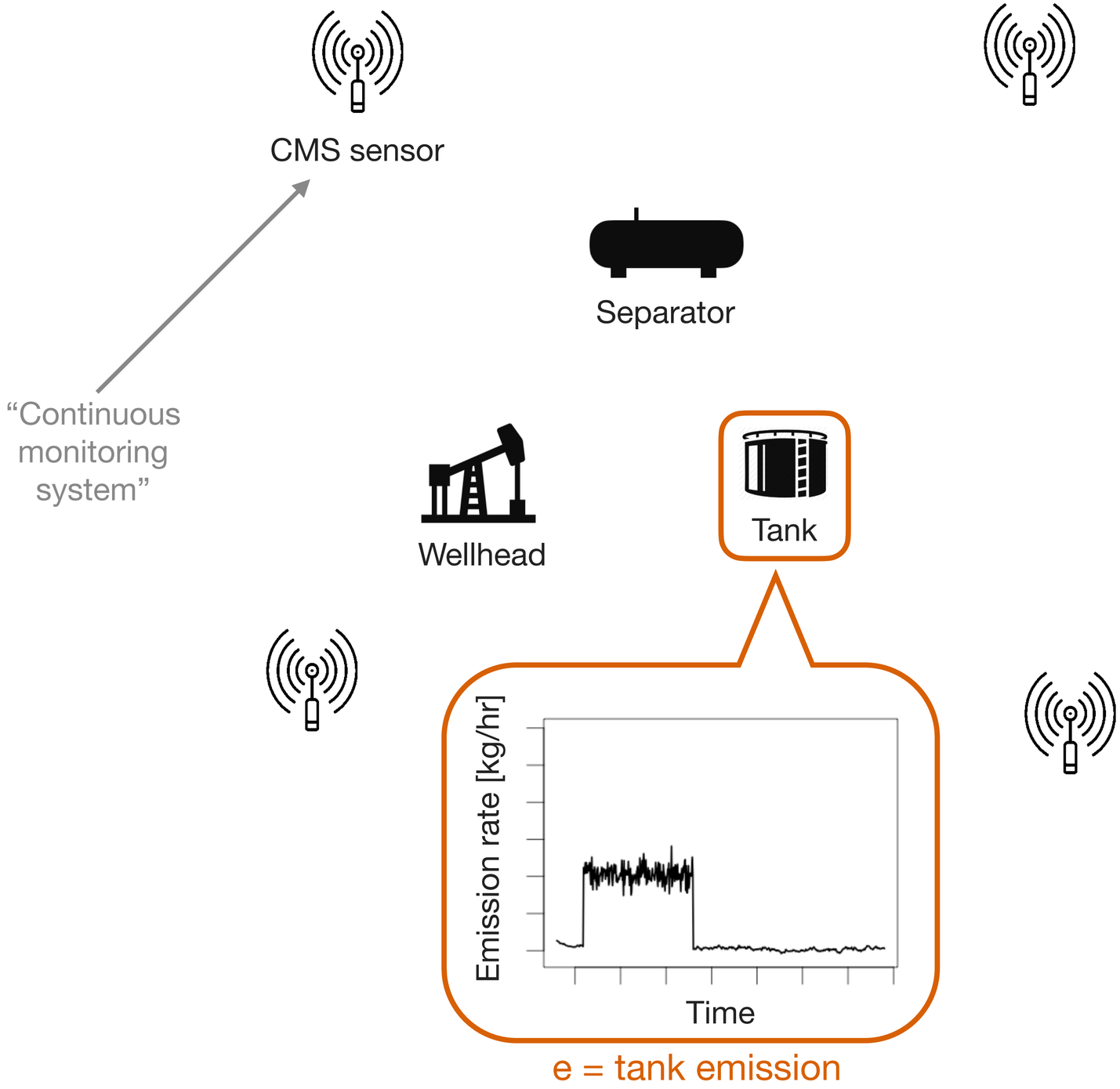
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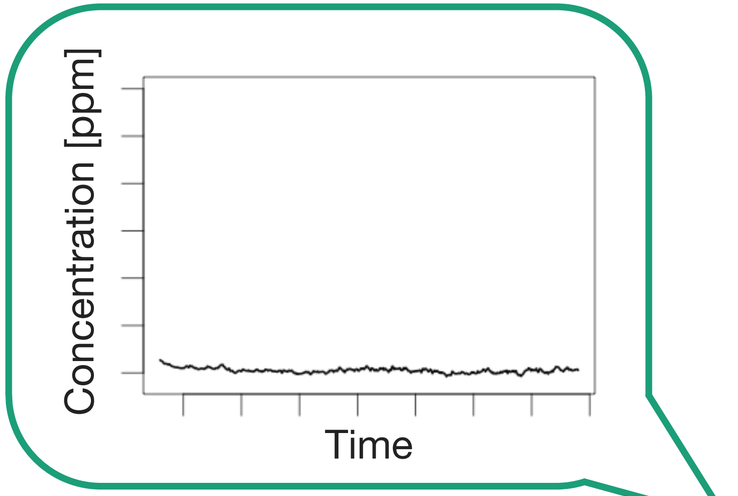
The single-source continuous monitoring inverse problem


CMS sensor
Separator
"Continuous monitoring system"
Wellhead
Tank
Emission rate [kg/hr]
Time
e = tank emission



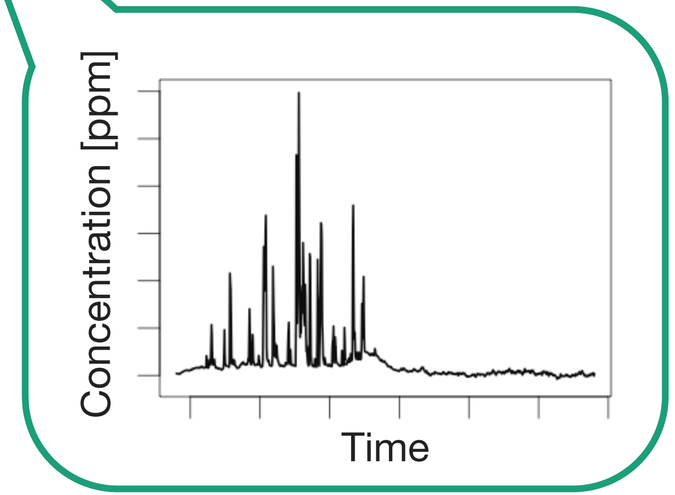
CMS sensor







Separator



d = concentration data

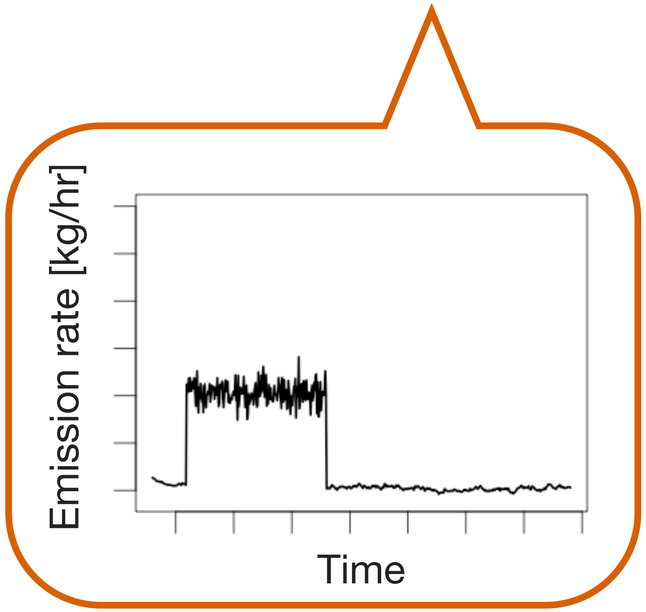


Wellhead



Tank





e = tank emission



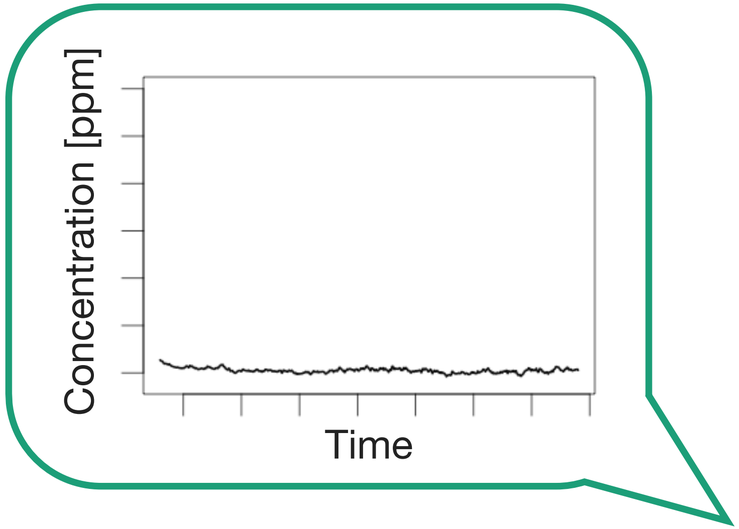
Wind direction



$d = F(e)$



CMS sensor





Separator



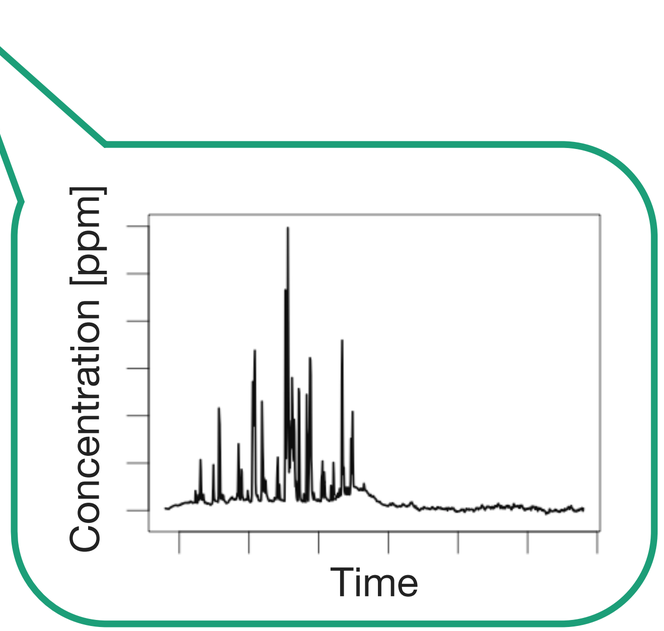
Wellhead



Tank







d = concentration data



Wind direction



$d = F(e)$

$e = F^{-1}(d)$

Open source framework for solving inverse problem

STEP 1:
Background removal
and event detection

→

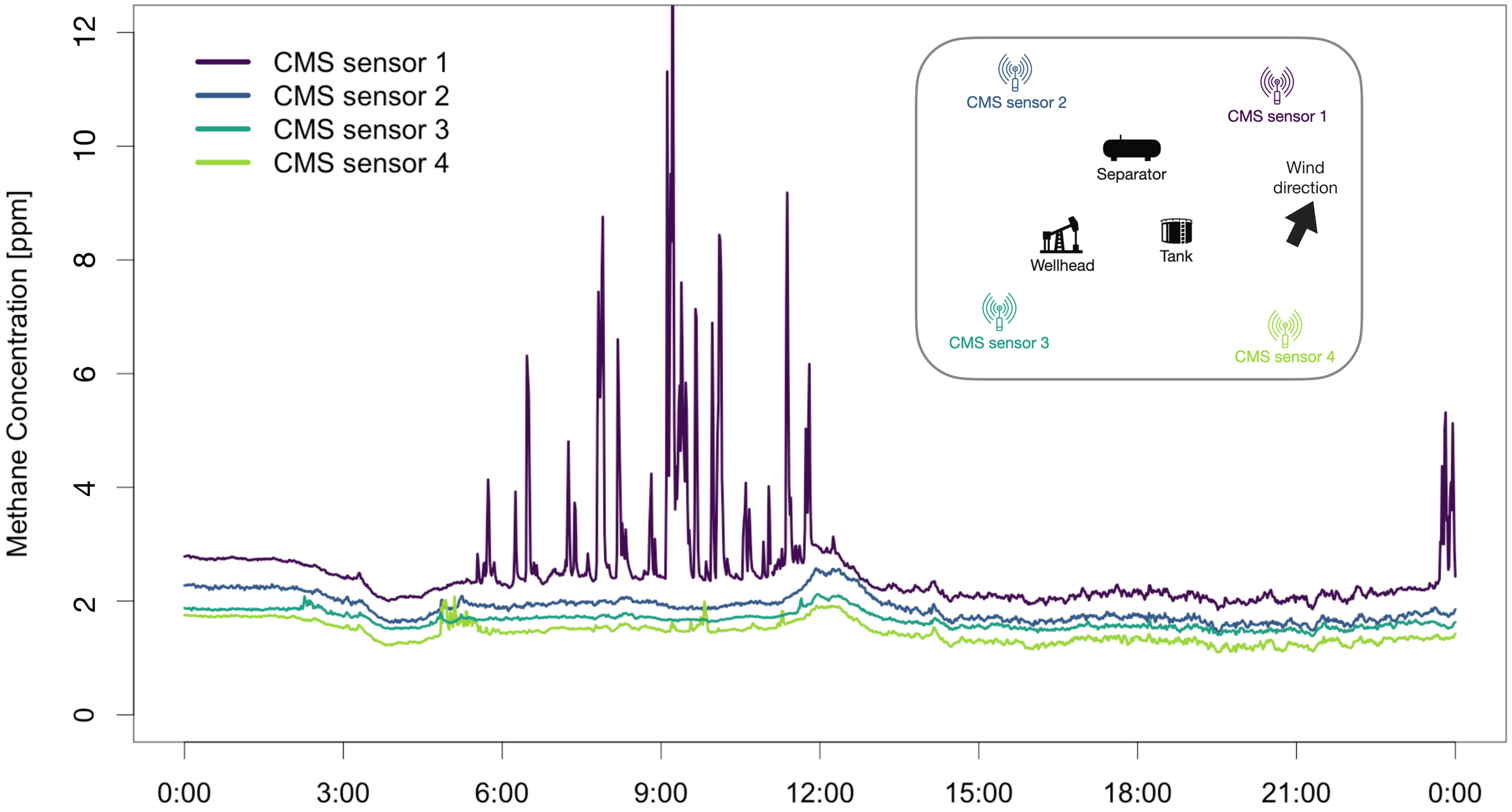
STEP 2:
Simulation

→

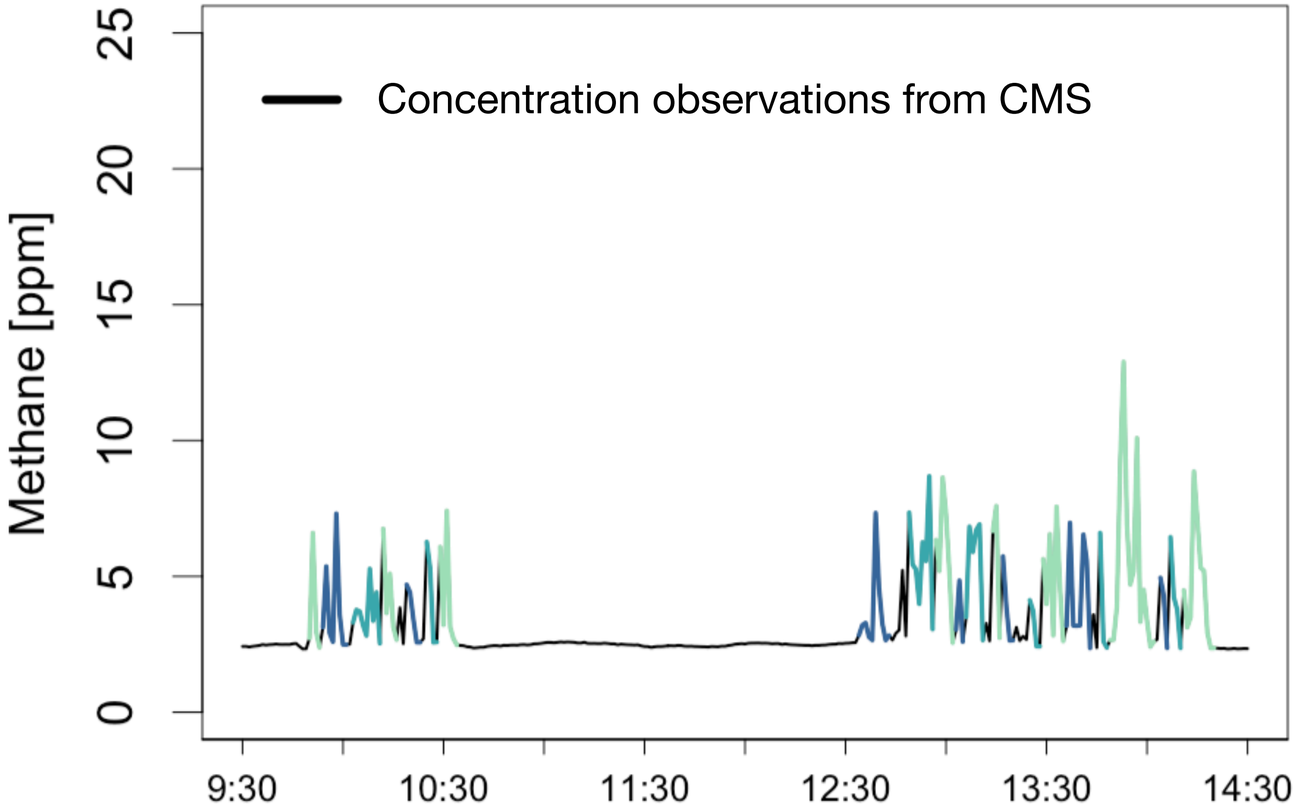
STEP 3:
Localization

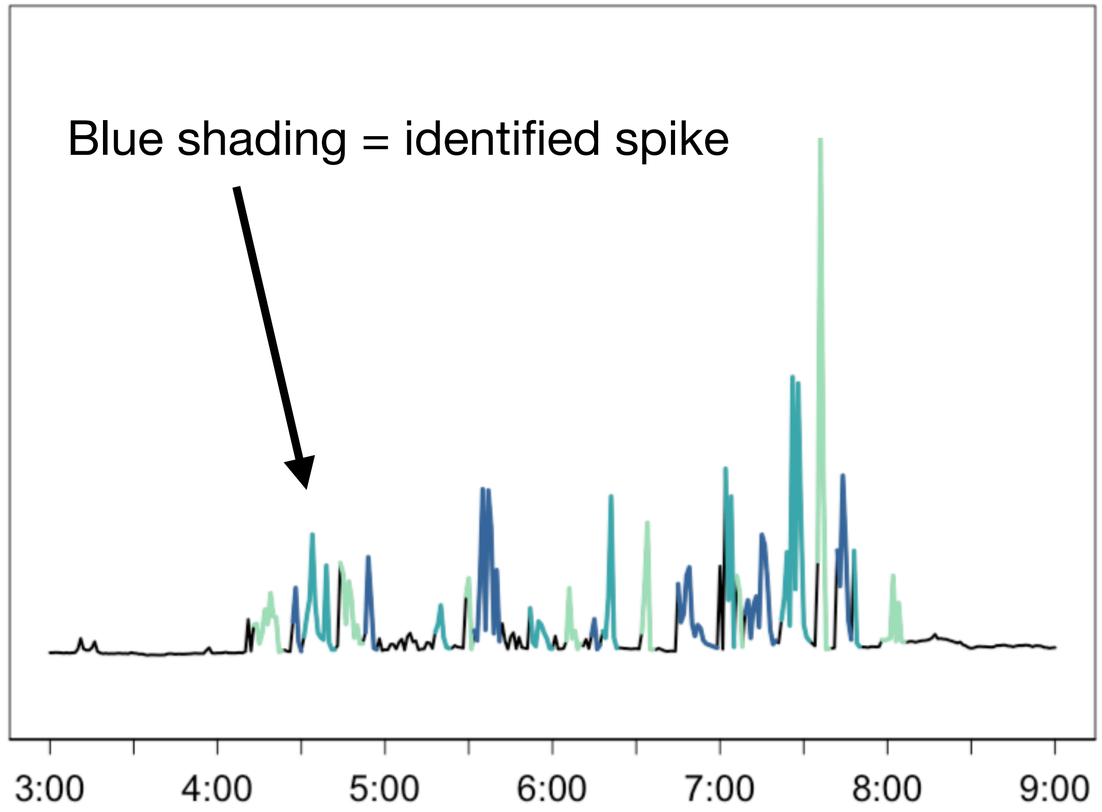
→

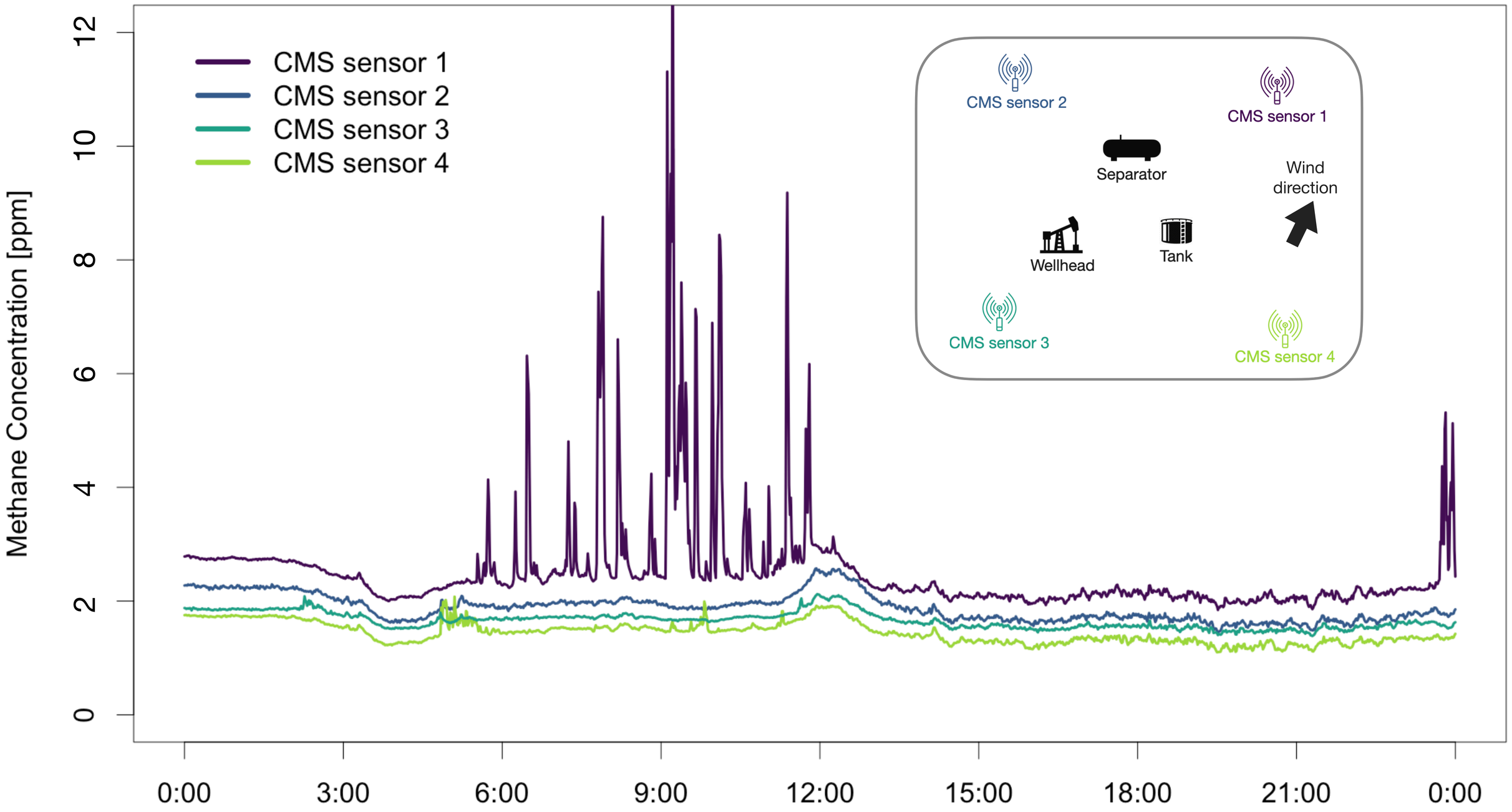
STEP 4:
Quantification

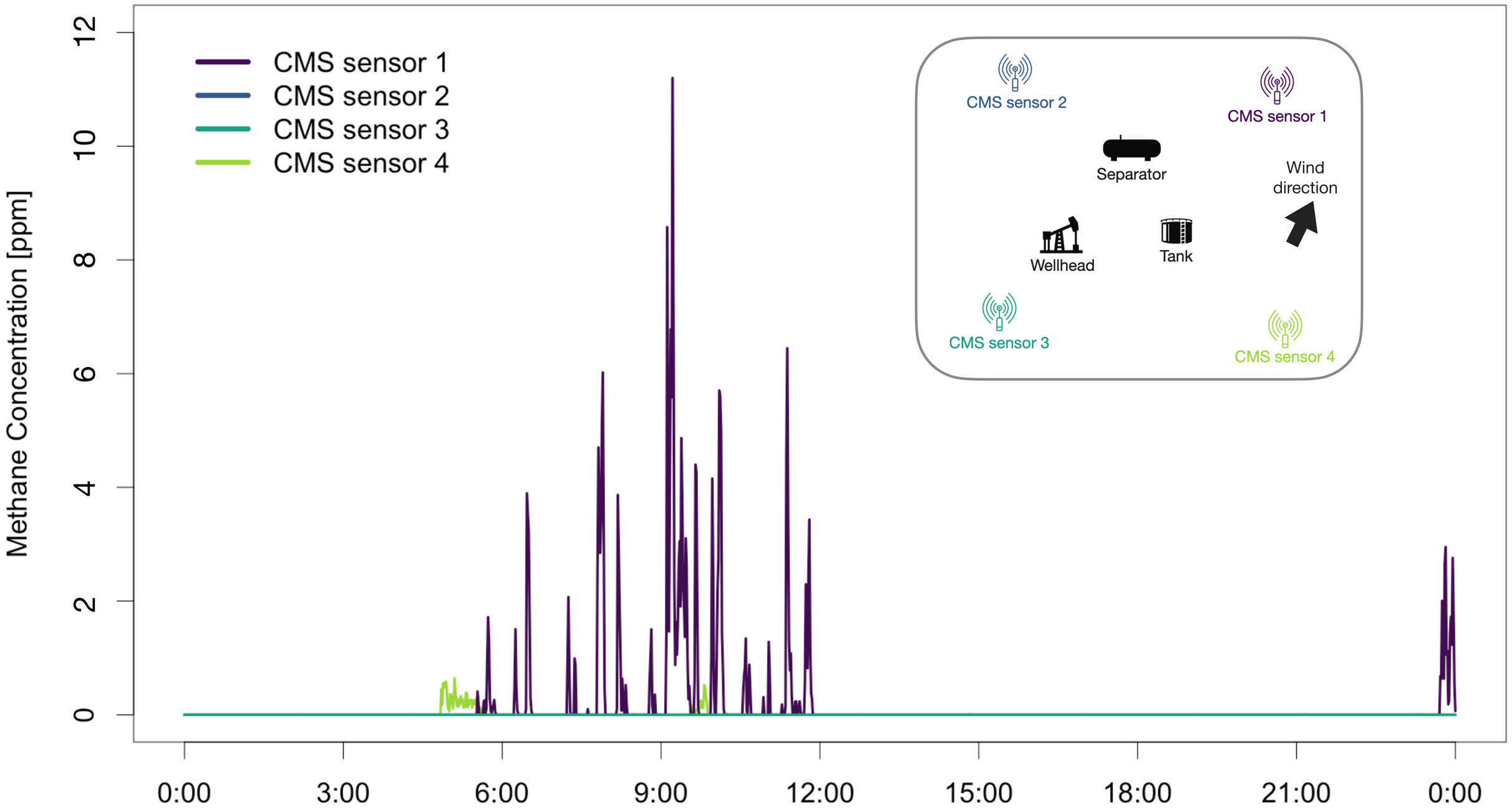

CMS sensor 1
CMS sensor 2
CMS sensor 3
CMS sensor 4
Methane Concentration [ppm]
0
2
4
6
8
10
12
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4

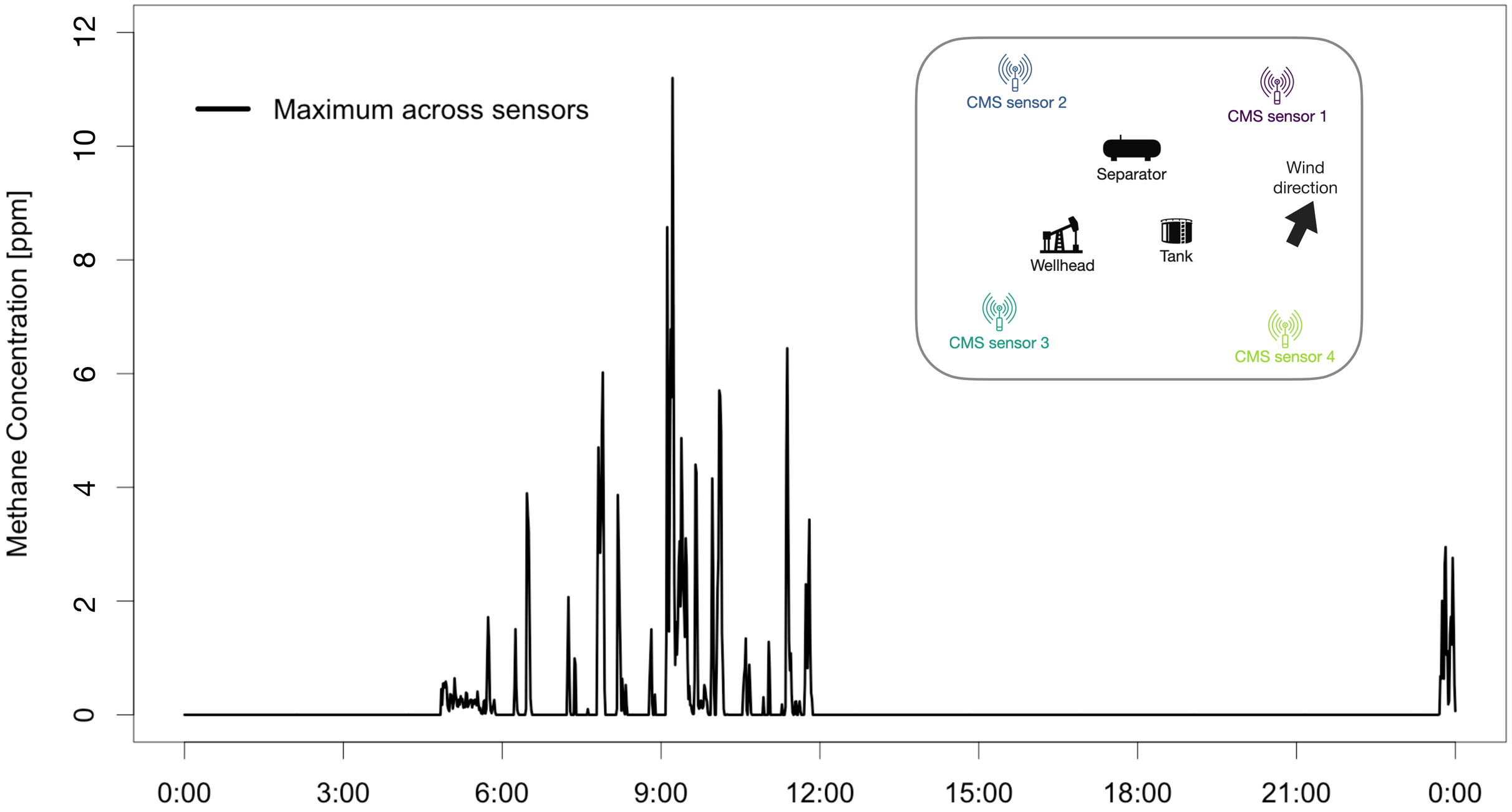
Spike detection algorithm examples

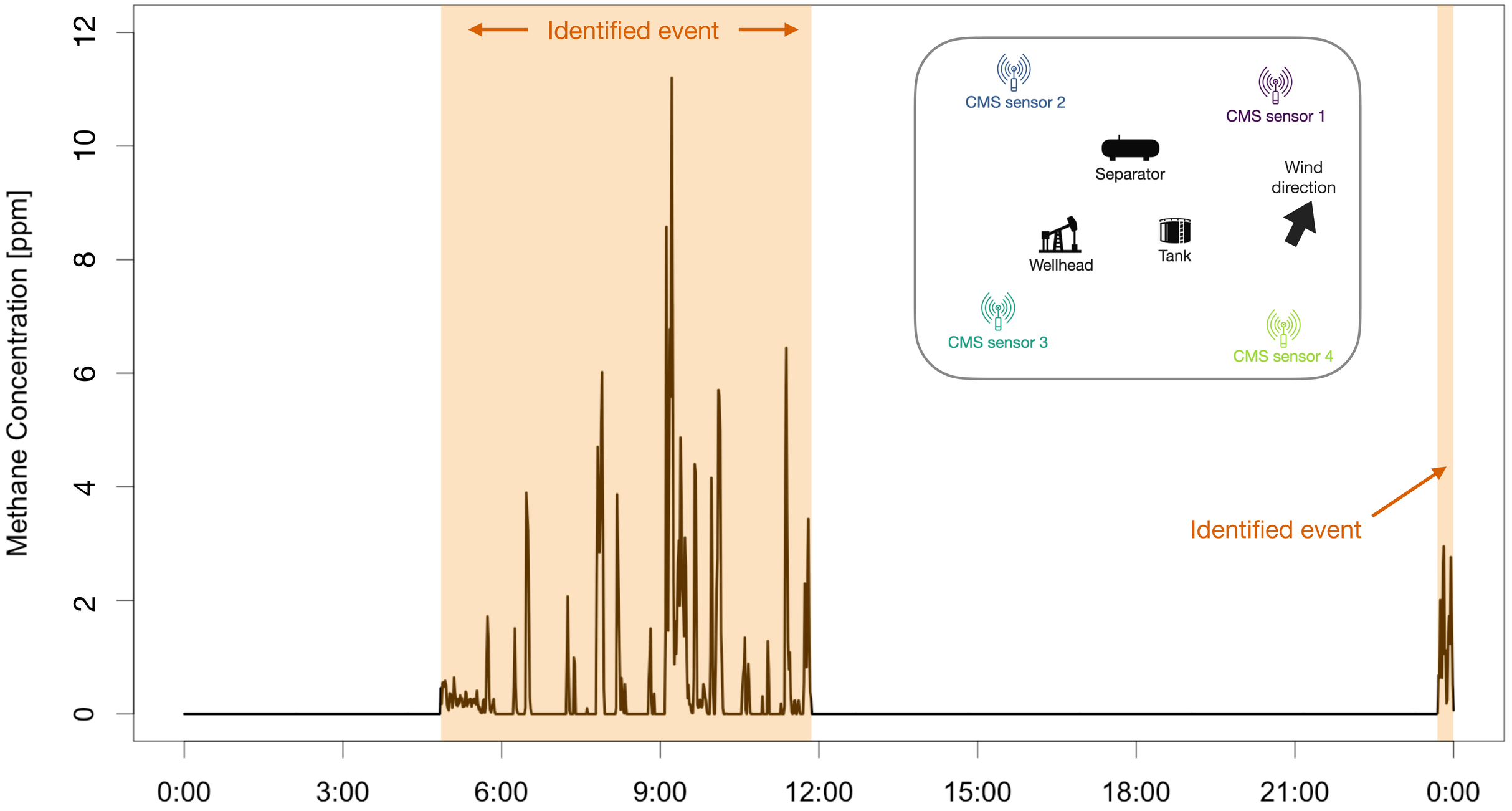


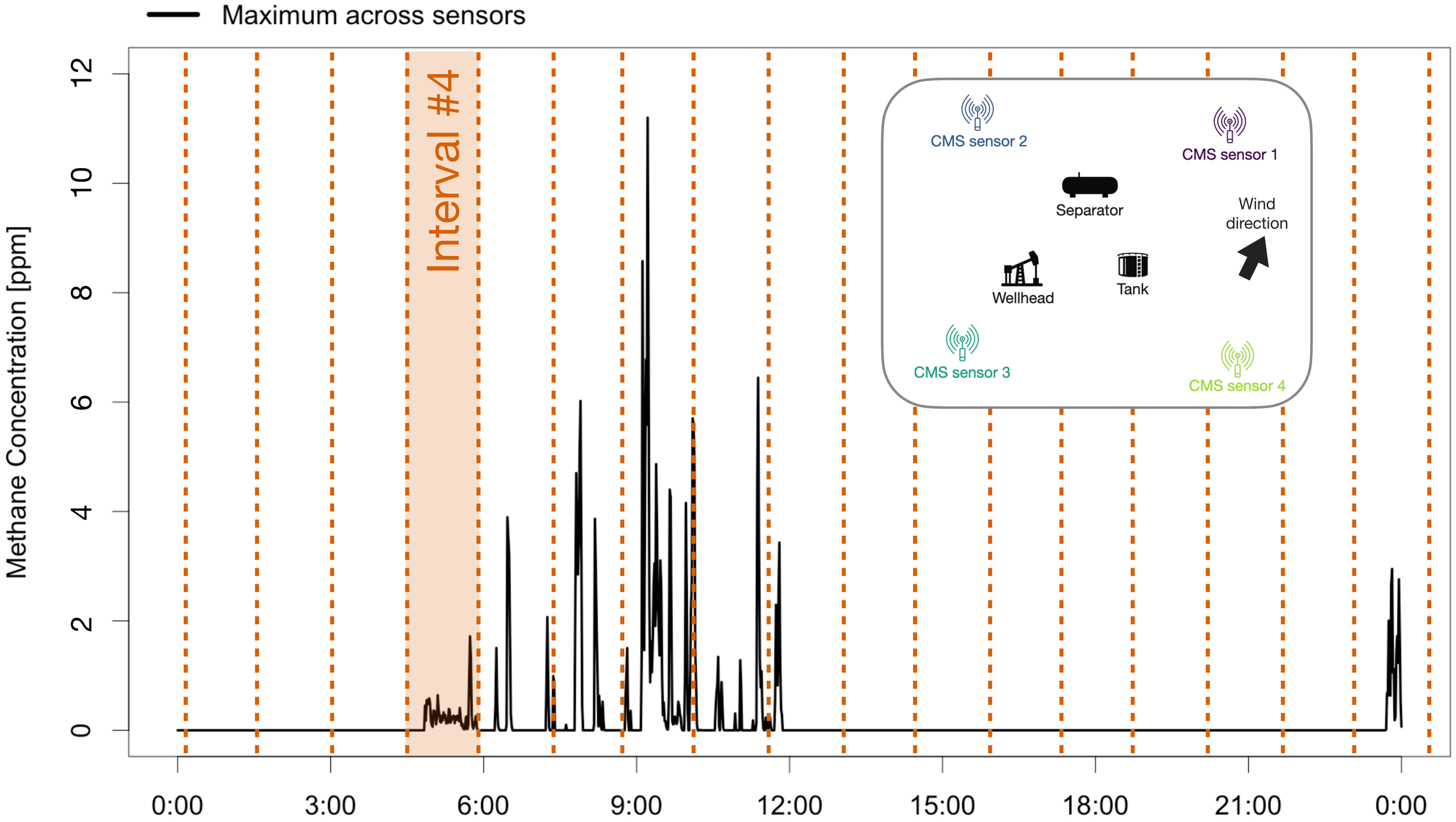



CMS sensor 1
CMS sensor 2
CMS sensor 3
CMS sensor 4
Methane Concentration [ppm]
0
2
4
6
8
10
12
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4


CMS sensor 1
CMS sensor 2
CMS sensor 3
CMS sensor 4
Methane Concentration [ppm]
0
2
4
6
8
10
12
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4

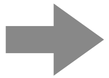

Maximum across sensors
Methane Concentration [ppm]
0
2
4
6
8
10
12
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4


Identified event
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4
Identified event
Methane Concentration [ppm]
0
2
4
6
8
10
12
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00

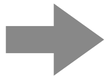

Maximum across sensors
Methane Concentration [ppm]
0
2
4
6
8
10
12
Interval #4
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4

Open source framework for solving inverse problem

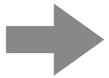
STEP 1:
Background removal
and event detection



STEP 2:
Simulation



STEP 3:
Localization

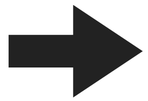


STEP 4:
Quantification

Gaussian plume model:

models the transport of methane by assuming that everything is steady state

Wind direction





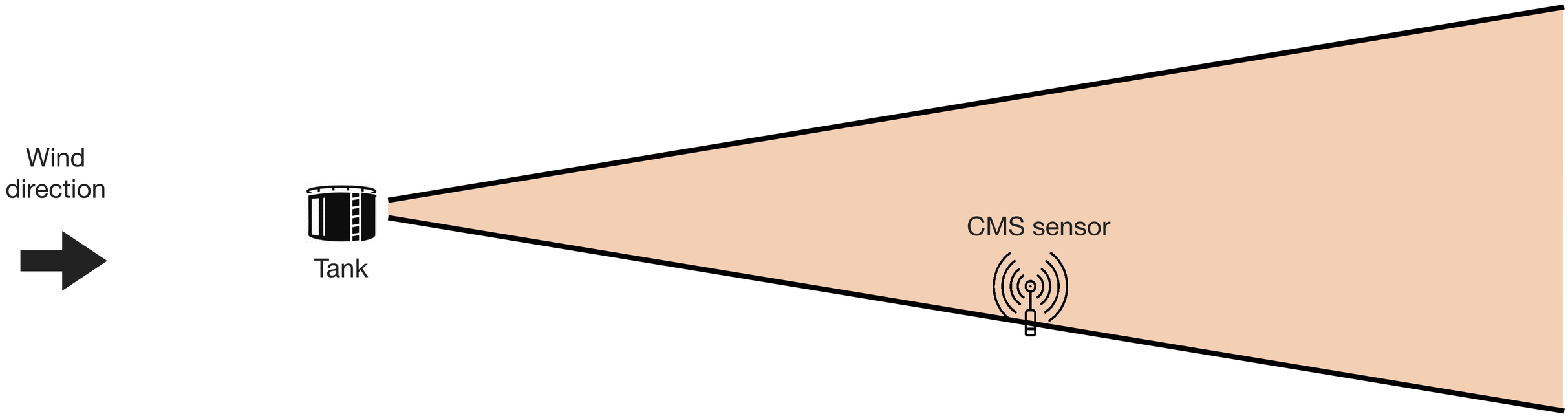
Tank

CMS sensor



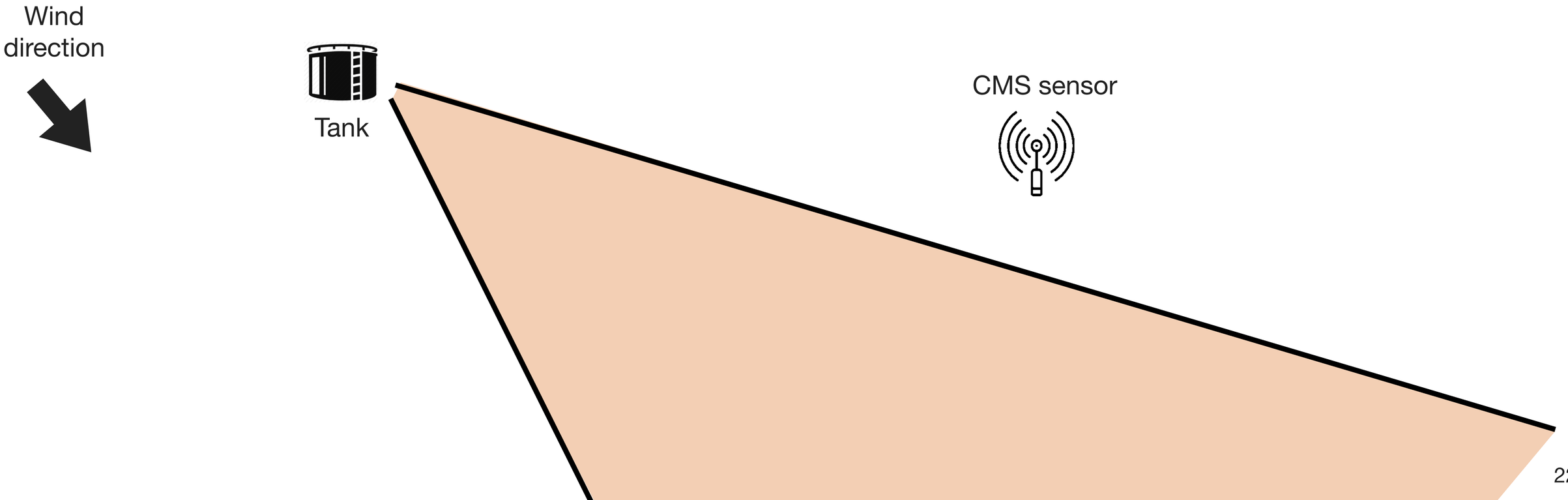
Gaussian plume model:

models the transport of methane by assuming that everything is steady state



Gaussian plume model:

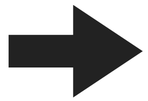
models the transport of methane by assuming that everything is steady state



Gaussian puff model:

approximates a continuous release of methane as a sum of many small “puffs”

Wind direction





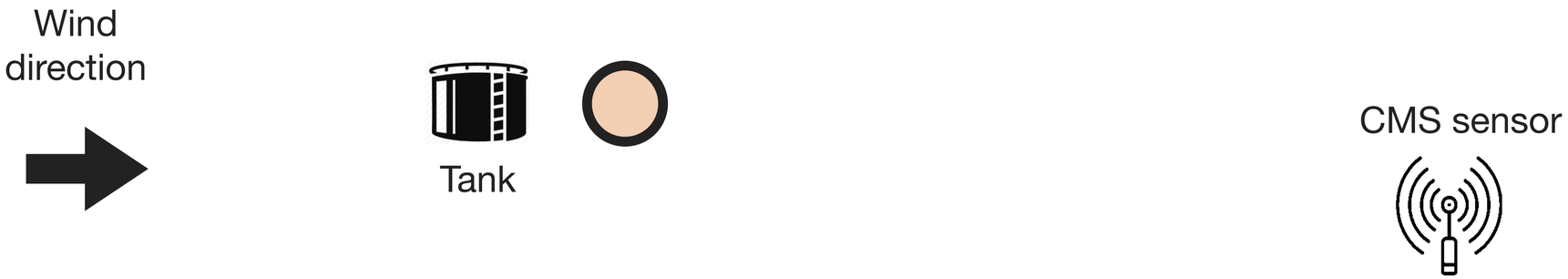
Tank

CMS sensor



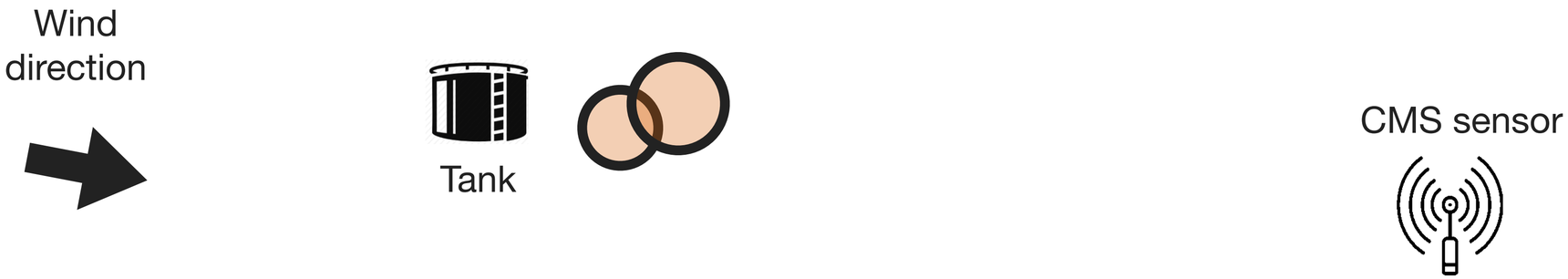
Gaussian puff model:

approximates a continuous release of methane as a sum of many small “puffs”



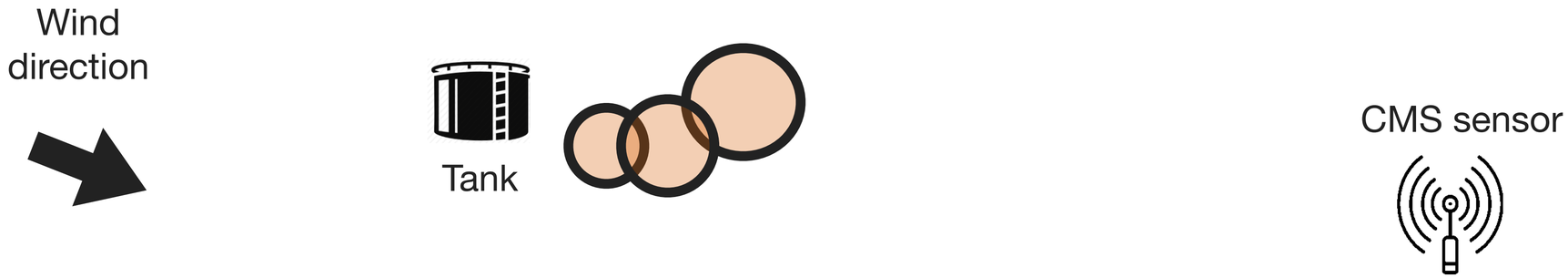
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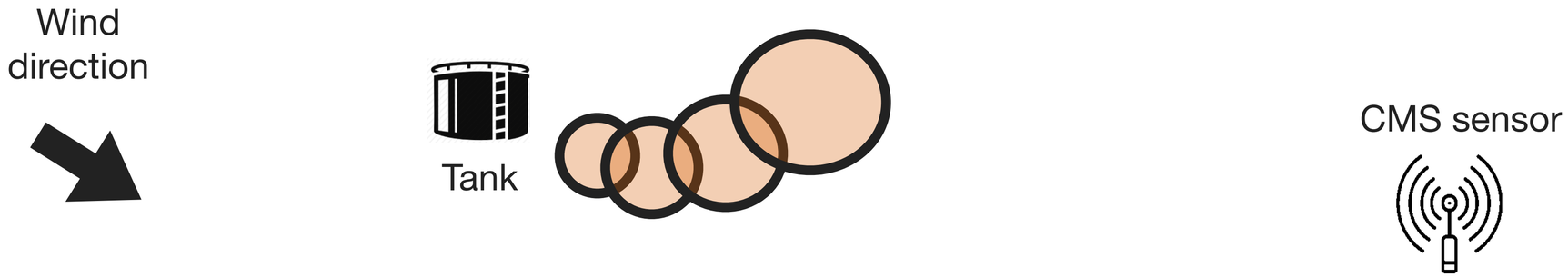
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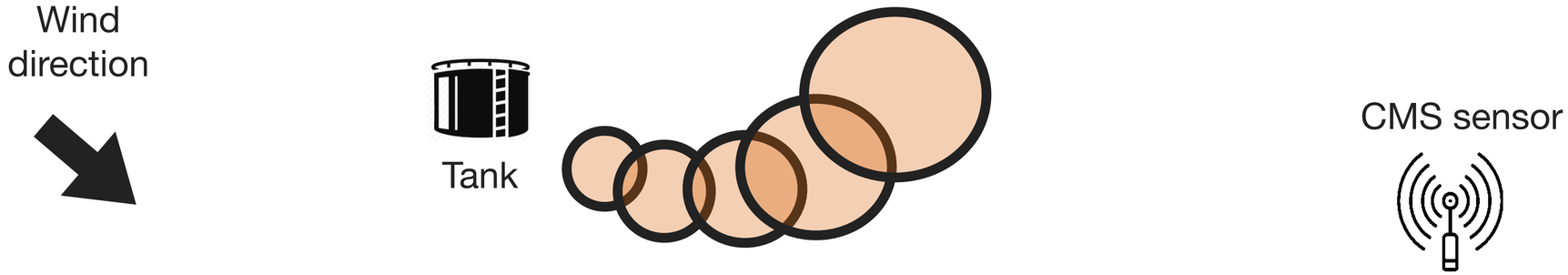
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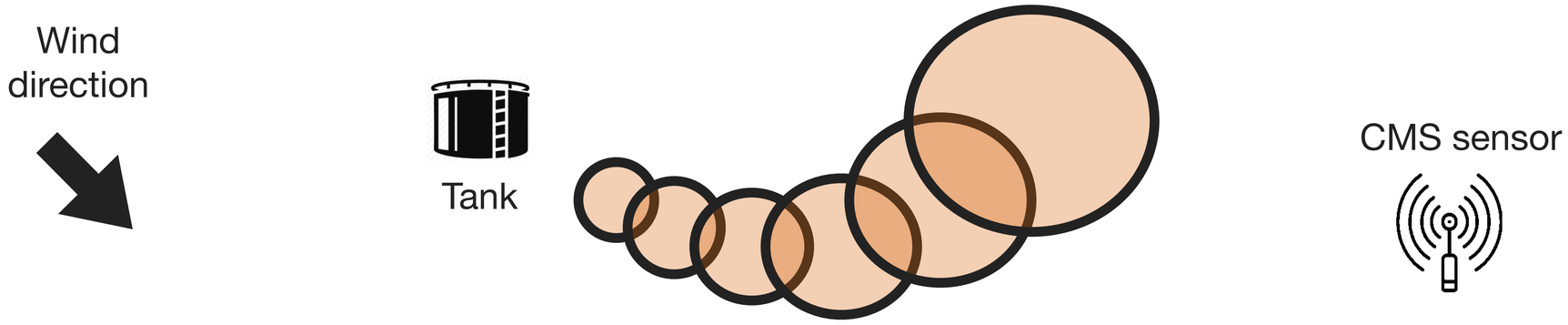
Gaussian puff model:

approximates a continuous release of methane as a sum of many small “puffs”



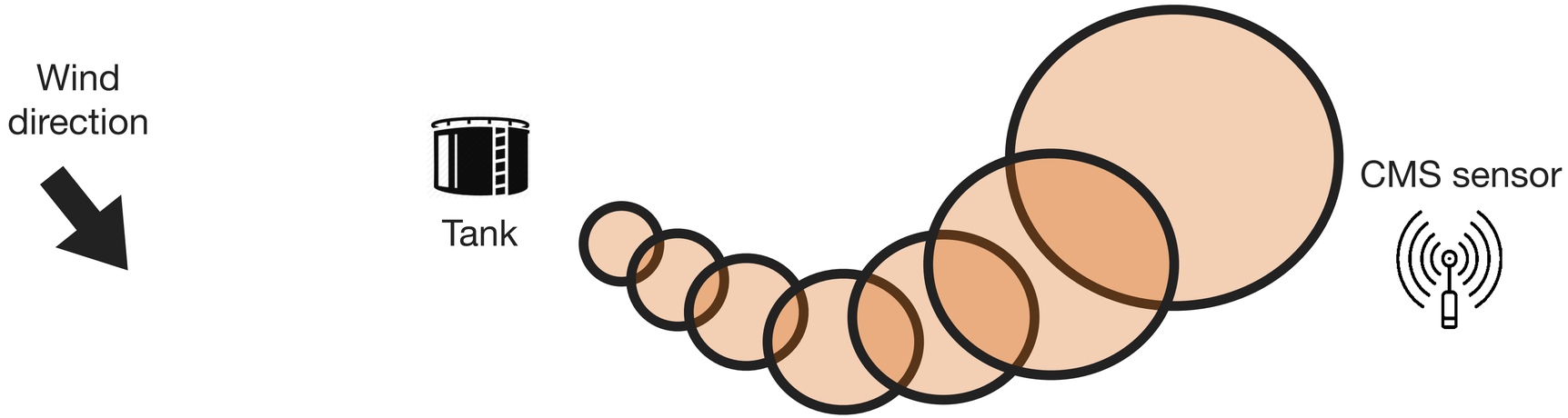
Gaussian puff model:

approximates a continuous release of methane as a sum of many small “puffs”



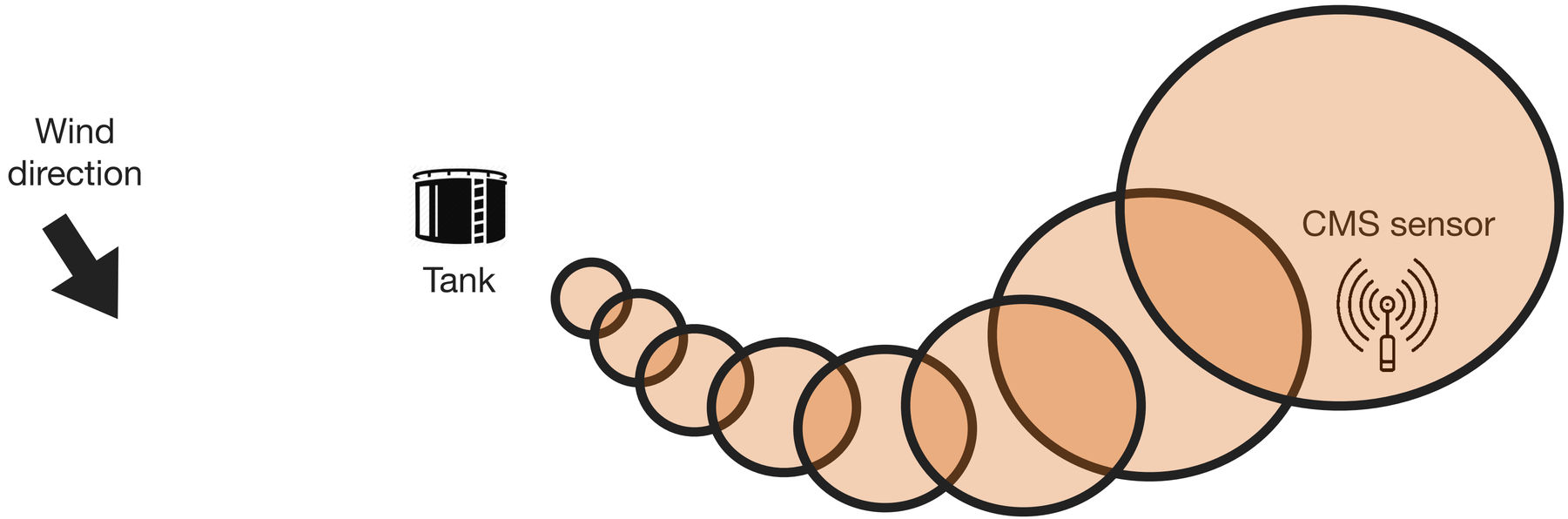
Gaussian puff model:

approximates a continuous release of methane as a sum of many small “puffs”

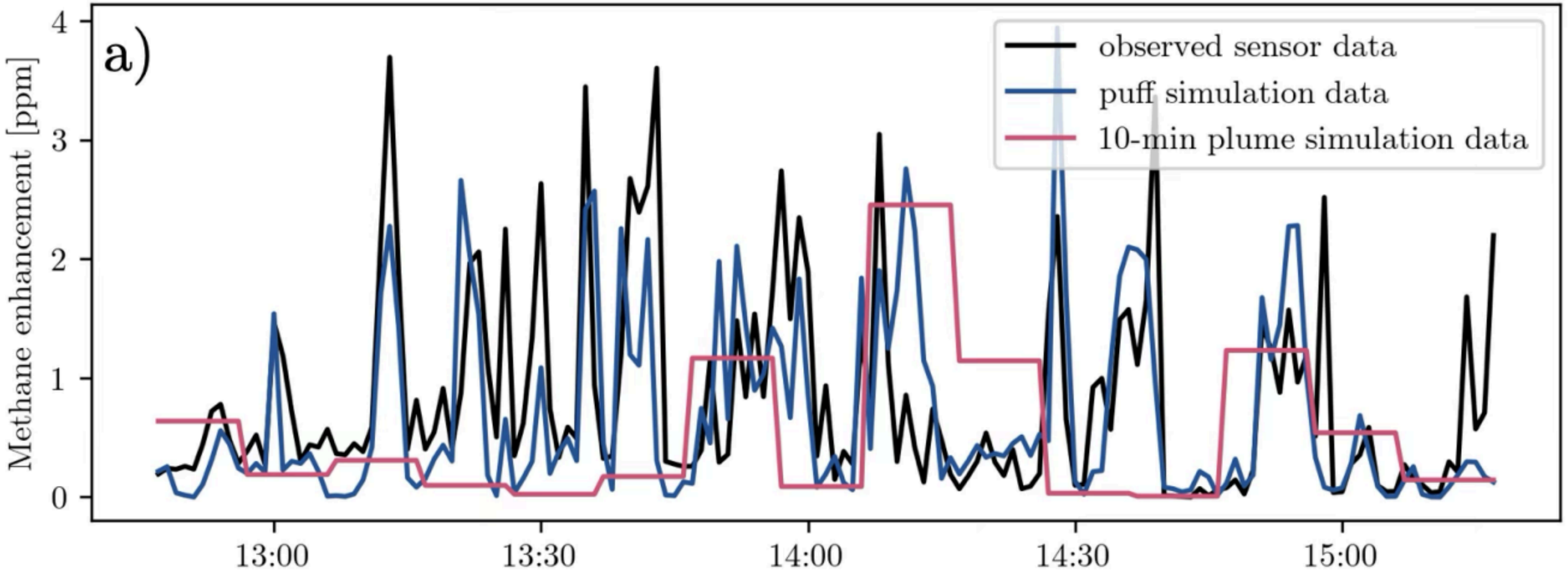


Gaussian puff model:

approximates a continuous release of methane as a sum of many small “puffs”



The Gaussian puff can leverage high frequency wind data, while the Gaussian plume requires a temporal average.



Gaussian puff model: mathematical definition

Set up coordinate system so that source is at (0,0,H) and positive x-axis aligns with downwind vector

Total volume of methane contained in puff p

$$c_p(x, y, z, t, Q) = \frac{Q}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Predicted methane concentration at sensor location (x,y,z) and time t from puff p

Exponential decay in concentration in horizontal plane (x, y)

Exponential decay in concentration in vertical dimension (z)

Gaussian puff model: mathematical definition

Set up coordinate system so that source is at (0,0,H) and positive x-axis aligns with downwind vector

Total concentration at (x, y, z, t)

$$c(x, y, z, t, Q) = \sum_{p=1}^{P} c_p(x, y, z, t, Q)$$

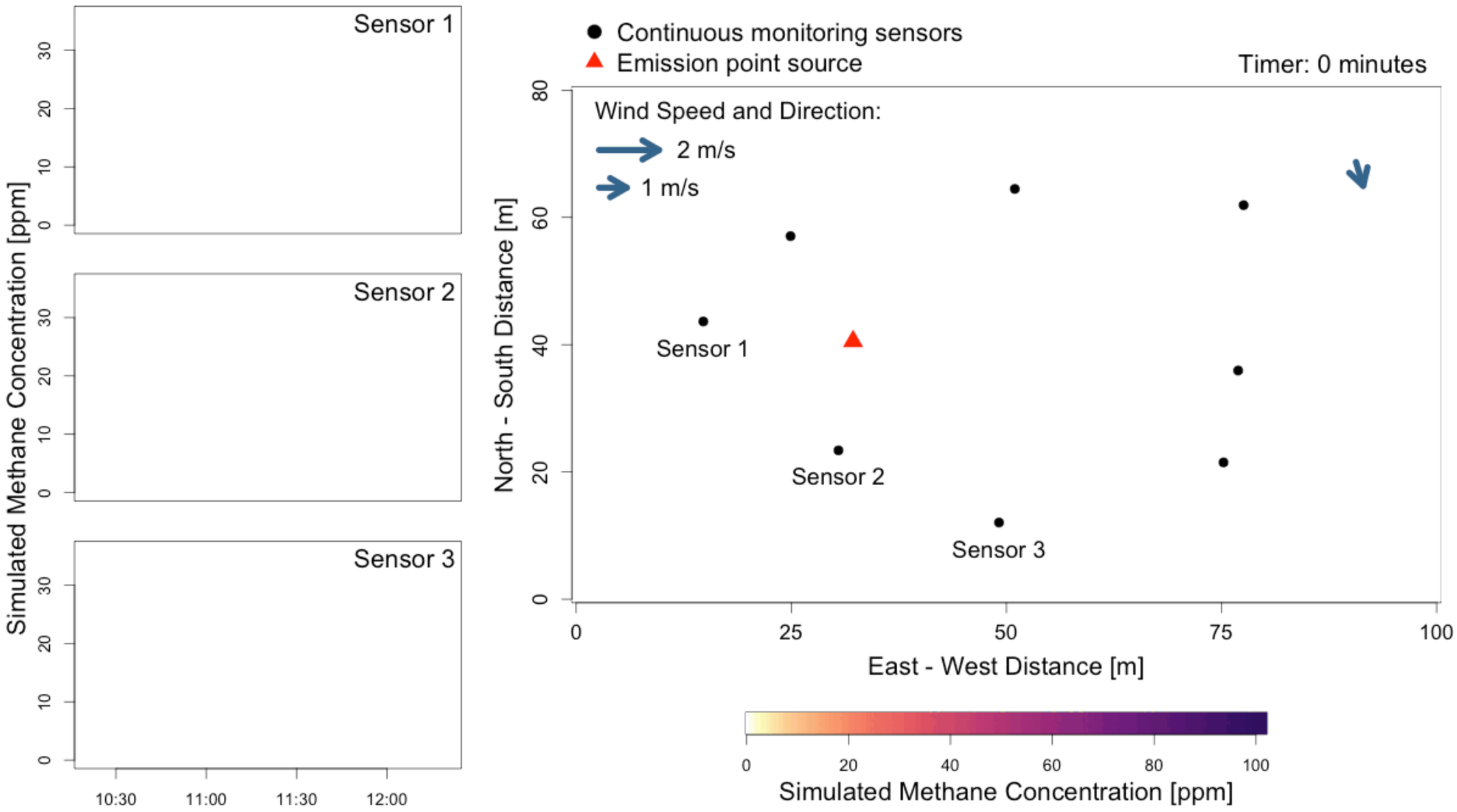
Total volume of methane contained in puff p

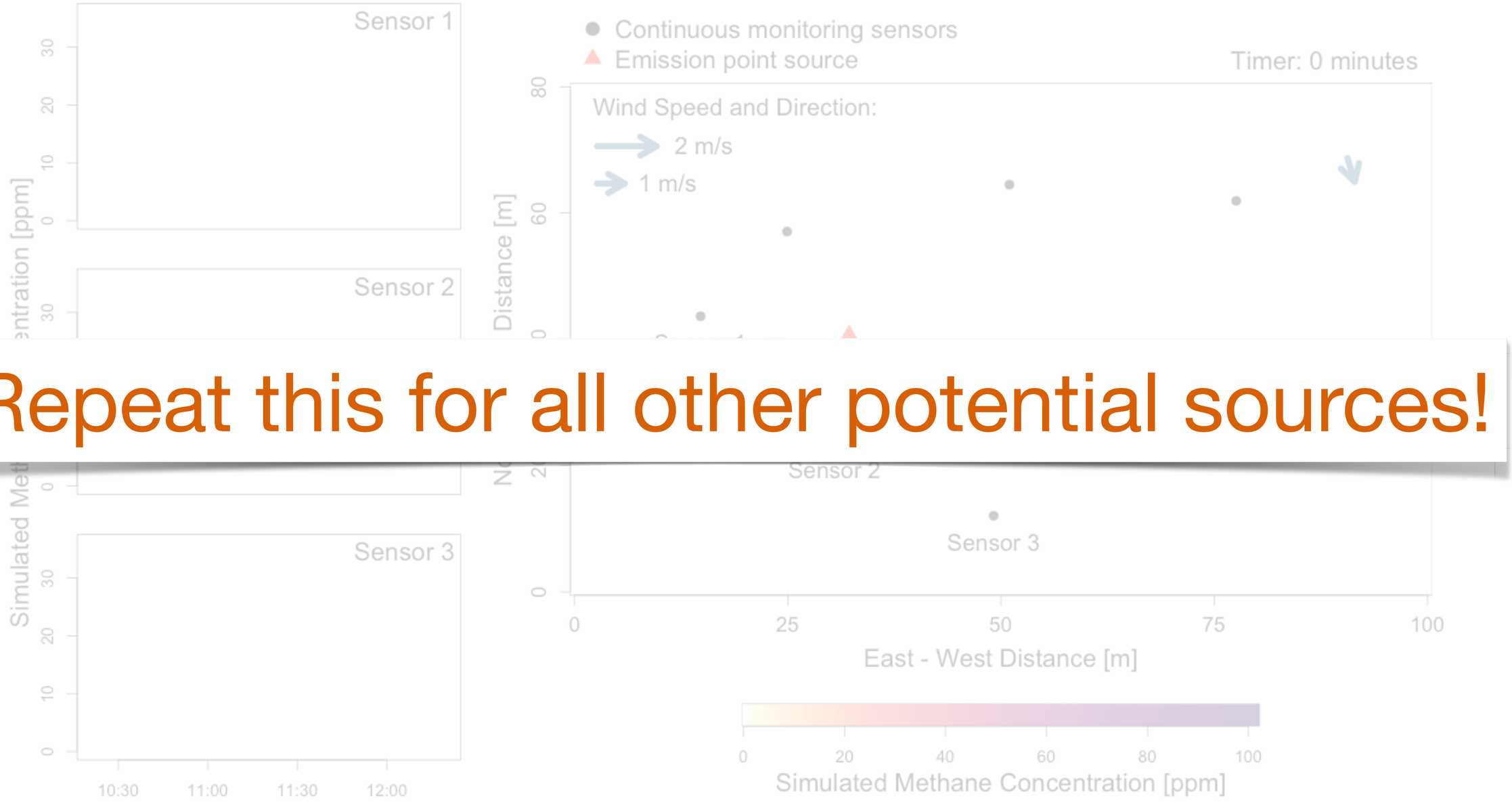
$$c_p(x, y, z, t, Q) = \frac{Q}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Predicted methane concentration at sensor location (x,y,z) and time t from puff p

Exponential decay in concentration in horizontal plane (x, y)

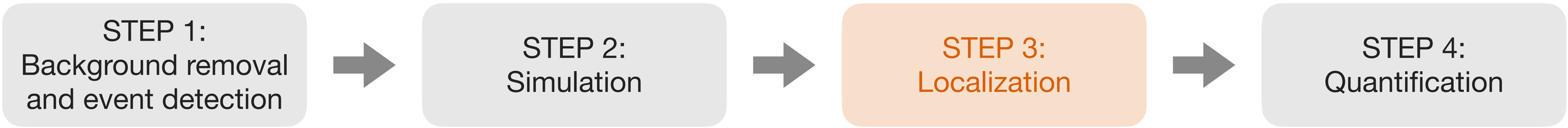
Exponential decay in concentration in vertical dimension (z)


Sensor 1
Sensor 2
Sensor 3
Simulated Methane Concentration [ppm]
0
10
20
30
10:30
11:00
11:30
12:00
Continuous monitoring sensors
Emission point source
Timer: 0 minutes
Wind Speed and Direction:
2 m/s
1 m/s
North - South Distance [m]
0
20
40
60
80
0
25
50
75
100
East - West Distance [m]
Sensor 1
Sensor 2
Sensor 3
0
20
40
60
80
100
Simulated Methane Concentration [ppm]



Repeat this for all other potential sources!

Open source framework for solving inverse problem





CMS sensor



Separator



Wellhead



Tank

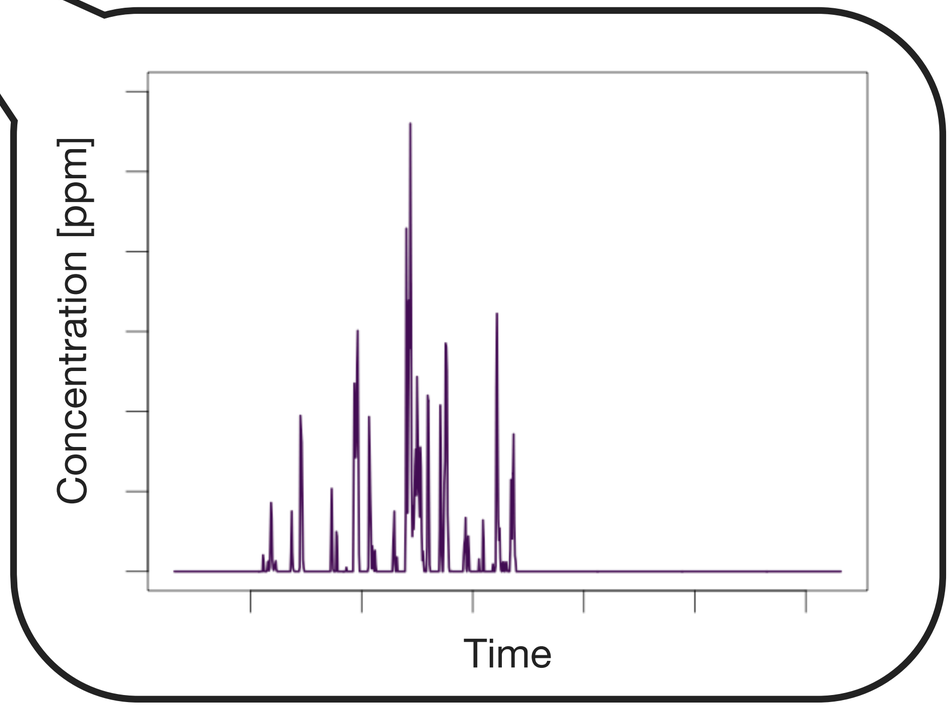




Wind direction







Background-removed observations

CMS sensor

Simulation emission source

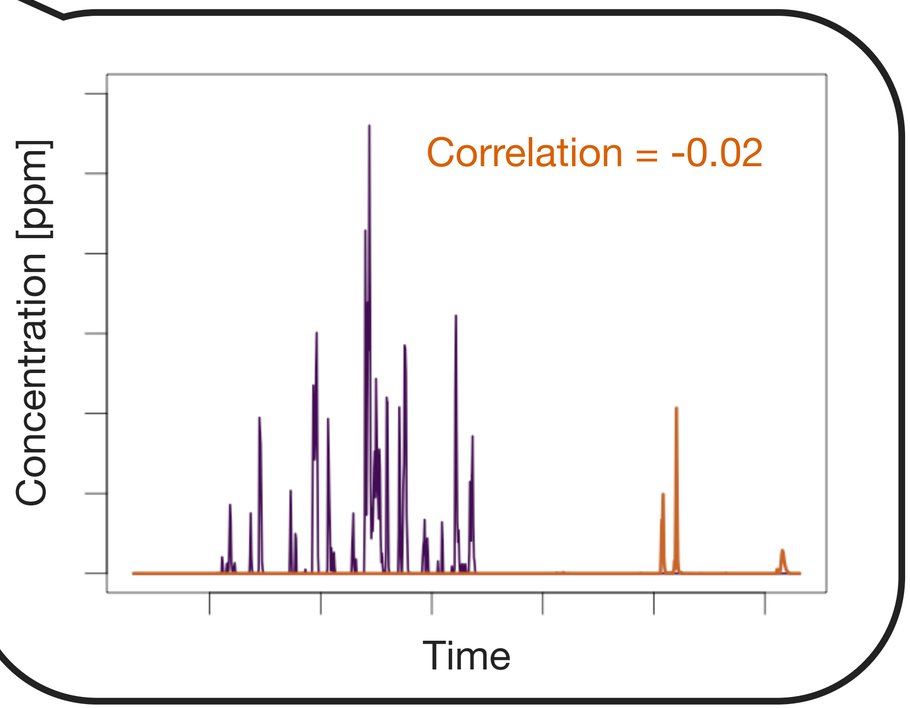
Separator

Wellhead

Tank

Wind direction

Pick source estimate using correlation coefficient



Background-removed observations

Simulated concentrations

CMS sensor

Separator

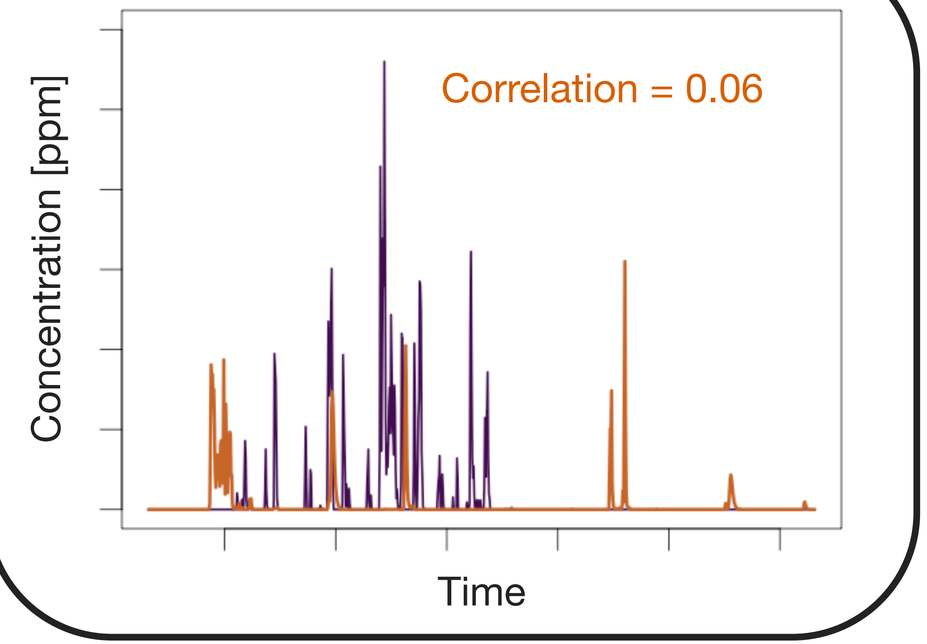
Wellhead

Simulation emission source

Tank

Wind direction

Pick source estimate using correlation coefficient



Background-removed observations

Simulated concentrations

CMS sensor

Separator

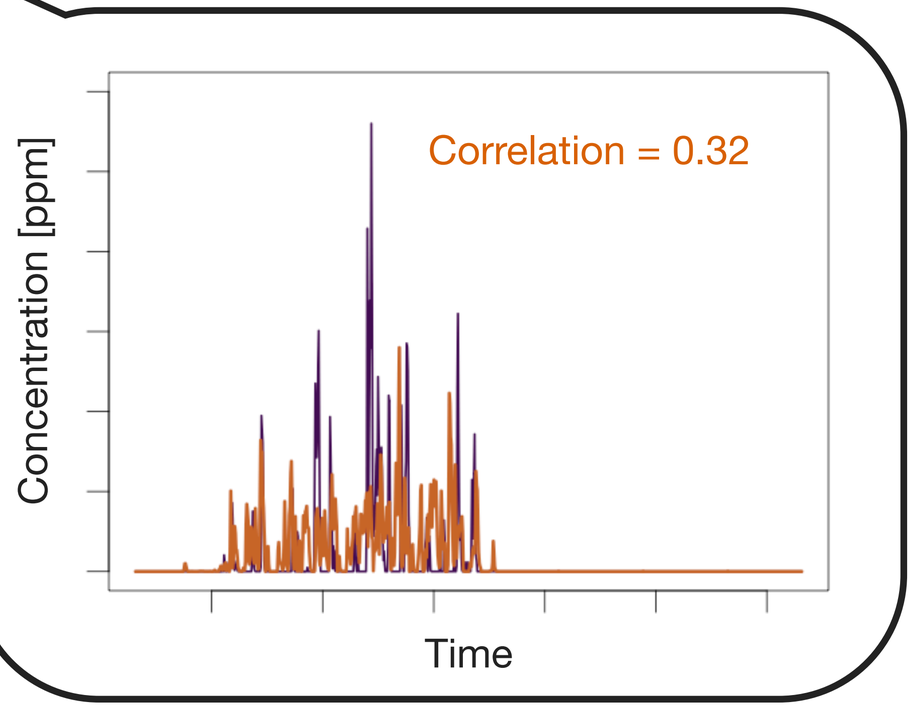
Wellhead

Tank

Simulation emission source

Wind direction

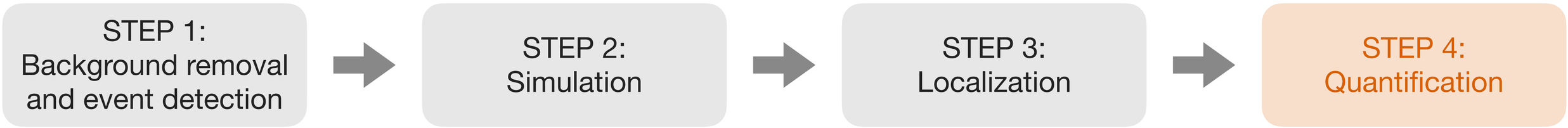
Pick source estimate using correlation coefficient



Background-removed observations

Simulated concentrations

Open source framework for solving inverse problem



Simulation is a linear function of emission rate

Volume of methane contained in puff p

$$c_p(x, y, z, t, Q) = Q \frac{1}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Concentration contribution of puff p

$$c(x, y, z, t, Q) = \sum_{p=1}^{P} c_p(x, y, z, t, Q)$$

Total concentration at (x, y, z, t)

Simulation is a linear function of emission rate

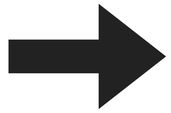
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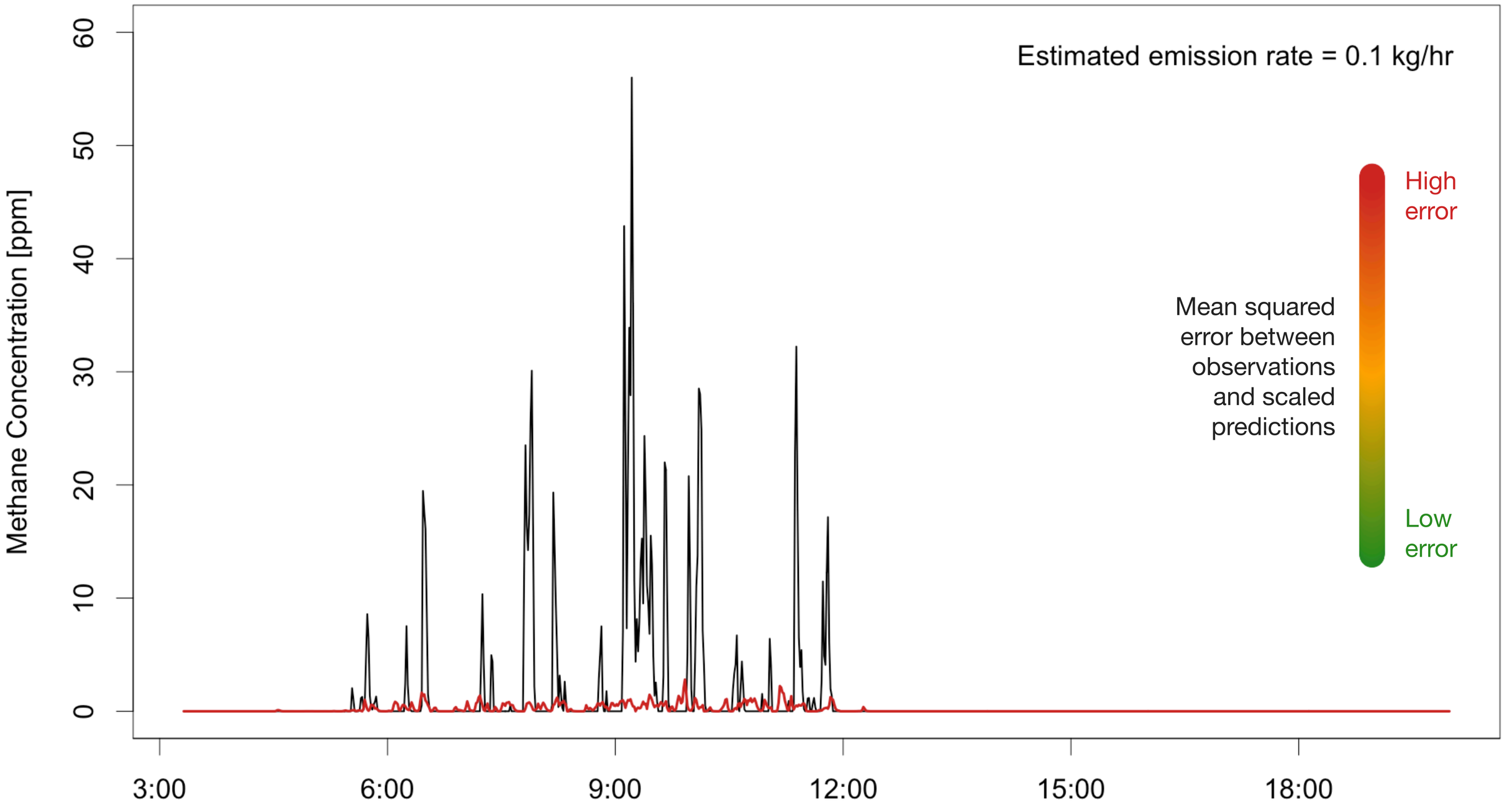


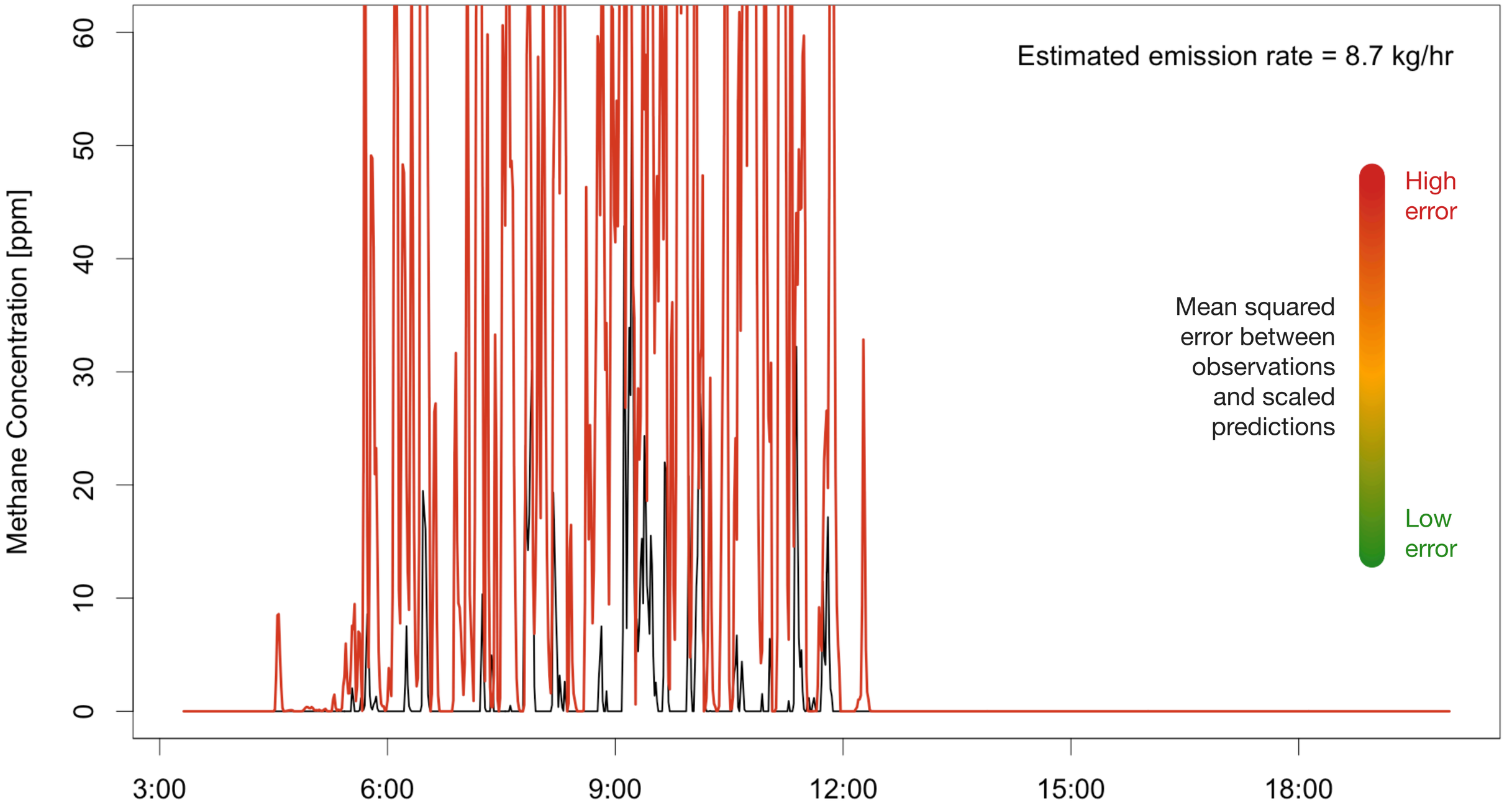
Concentration data

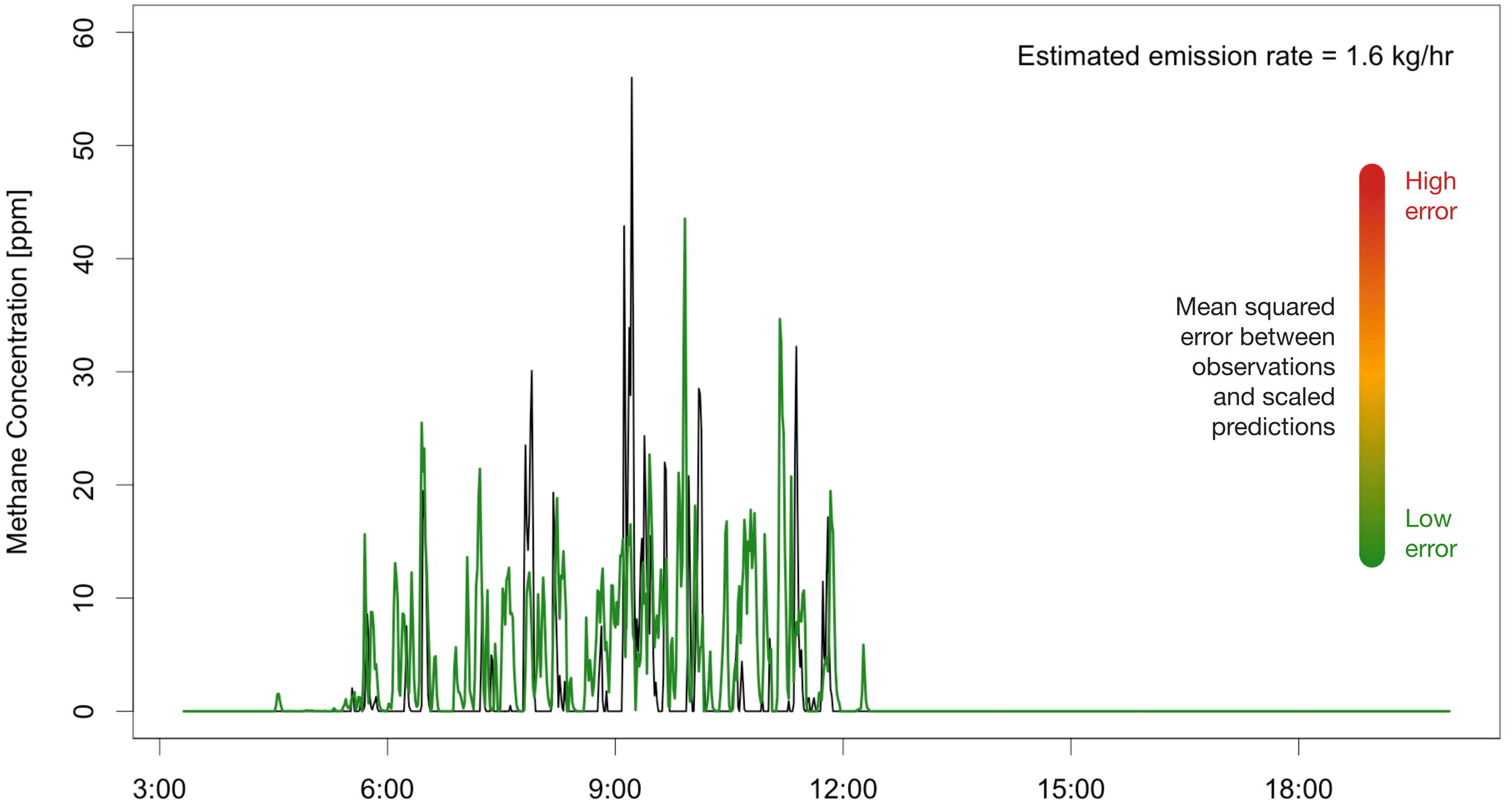
$$\hat{Q} = \underset{Q}{\operatorname{argmin}} \left\{ \frac{1}{n} \sum_{t=1}^{n} \left(d(x, y, z, t) - c(x, y, z, t, Q) \right)^2 \right\}$$

Emission rate estimate

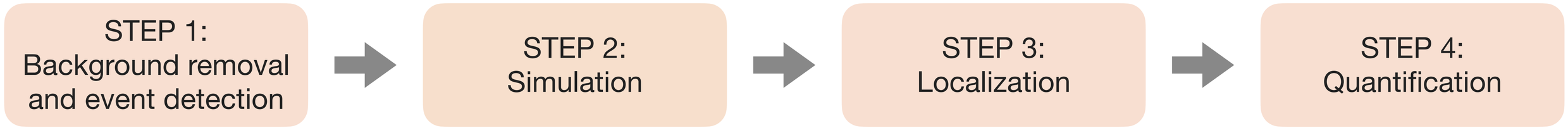
Simulated concentrations


Estimated emission rate = 0.1 kg/hr
Methane Concentration [ppm]
0
10
20
30
40
50
60
3:00
6:00
9:00
12:00
15:00
18:00
Mean squared error between observations and scaled predictions
High error
Low error

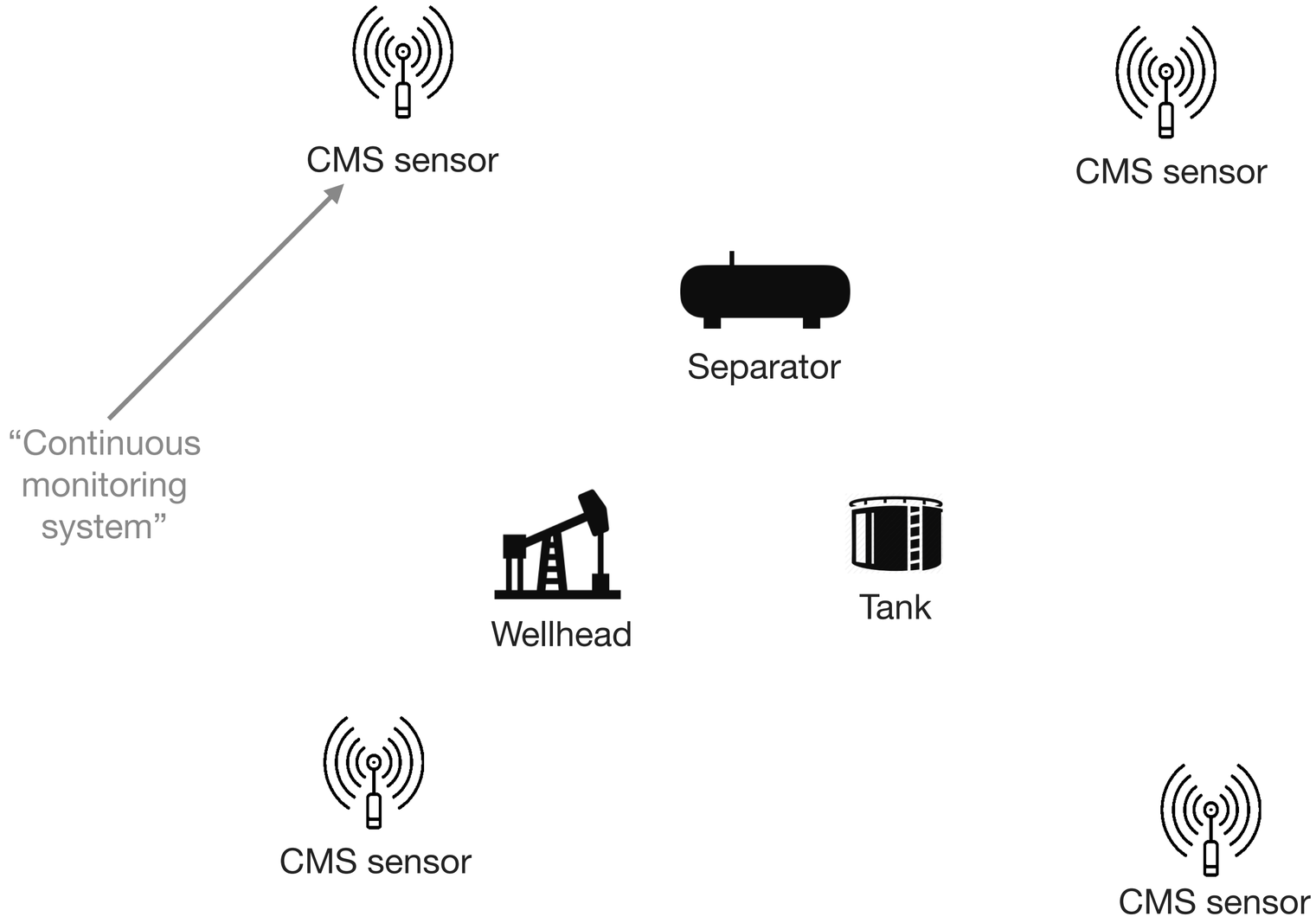

Estimated emission rate = 8.7 kg/hr
Methane Concentration [ppm]
0
10
20
30
40
50
60
3:00
6:00
9:00
12:00
15:00
18:00
Mean squared error between observations and scaled predictions
High error
Low error


Estimated emission rate = 1.6 kg/hr
Methane Concentration [ppm]
0
10
20
30
40
50
60
3:00
6:00
9:00
12:00
15:00
18:00
Mean squared error between observations and scaled predictions
High error
Low error

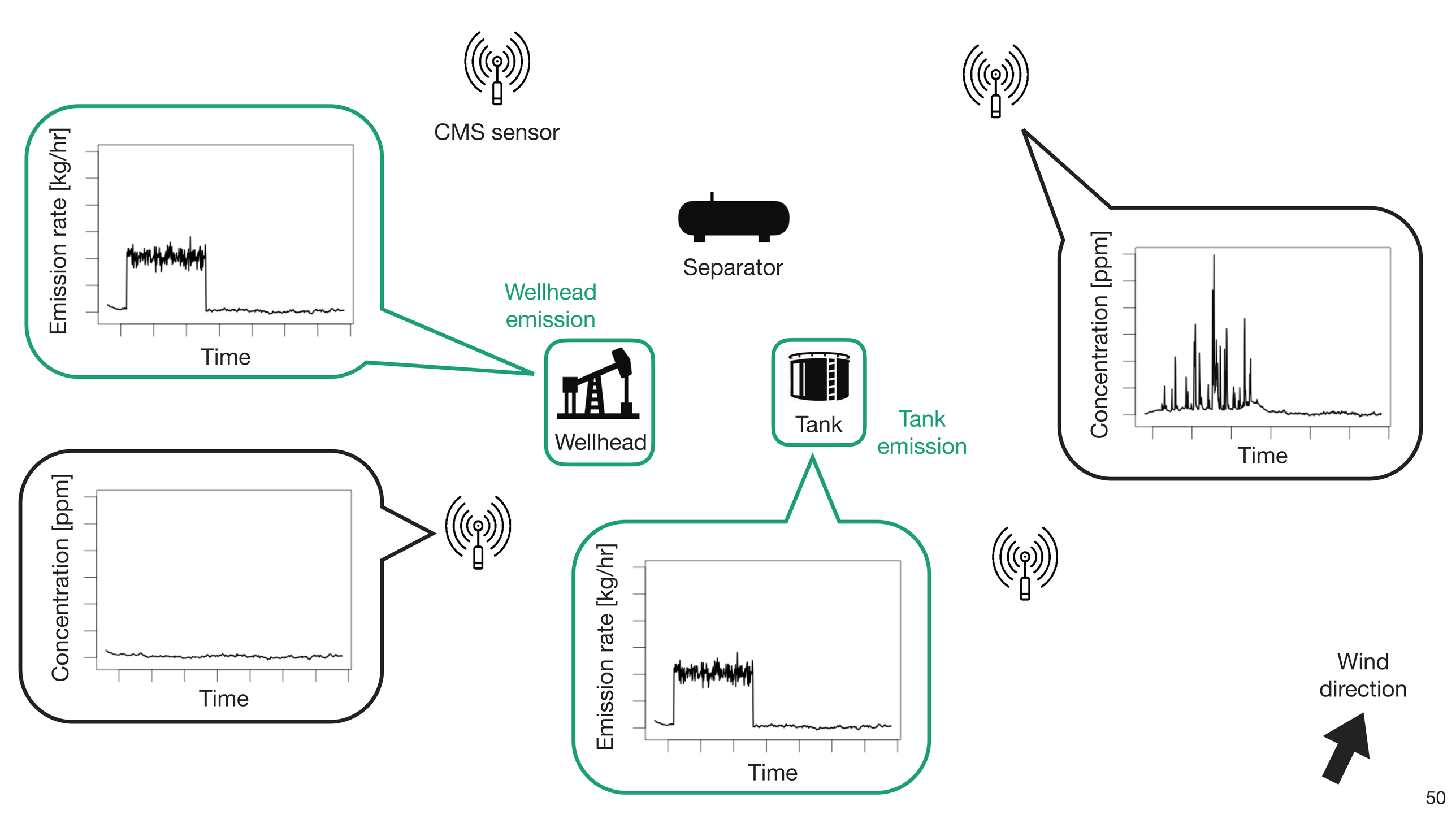
Open source framework for solving inverse problem



Daniels, Jia, and Hammerling (2024). Elementa.



The multi-source continuous monitoring inverse problem


CMS sensor
Separator
Wellhead emission
Wellhead
Tank
Tank emission
Emission rate [kg/hr]
Time
Concentration [ppm]
Time
Concentration [ppm]
Time
Emission rate [kg/hr]
Time
Wind direction

Bayesian model hierarchy

n = number of observations
p = number of potential sources

Assume a multiple linear regression model at the data level

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \ldots, y_n\}, \beta \equiv \{\beta_1, \ldots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Concentration observations from CMS sensors

Emission rates for each source

Simulated concentrations from forward model, with each column assuming a different source

Bayesian model hierarchy

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Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

$$\epsilon \sim N(0, \sigma^2 R)$$

Within an interval!

Bayesian model hierarchy

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Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

$$\epsilon \sim N(0, \sigma^2 R)$$

Let the errors follow an AR(1) process such that

$$\epsilon_i = r\epsilon_{i-1} + w$$

Autocorrelation coefficient

Gaussian white noise

Bayesian model hierarchy

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Assume a multiple linear regression model at the data level

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \ldots, y_n\}, \beta \equiv \{\beta_1, \ldots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

$$\epsilon \sim N(0, \sigma^2 R)$$

Let the errors follow an AR(1) process such that

$$\epsilon_i = r\epsilon_{i-1} + w$$

This gives us: $y \sim N(X\beta, \sigma^2 R)$

Bayesian model hierarchy

n = number of observations
p = number of potential sources

Given an AR(1) process for ϵ, the correlation matrix is

$$R = \begin{bmatrix} 1 & r & r^2 & \dots & r^{n-1} \\ r & 1 & r & \dots & \vdots \\ r^2 & r & 1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r^{n-1} & \dots & \dots & \dots & 1 \end{bmatrix}$$

Bayesian model hierarchy

n = number of observations
p = number of potential sources

Given an AR(1) process for ϵ, the correlation matrix is

$$R = \begin{bmatrix} 1 & r & r^2 & \dots & r^{n-1} \\ r & 1 & r & \dots & \vdots \\ r^2 & r & 1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r^{n-1} & \dots & \dots & \dots & 1 \end{bmatrix}$$

which has closed form expressions for the inverse and determinant:

$$R^{-1} = \frac{1}{(1-r^2)} \begin{bmatrix} 1 & -r & 0 & \dots & 0 \\ -r & 1+r^2 & -r & \dots & \vdots \\ 0 & -r & 1+r^2 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 1 \end{bmatrix} \quad \text{and} \quad |R| = (1-r^2)^{n-1}$$

Bayesian model hierarchy

n = number of observations
p = number of potential sources

Data-level: $y = X\beta + \epsilon$

$\epsilon \sim N(0, \sigma^2 R)$

The remainder of the hierarchy takes the following form

Spike-and-slab prior allows samples to be identically zero

$$\beta_i \sim \begin{cases} 0, & z_i = 0 \\ \text{Exp}(\tau_i^2 \sigma^2), & z_i = 1 \end{cases}$$

"Slab" component is non-negative

$$z_i \sim \text{Bernoulli}(\theta_i)$$

Proportion of samples where $z_i = 1$ gives posterior probability that source i is emitting

$$\theta_i \sim \text{Beta}(a_i, b_i)$$

$$\tau_i^2 \sim \text{Inv-Gamma}(c_i, d_i)$$

a_i, b_i, c_i, d_i can contain operator insight

$$\sigma^2 \sim \text{Inv-Gamma}(\nu/2,\ \nu/2)$$

$$\nu \sim \text{Inv-Gamma}(\alpha_1, \alpha_2)$$

$$r \sim \text{Uniform}(0, 1)$$

Bayesian model hierarchy

$$\beta_i \sim \begin{cases} 0, & z_i = 0 \\ \text{Exp}(\tau_i^2), & z_i = 1 \end{cases}$$

$$z_i \sim \text{Bernoulli}(\theta_i)$$

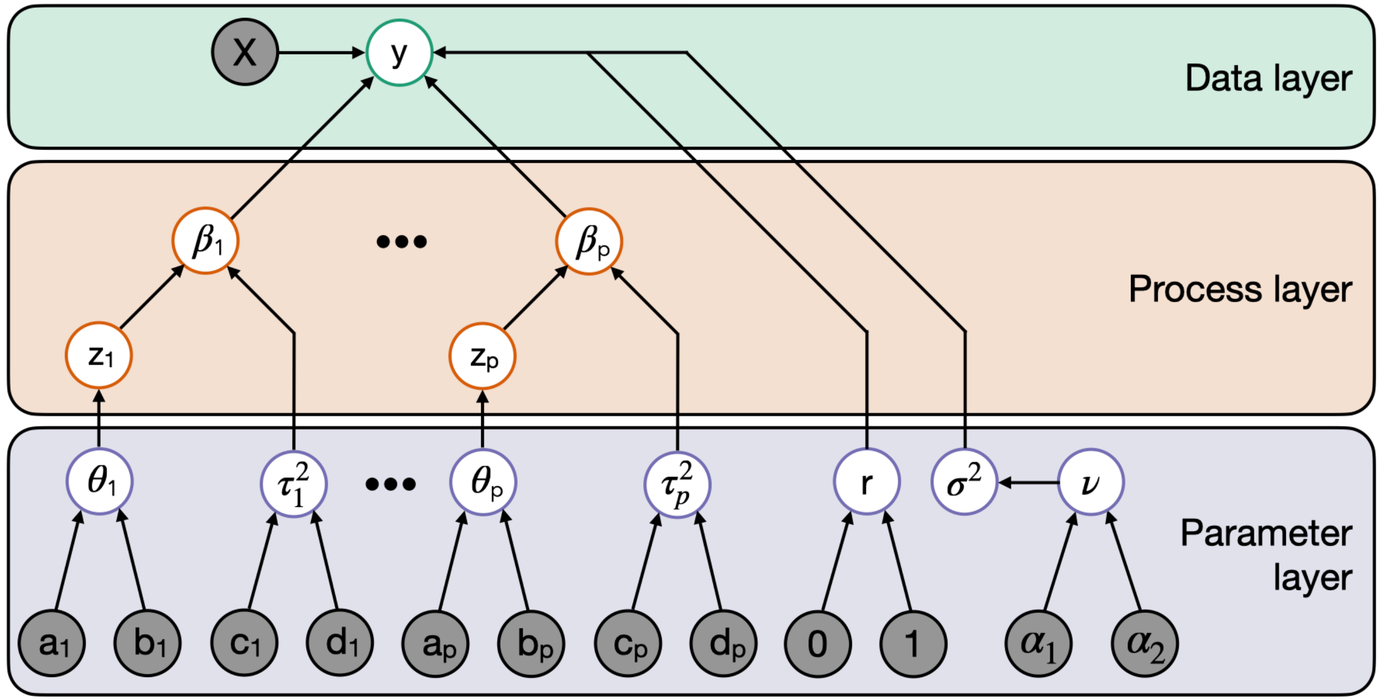
$$\theta_i \sim \text{Beta}(a_i, b_i)$$

$$\tau_i^2 \sim \text{Inv-Gamma}(c_i, d_i)$$

$$\sigma^2 \sim \text{Inv-Gamma}(\nu/2,\ \nu/2)$$

$$\nu \sim \text{Inv-Gamma}(\alpha_1, \alpha_2)$$

$$r \sim \text{Uniform}(0, 1)$$



Sampling from the posterior

We can derive Gibbs updates for all parameters except ν.

$$\theta_i|\xi \sim \text{Beta}(z_i + a_i, 1 - z_i + b_i)$$

$$\sigma^2|\xi \sim \text{Inv-Gamma}\left(\frac{\nu}{2} + \frac{n}{2}, \frac{\nu}{2} + \frac{1}{2}(y - X\beta)^T R^{-1}(y - X\beta)\right)$$

$$r|\xi \sim \begin{cases} N(X\beta, \sigma^2 R) & 0 < r < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_i^2|\xi \sim \text{Inv-Gamma}\left(1 + c_i, \beta_i + d_i\right)$$

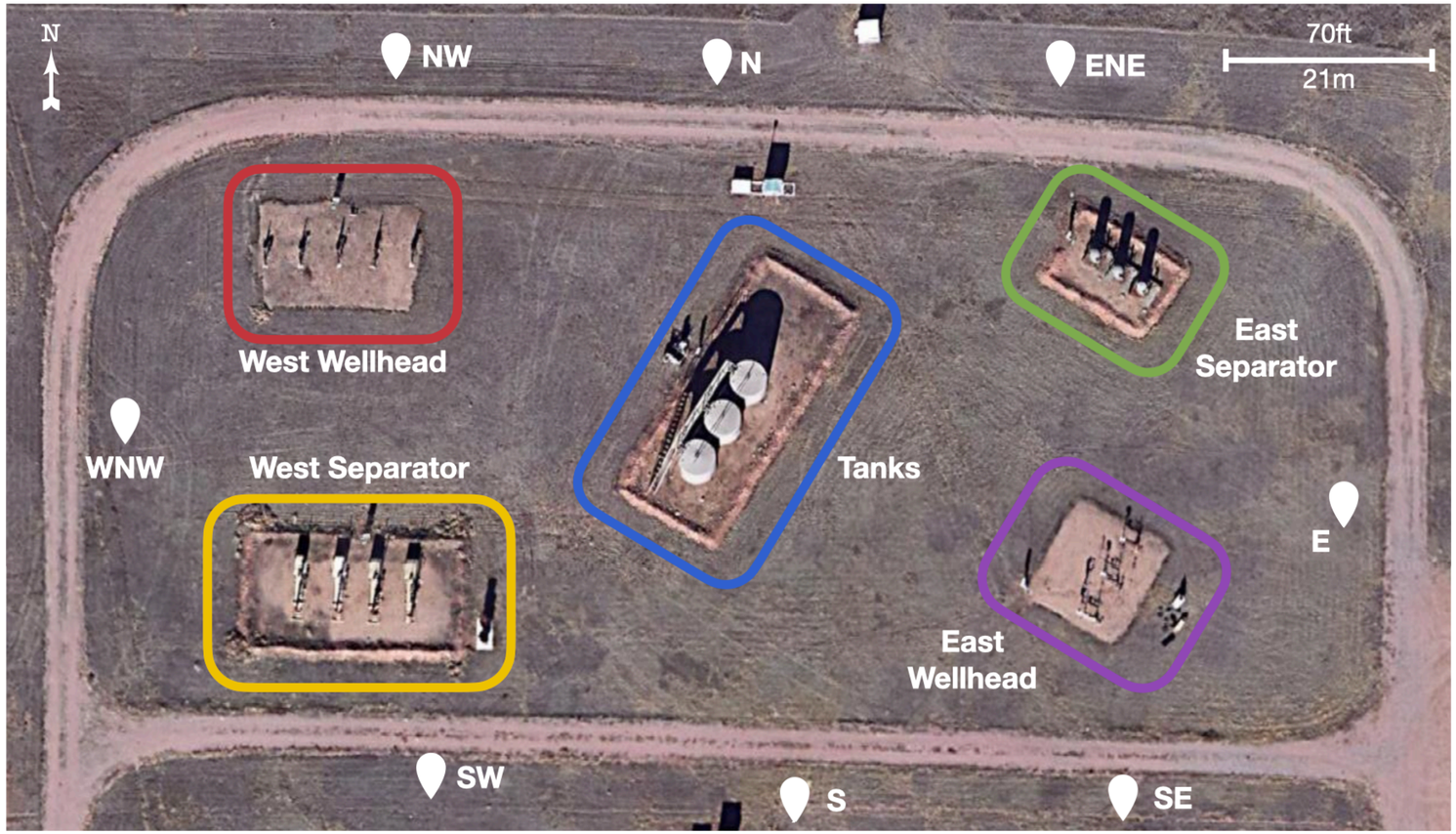
$$\beta_i|\xi \sim \begin{cases} 0 & z_i = 0 \\ \mathcal{N}\left(\left(\frac{X^T R^{-1} X}{\sigma^2}\right)^{-1}\left(\frac{X^T R^{-1} y}{\sigma^2} - \frac{e_i}{\tau_i^2}\right), \left(\frac{X^T R^{-1} X}{\sigma^2}\right)^{-1}\right) & z_i = 1 \end{cases}$$

$$z_i \sim \text{Bernoulli}\left(1 - \frac{1 - \theta_i}{(1 - \theta_i) + \theta_i\left(\frac{1}{\tau_i^2}\right)\exp\left(\frac{\left(\sum_{j=1}^n (w_j X_{j,i}^* + w_j^* X_{j,i}) - \frac{2\sigma^2}{\tau_i^2}\right)^2}{4\sigma^2 \sum_{j=1}^n X_{j,i} X_{j,i}^*}\right)\left(\frac{2\sigma^2\pi}{\sum_{j=1}^n X_{j,i} X_{j,i}^*}\right)^{1/2}\left(\frac{1}{2}\right)}\right)$$

$$\nu|\xi \sim ? \text{ (Use a Metropolis-Hastings step)}$$

Iterative samples from each full conditional gives you samples from the joint posterior!

Model evaluation on multi-source controlled release data



Methane Emissions Technology Evaluation Center (METEC)

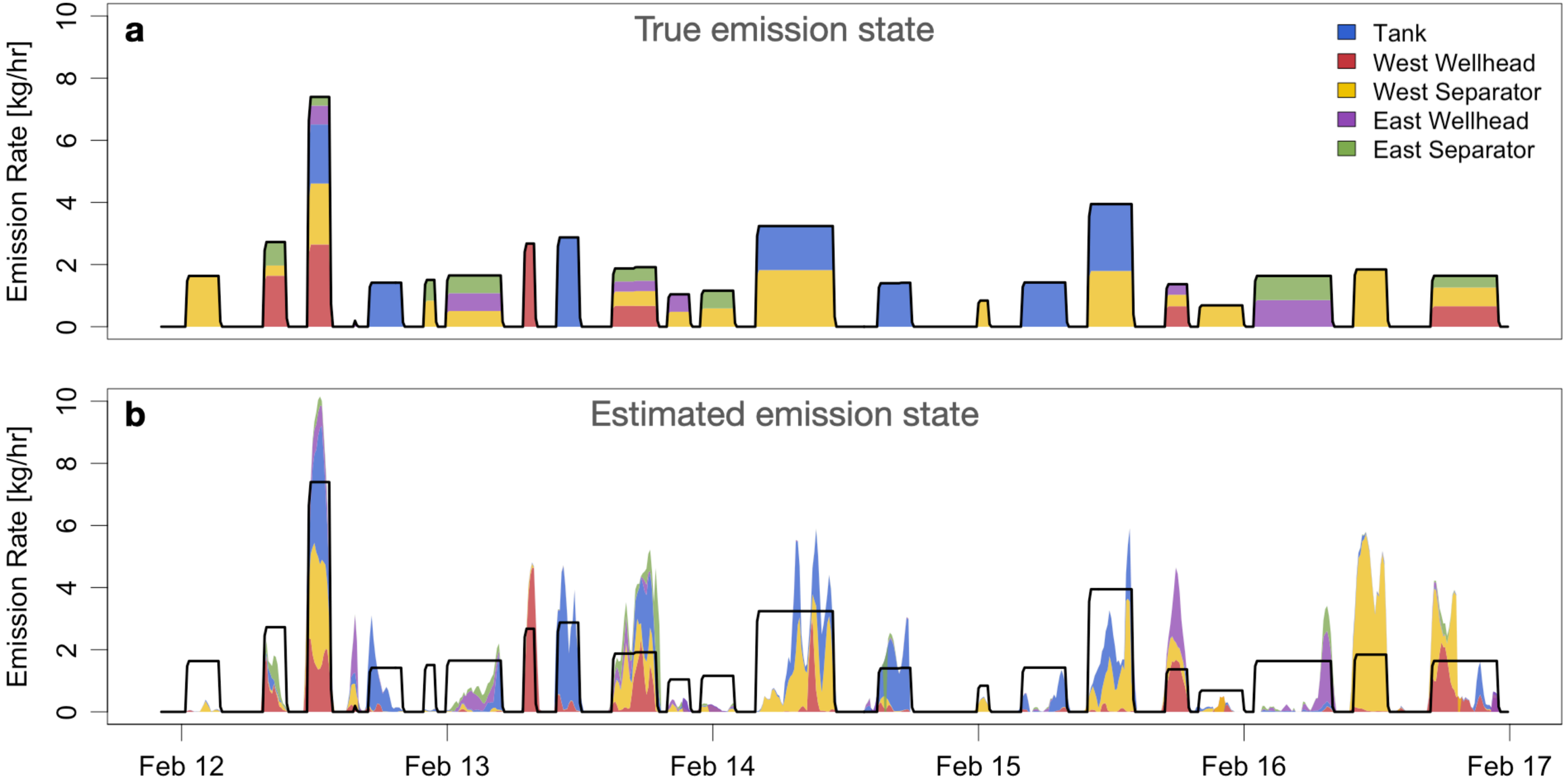
337 controlled releases:

- **99** (29%) single-source
- **238** (71%) multi-source

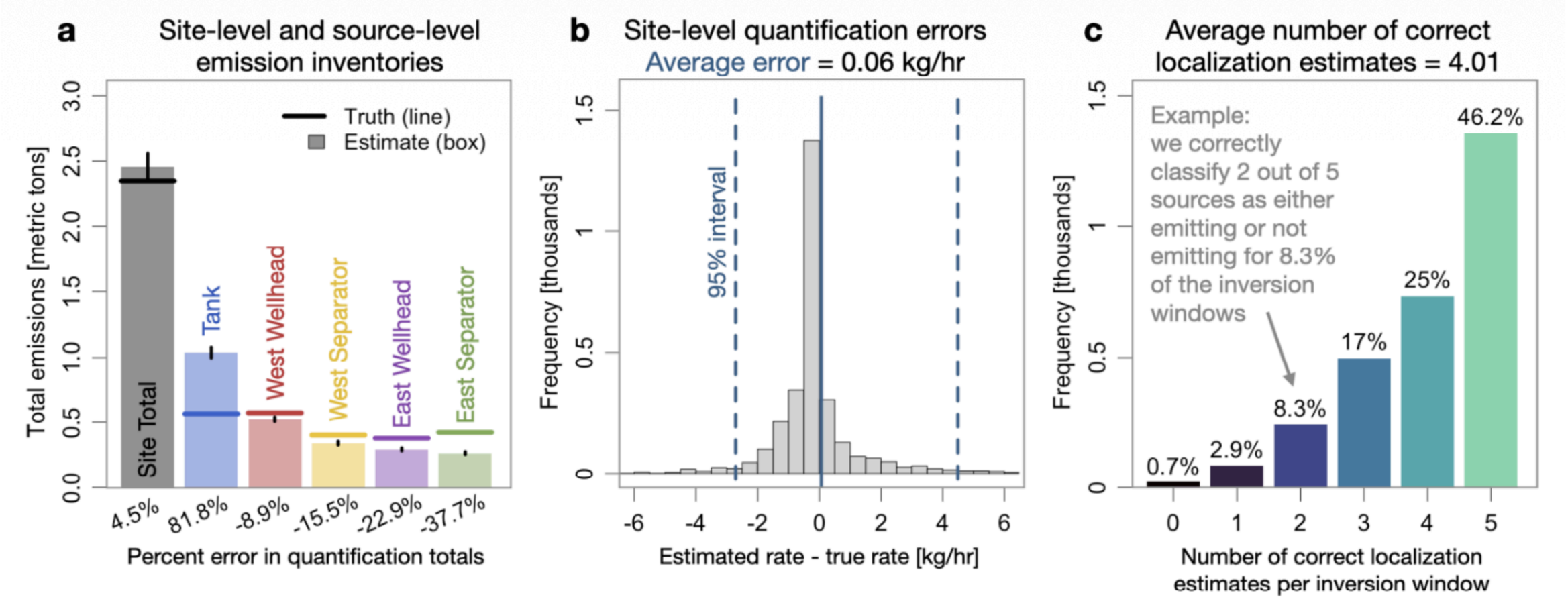
Emission rates range from **0.08** to **7.2** kg/hr

Emission durations range from **0.5** to **8** hours

Model evaluation on multi-source controlled release data

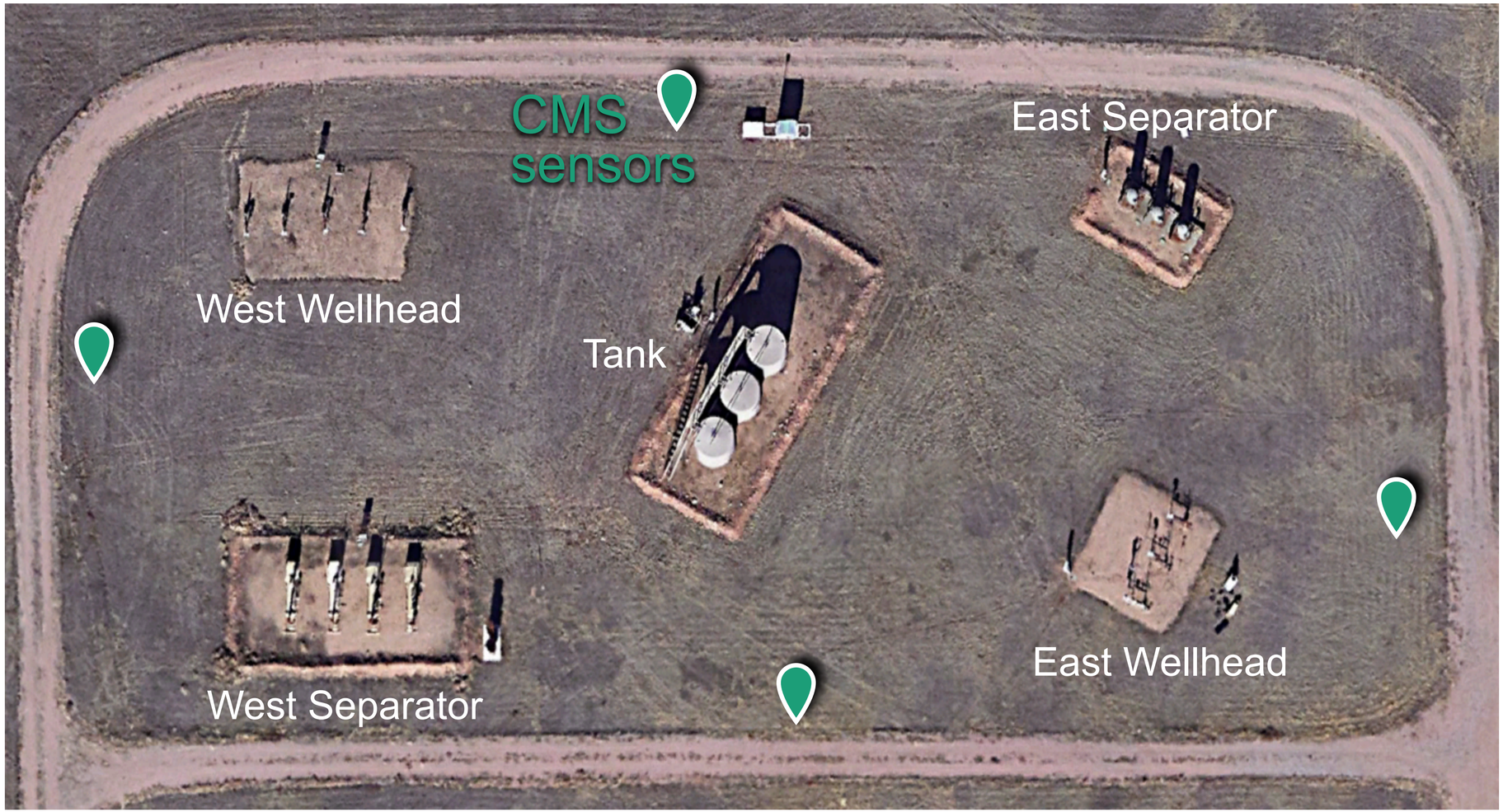


Model evaluation on multi-source controlled release data

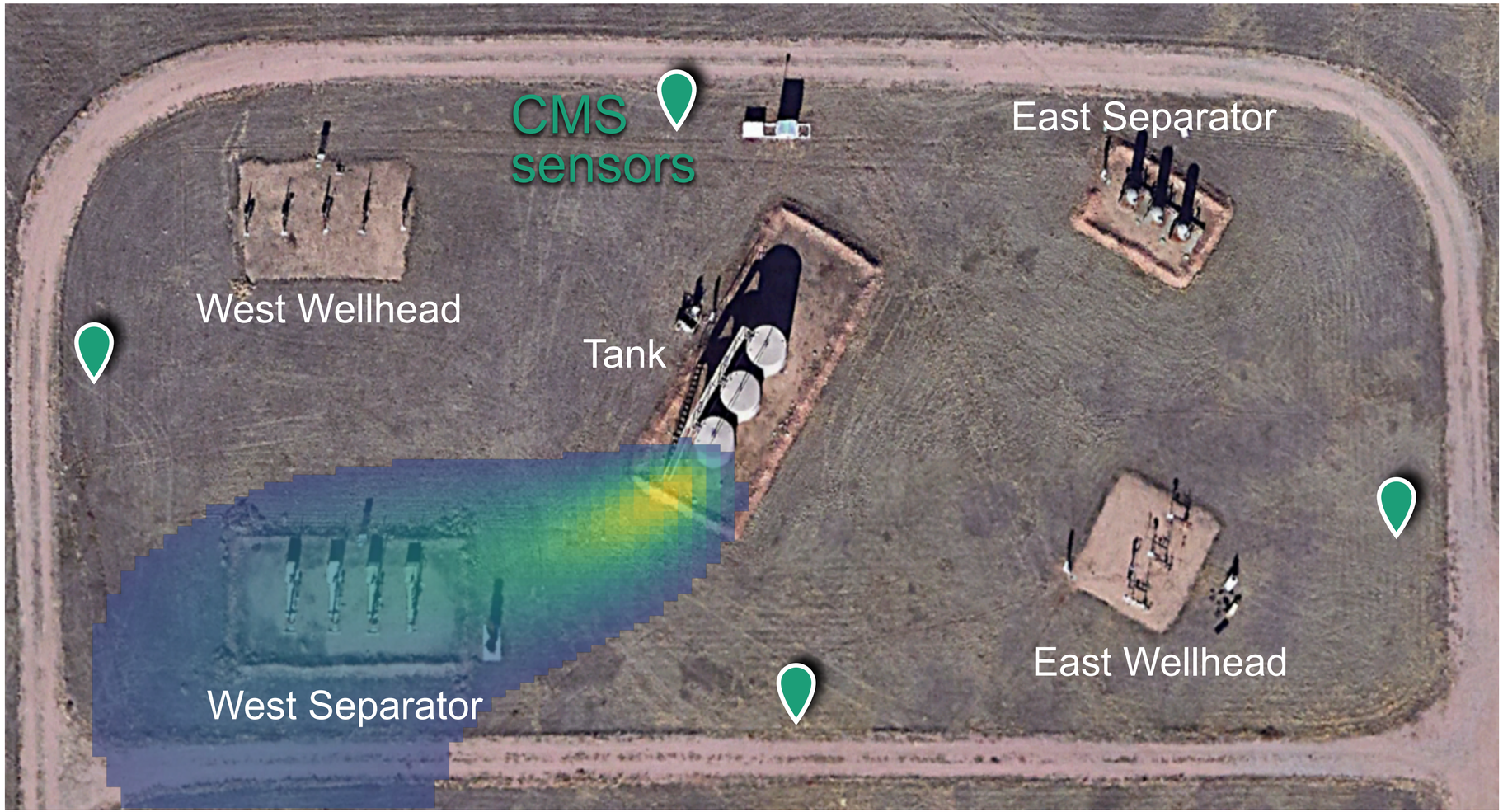


Daniels, Nychka, and Hammerling (2026). Annals of Applied Statistics.

Regardless of the inverse model… need to account for incomplete sensor coverage

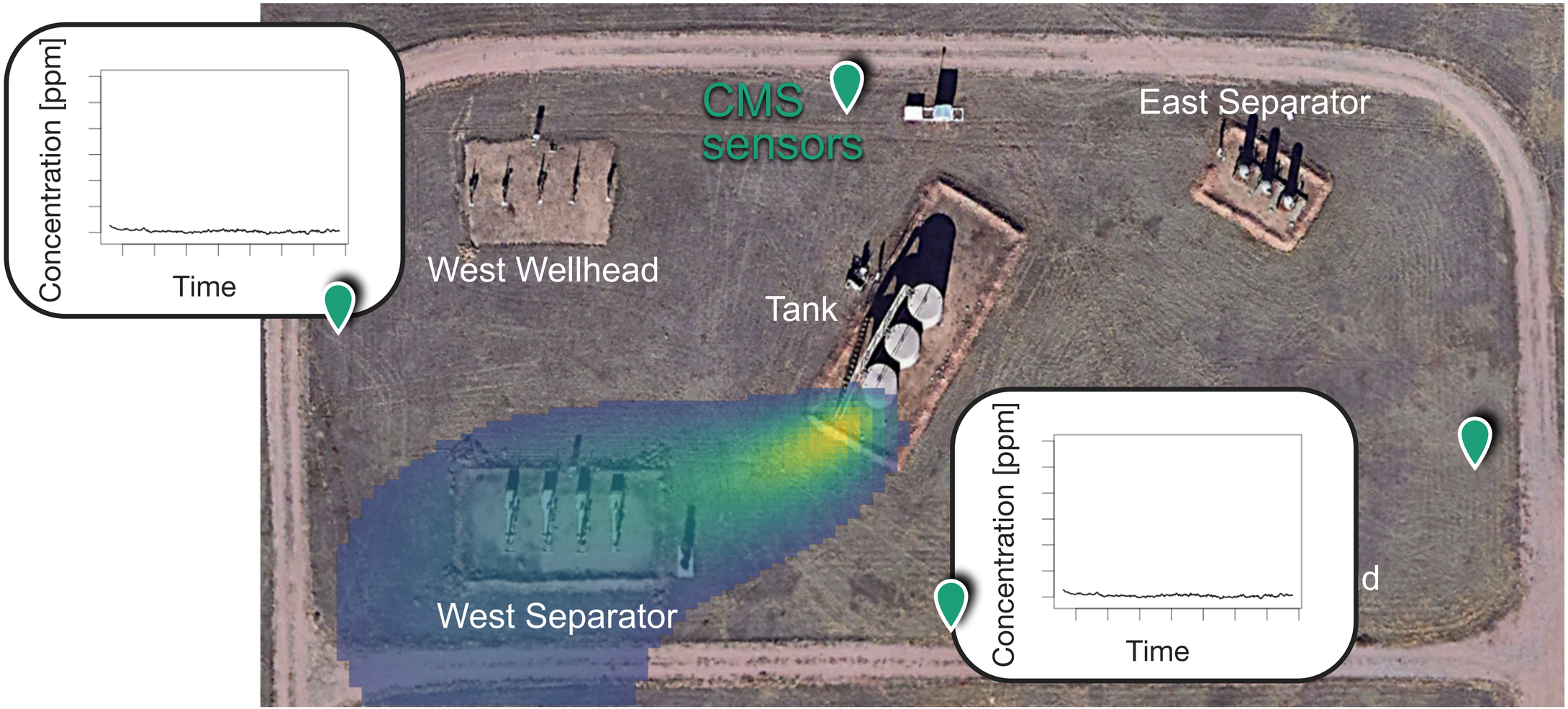


Regardless of the inverse model… need to account for incomplete sensor coverage



Wind direction

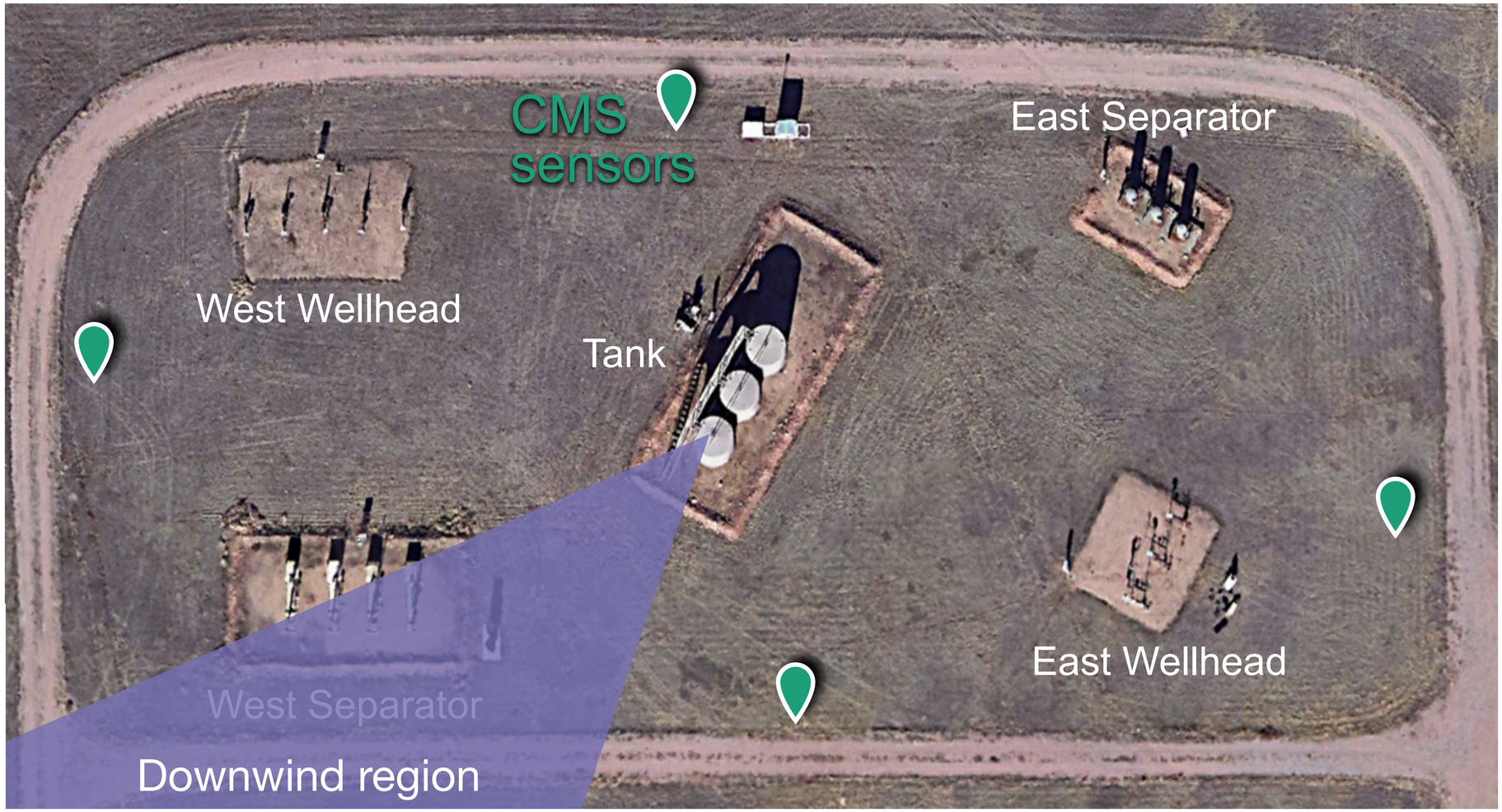
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Wind direction

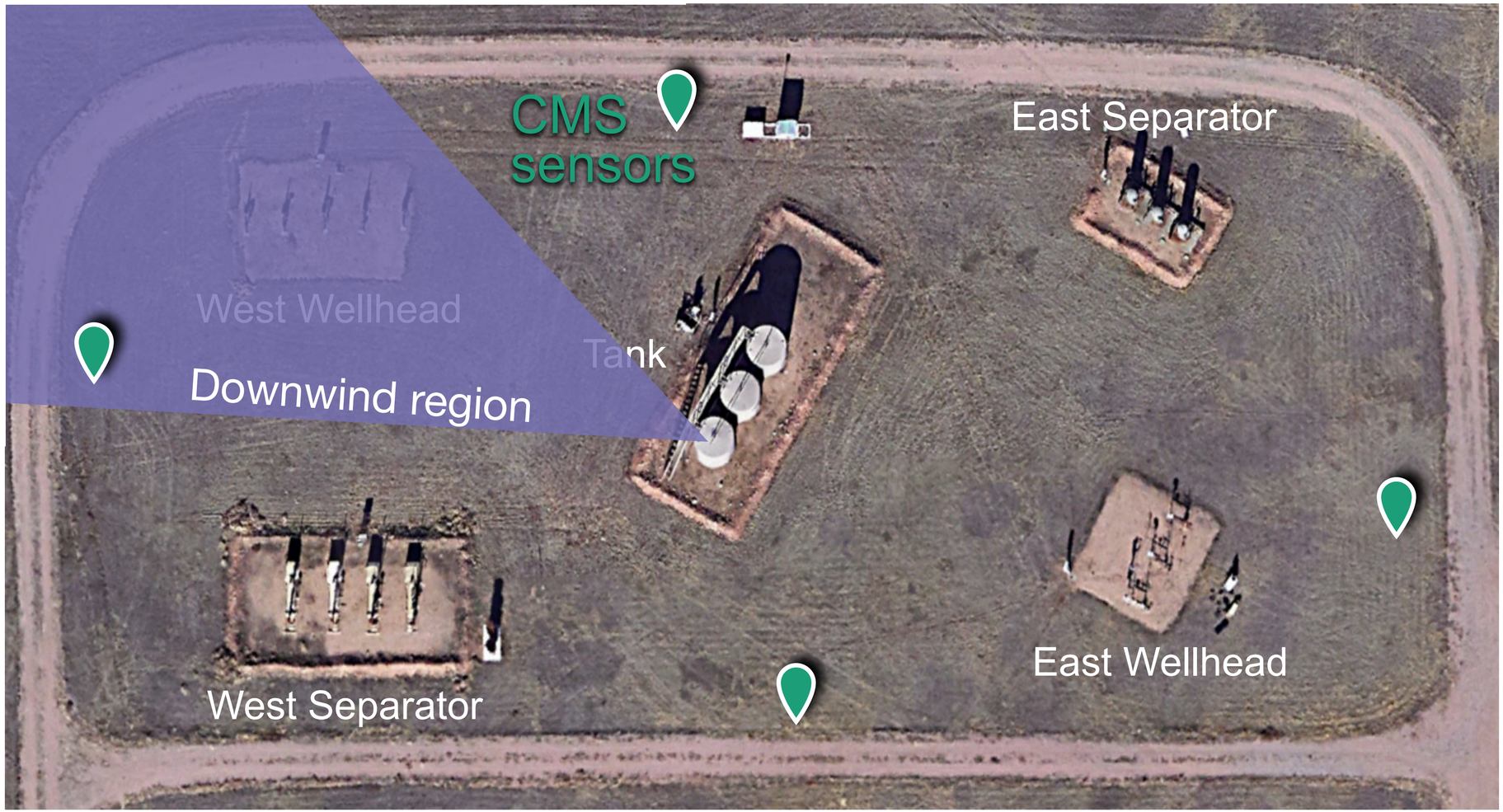
CMS do not provide emission information when the wind blows between sensors

However, we can estimate when no information periods happen!



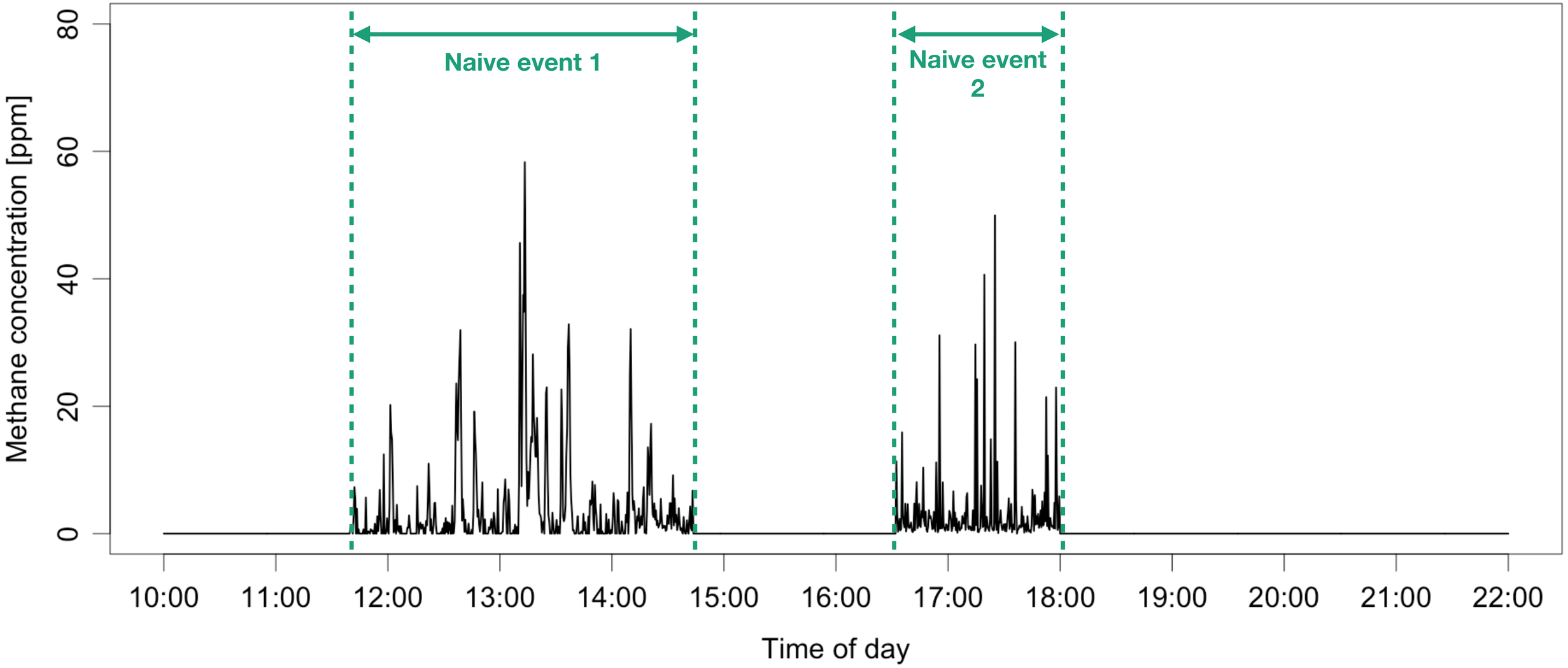
Downwind region **does not** overlap with CMS sensors = period of “**no information**”

However, we can estimate when no information periods happen!

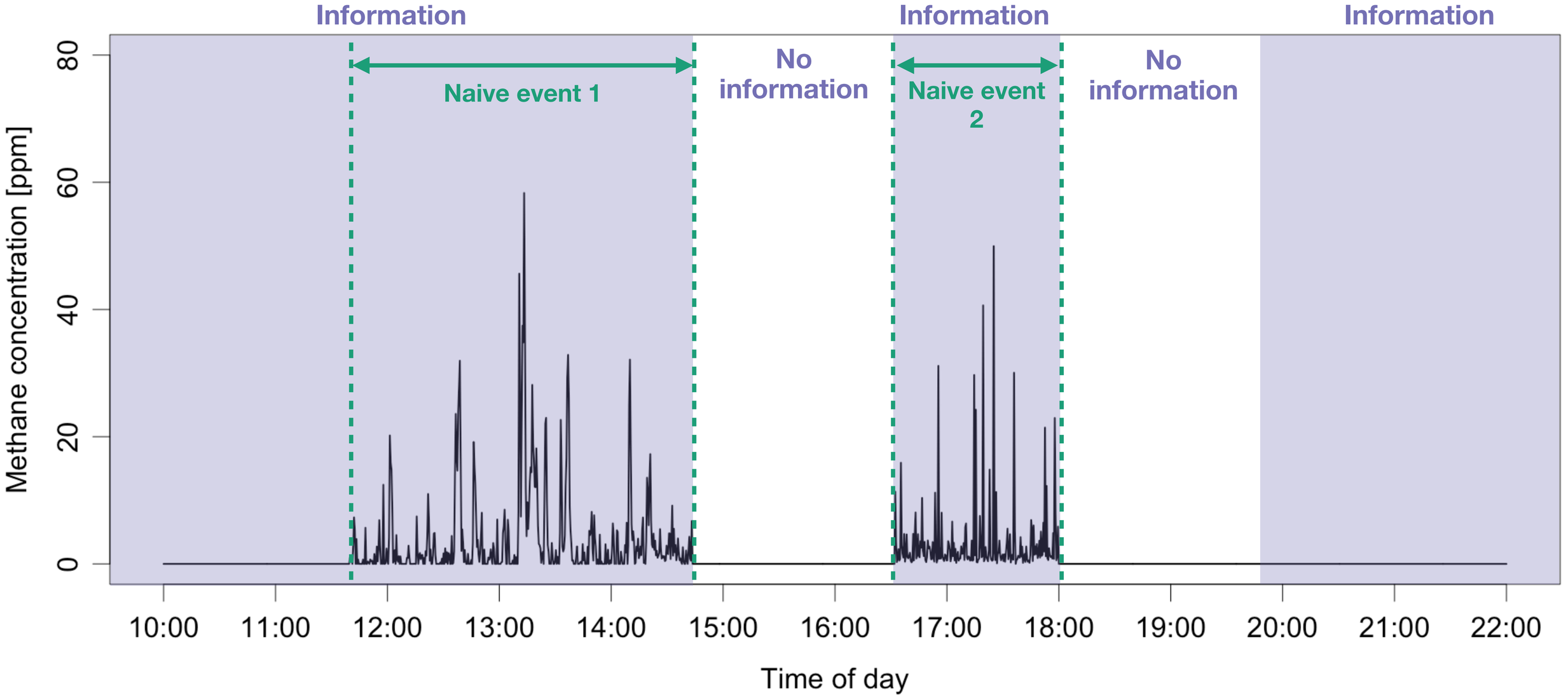


Downwind region **does** overlap with CMS sensors = period of “**information**”

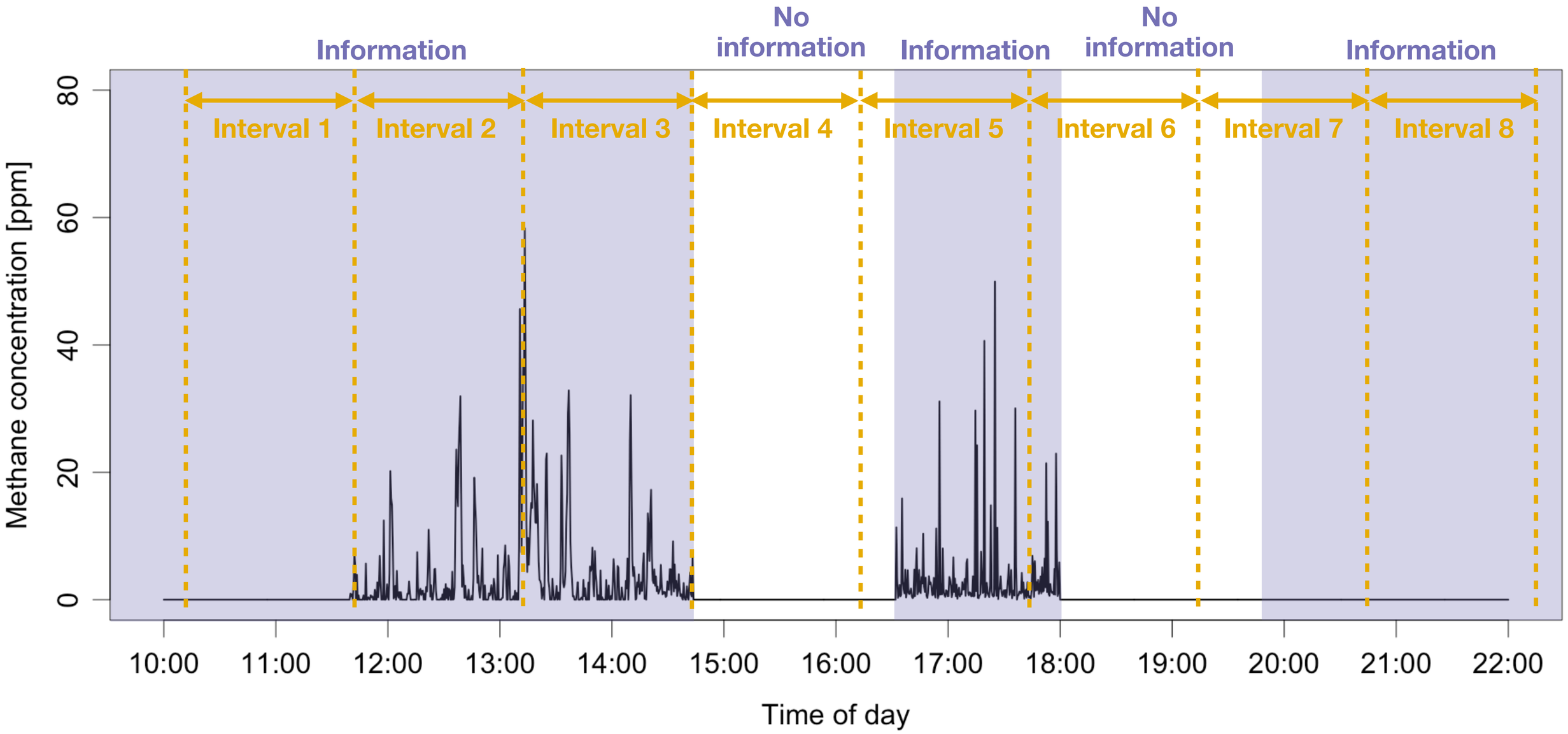
How do periods of information and no information present themselves in the data?



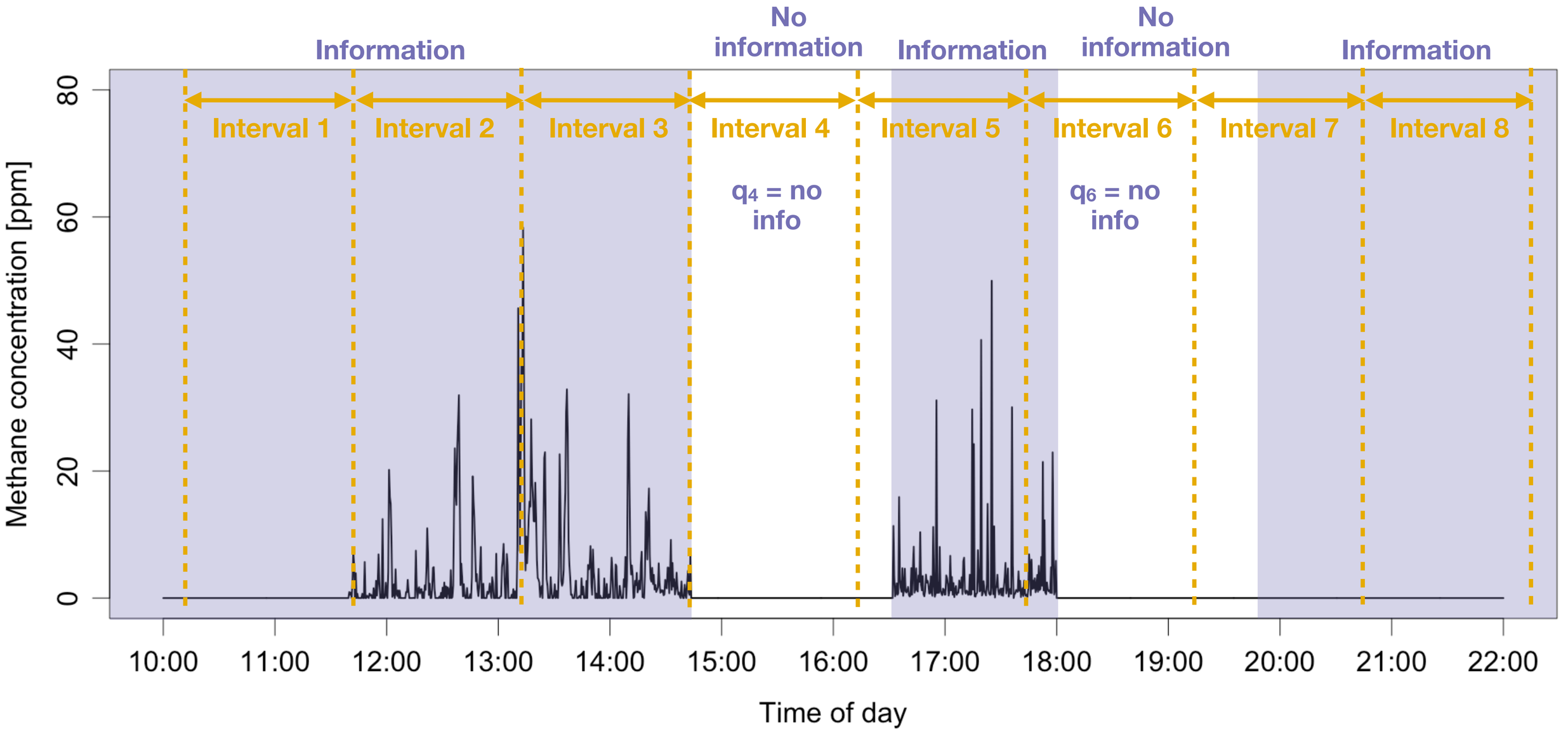
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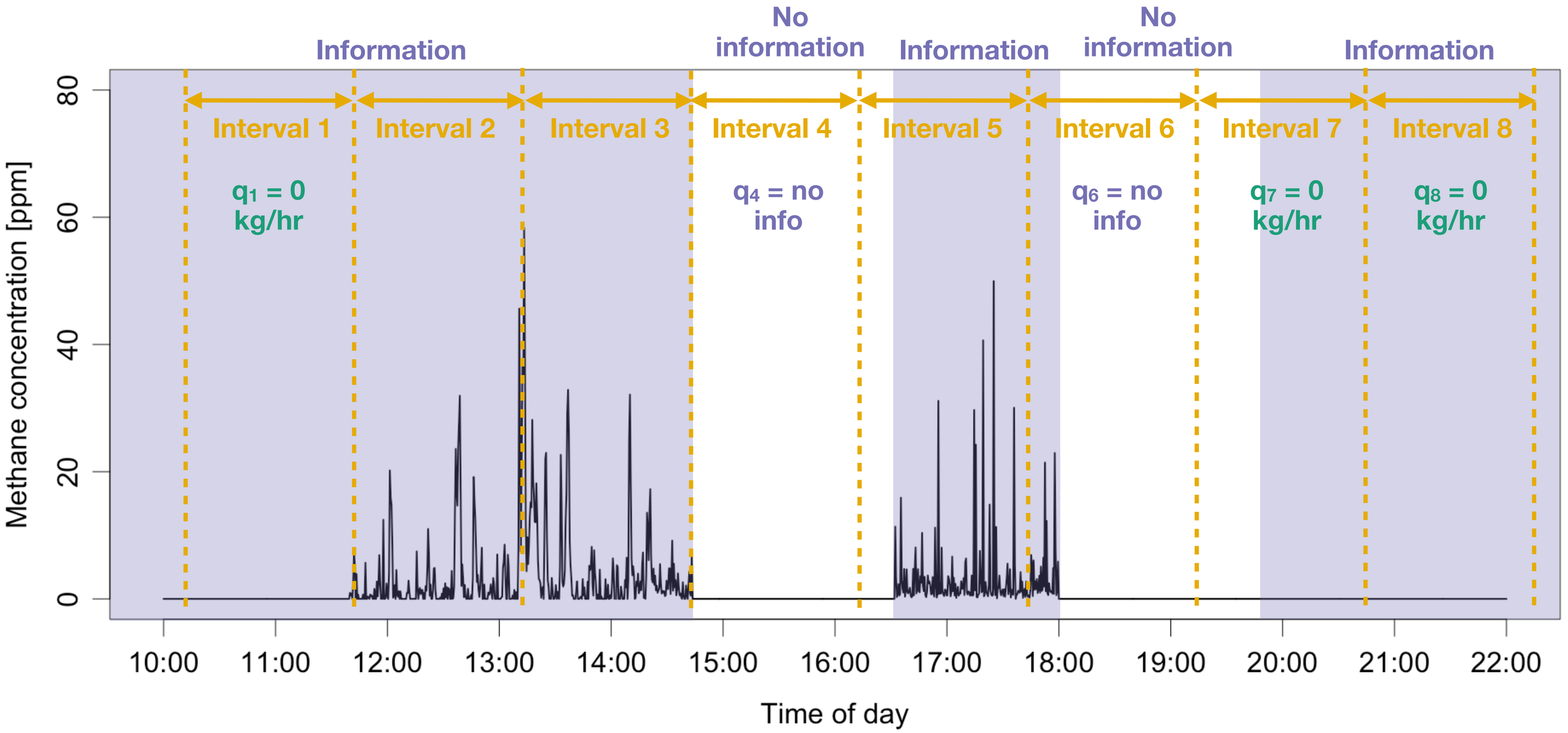
In practice, run the MDLQ (or DLQ) model on fixed intervals



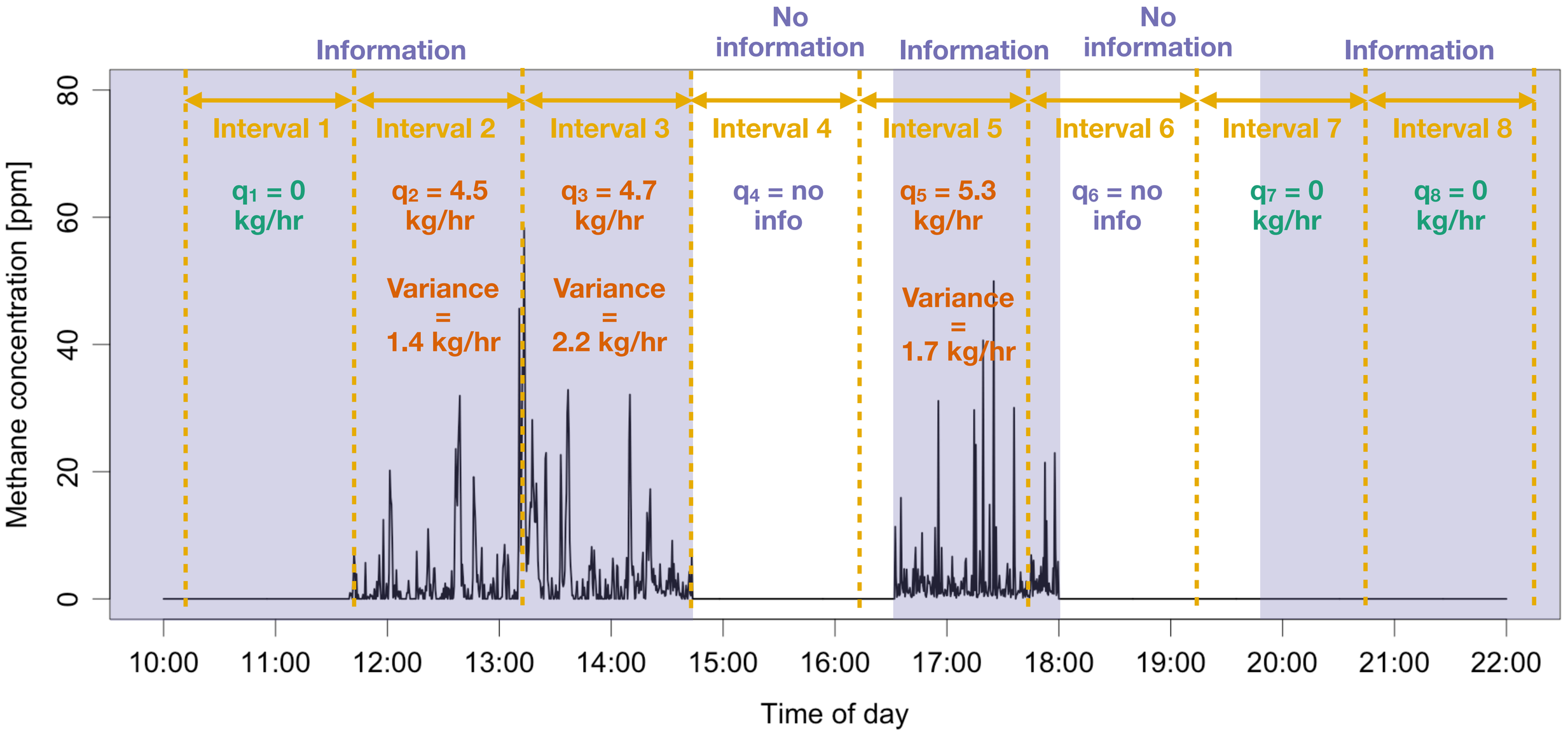
Whether an interval is no information,



Whether an interval is no information, zero emission rate,



Whether an interval is no information, zero emission rate, or non-zero emission rate depends on the data.



Improving the Gaussian puff model is an active area of research

Improving the Gaussian puff model is an active area of research

1. Obstacle-aware dispersion model that runs fast **[Andres]**
2. Better estimates of dispersion during nighttime and low wind **[Michael]**
3. More accurate method for aggregating high frequency wind measurements **[Andy]**

Andres Pruet
Colorado School of Mines



Michael Basanese
Colorado School of Mines

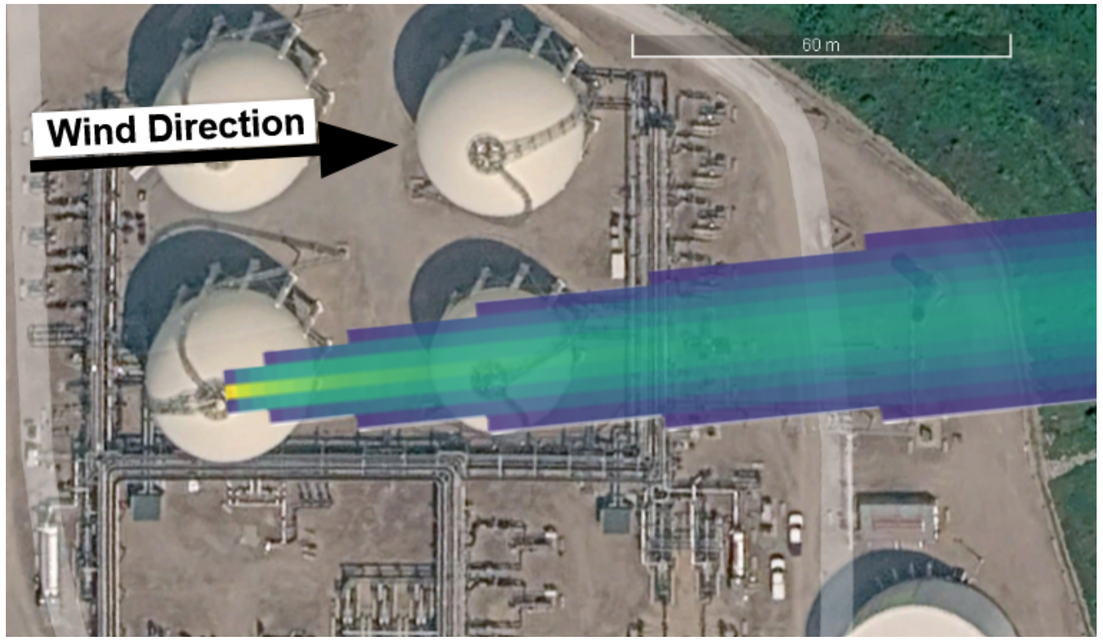


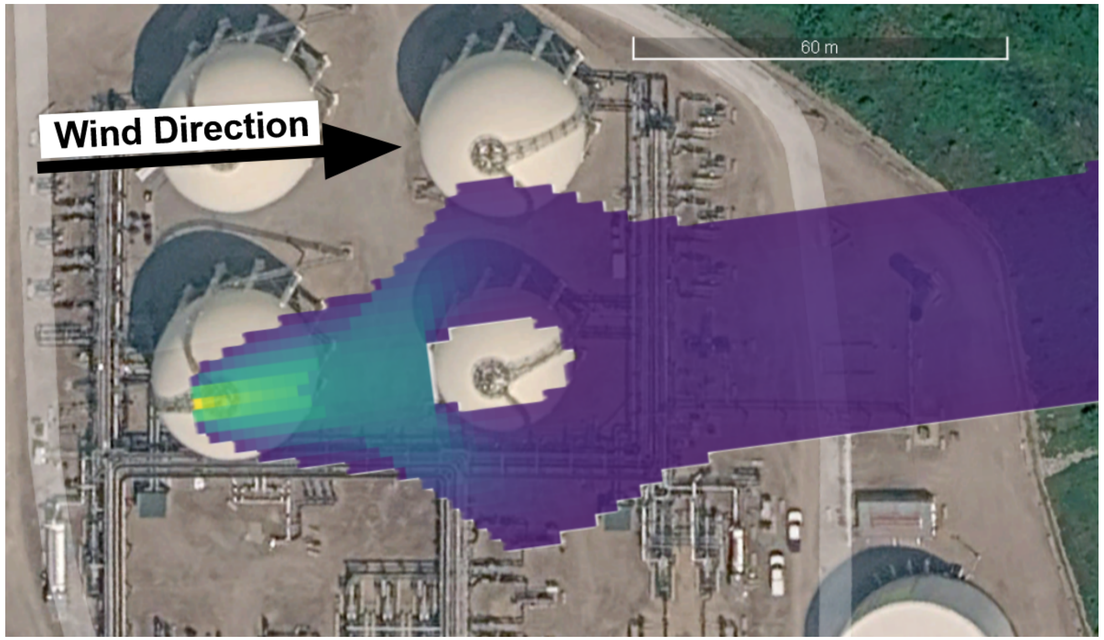
Andy Holmberg
Colorado School of Mines



Direction #1: Obstacle-aware dispersion model

- The current Gaussian puff model is not aware of obstacles, like tanks or buildings.

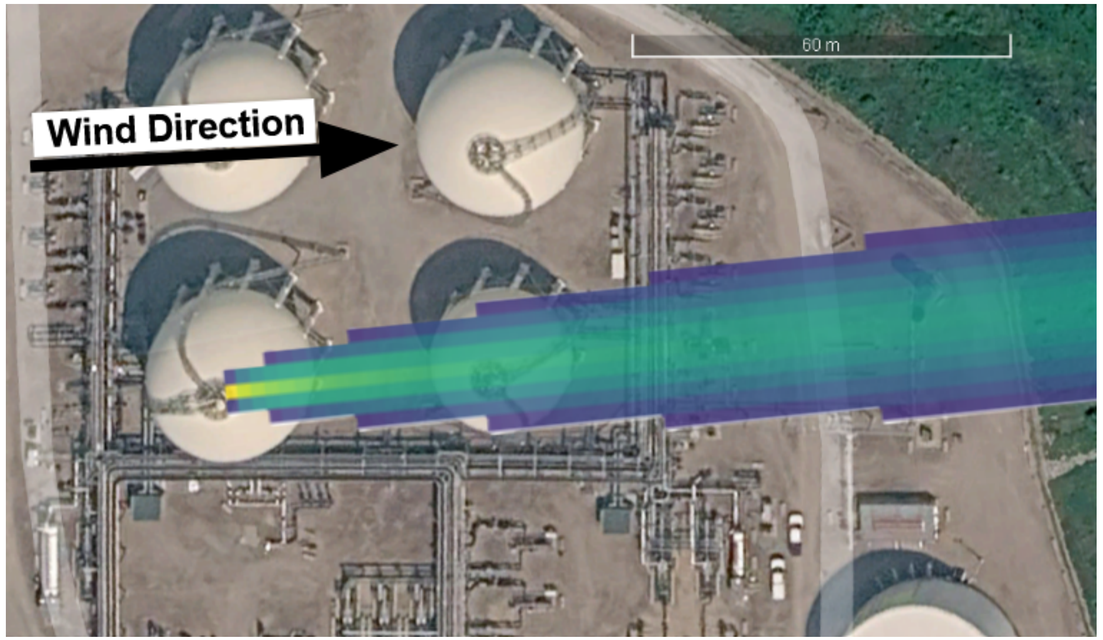


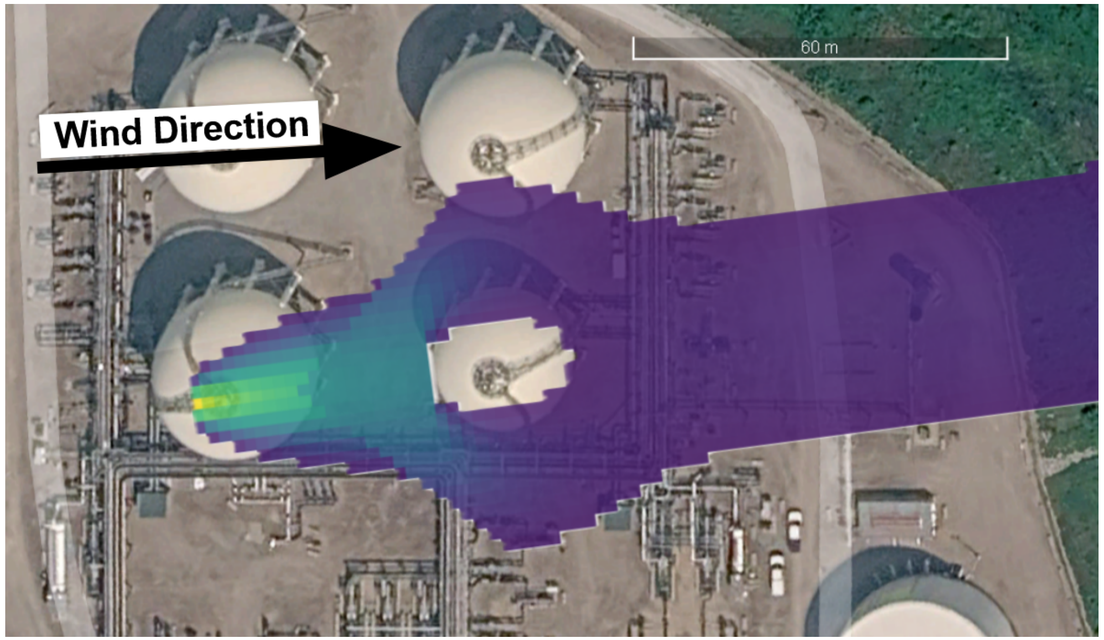


Gas processing facility. Google Earth.

Direction #1: Obstacle-aware dispersion model

- The current Gaussian puff model is not aware of obstacles, like tanks or buildings.
- Computational fluid dynamics (CFD) models are the gold standard for obstacle-aware transport, but they often require specialized expertise to run and are too computationally expensive to use in a production setting.





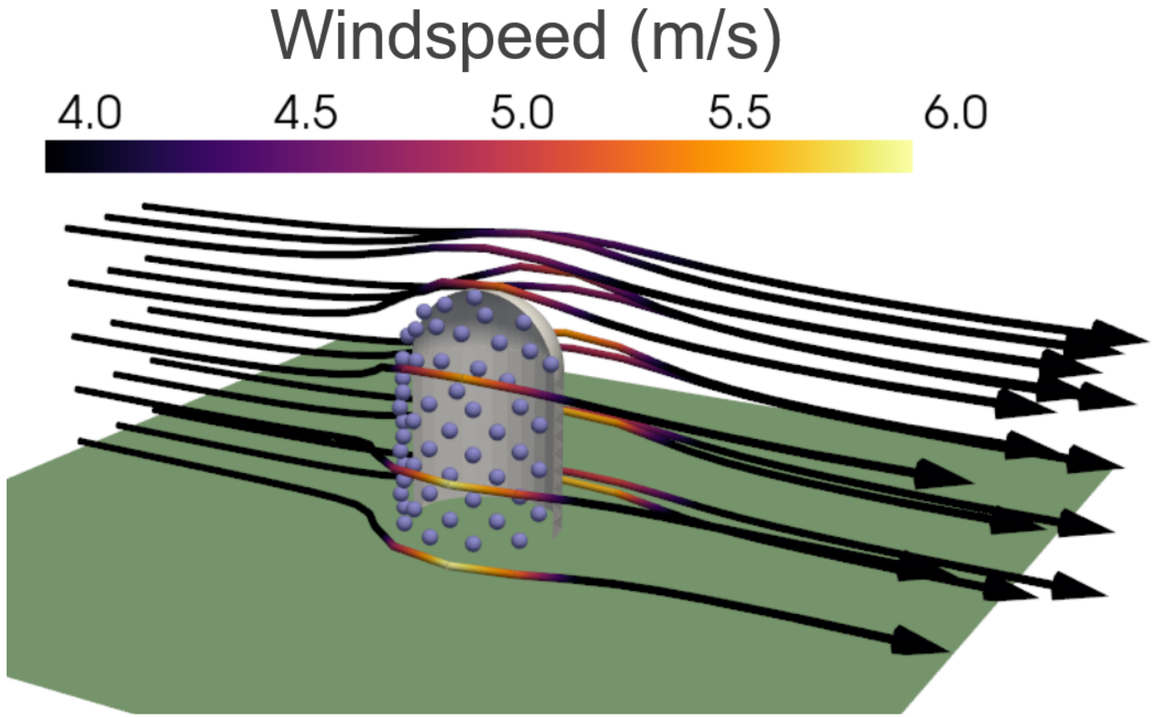
Gas processing facility. Google Earth.

Direction #1: Obstacle-aware dispersion model

Our approach

Given a wind field $\vec{u}$, a diffusion coefficient D, and an emission described by $S(\mathbf{x}, t)$, we solve for concentration C according to the Advection-Diffusion equation:

$$\frac{\partial C}{\partial t} + \nabla \cdot (C\vec{u} - D\nabla C) = S$$



- **Step 1:** Use a potential flow solution for the wind velocity to get a coarse obstacle-aware wind field
- **Step 2:** Simulate the concentration profile using a fast Fourier transform on a fixed background grid
- **Step 3:** After finding the background solution, apply a local correction near the surface of the obstacle

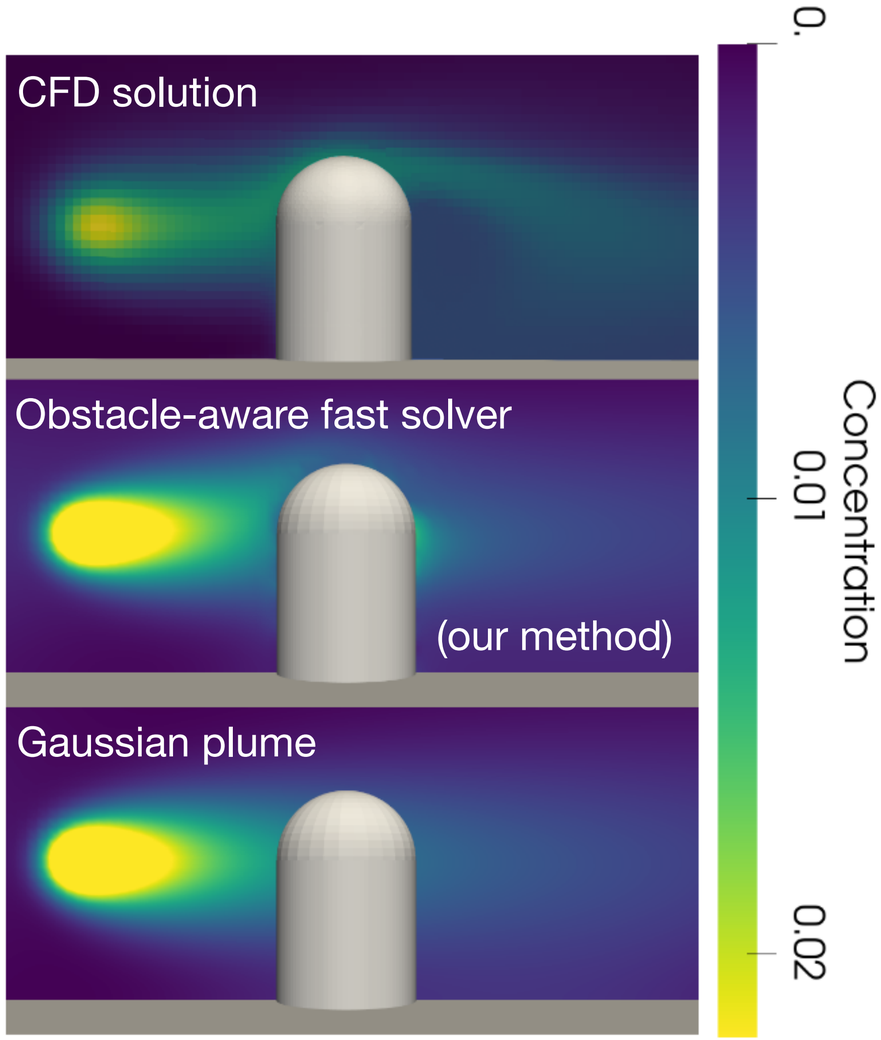
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Direction #1: Obstacle-aware dispersion model

EEMDL
Energy Emissions Modeling and Data Lab

2026 EEMDL Annual Conference & Meeting
Poster Session

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COLORADO SCHOOL OF MINES



Title: Integral Spectral Method for Advection-Diffusion in 3D

ABSTRACT

Atmospheric dispersion models are used in a variety of real world applications, from estimating anthropogenic methane emissions to informing air quality monitoring efforts. In applications such as emissions estimation, atmospheric dispersion models must be computationally efficient enough to run in near-realtime. In addition, dispersion models should account for large impenetrable obstacles on the domain, such as buildings, which disrupt the flow field and affect the dispersion of trace gasses. Currently, the fastest family of models, Gaussian dispersion models, do not account for obstacles, which can lead to highly inaccurate results, especially on industrial facilities which have large buildings and equipment downwind of the emission source. While there do exist models which are able to account for obstacles, like computational fluid dynamics models, these models are often too computationally expensive to be used in real-time applications. This research proposes an integral-equation formula for solving the advection-diffusion problem in three dimensions while minimizing computational cost. The method makes use of a Fourier-Chebyshev grid to parallelize the problem, and it uses an immersed-boundary method to account for obstacles on the domain, while accurately accounting for the effect on both the wind field and the diffusion of gasses.

1. Background

- Fast and accurate atmospheric dispersion models are needed to estimate emissions and inform policy.
- Current models fail to account for obstacles on the domain, or are prohibitively slow.
- We propose a fast, obstacle-aware PDE solver.

2. Objectives

- Model the dispersion of trace gasses, while accounting for obstacles on the domain.
- Accuracy should be similar to the CFD model, while greatly reducing computational cost.

3. Methodology

- Our goal is to solve for the concentration C according to the Advection-Diffusion equation:

$$\frac{\partial C}{\partial t} + \nabla \cdot (C\vec{u} - D\nabla C) = S \qquad (1)$$

Where the wind $\vec{u}$, diffusion constant D, and source term S (emission rate and location) are provided.

- We use Fourier transforms in the x and y dimensions to obtain the inhomogeneous, transformed ODE:

$$\hat{C}_{zz}(z) - k^2\hat{C}(z) = f(z)$$

This ODE is solved using Chebyshev transforms, and then we use inverse transforms in all 3 dimensions to get C_P, a solution to (1) in the absence of obstacles.

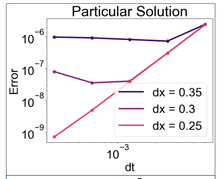
- We modify the particular solution with a homogenous update C_H, to account for the obstacle. The homogeneous solution is found using the Method of Fundamental Solutions:

$$C_H(\mathbf{x}) = \sum_{j=1}^{N} \alpha_j G_{\mathbf{x}_j}(\mathbf{x})$$

- Where $G_{\mathbf{x}_j}(\mathbf{x})$ is a fundamental solution of the modified Helmholtz equation.

4. Results

- Accuracy of the particular solution increases cubically with time resolution, and exponentially with spatial resolution.



- Conservation of mass is satisfied more accurately (the line closest to 1.0) by a model which accounts for the obstacle's effect on the wind field as well as turbulent diffusion.

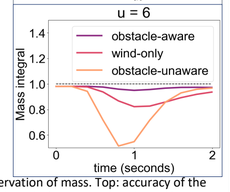
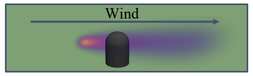


Figure 1: Accuracy and conservation of mass. Top: accuracy of the particular solution (with no obstacles) compared to the analytic solution for a Gaussian puff of gas. Bottom: conservation of mass over time, compared between three models.

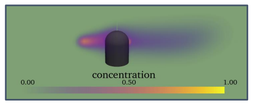
5. Conclusions

- This framework can solve for the concentration field of a trace gas or pollutant, on a 3D domain with obstacles.

- Naive model



- Obstacle aware model



- Computation is optimized for GPU usage, and linear systems can be prefactored to efficiently simulate multiple source locations.
- This method relies on modeling obstacles with smooth boundaries (like cylinders and spheres) but doesn't perform well on obstacles with sharp edges.

6. Next Steps

- More research must be done comparing the results of this model to the CFD model, in order to inform the selection of parameters like diffusion constant D.
- The model should be implemented into an inverse problem framework and used for emission detection, localization, and quantification on oil-and-gas sites with significant obstructions, like buildings and tanks.

Colorado State University

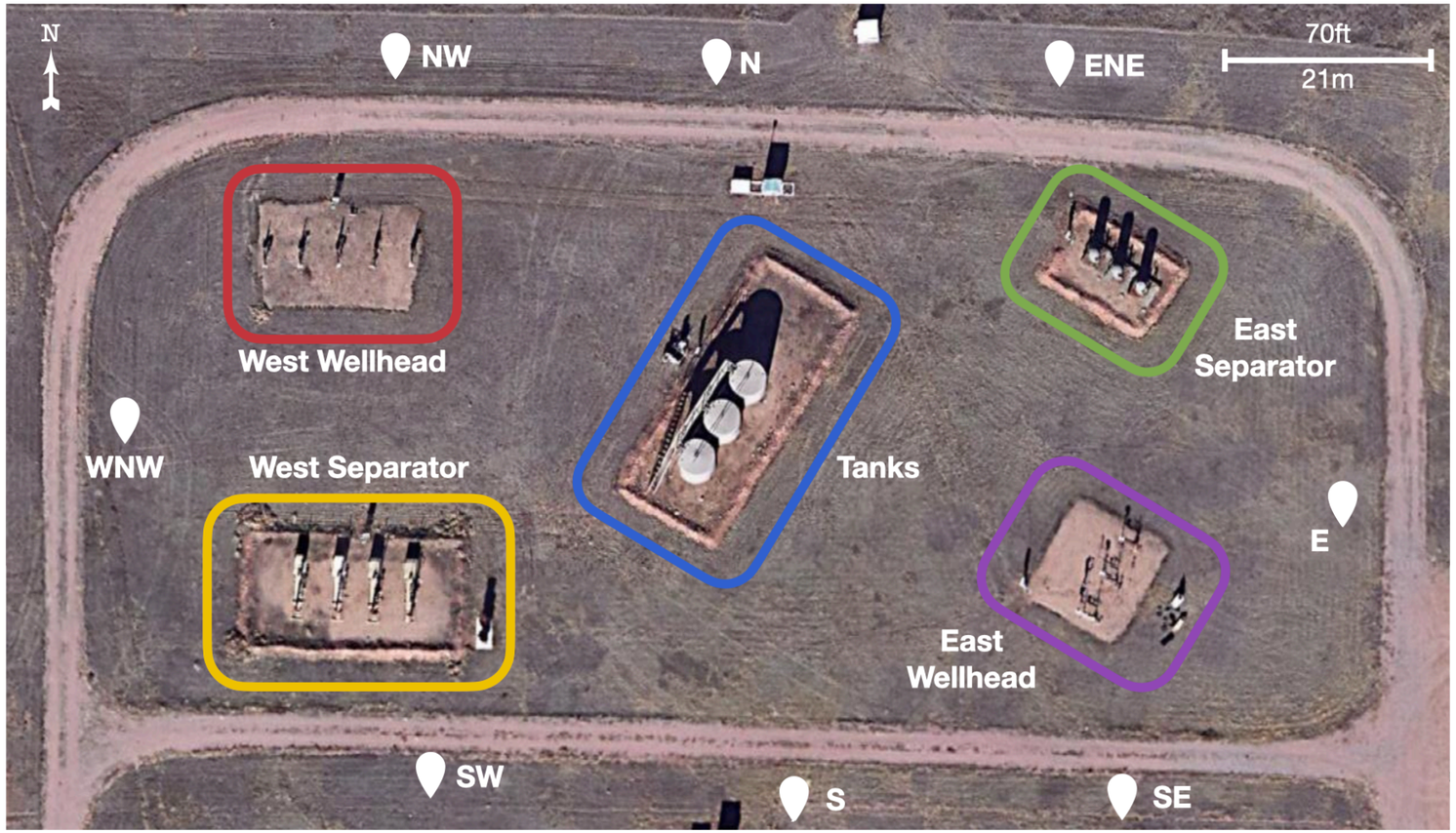
The University of Texas at Austin
Cockrell School of Engineering

See Andres Pruet's poster for more details!

"Integral spectral method for advection-diffusion in 3D"

Direction #2: Better estimates of dispersion

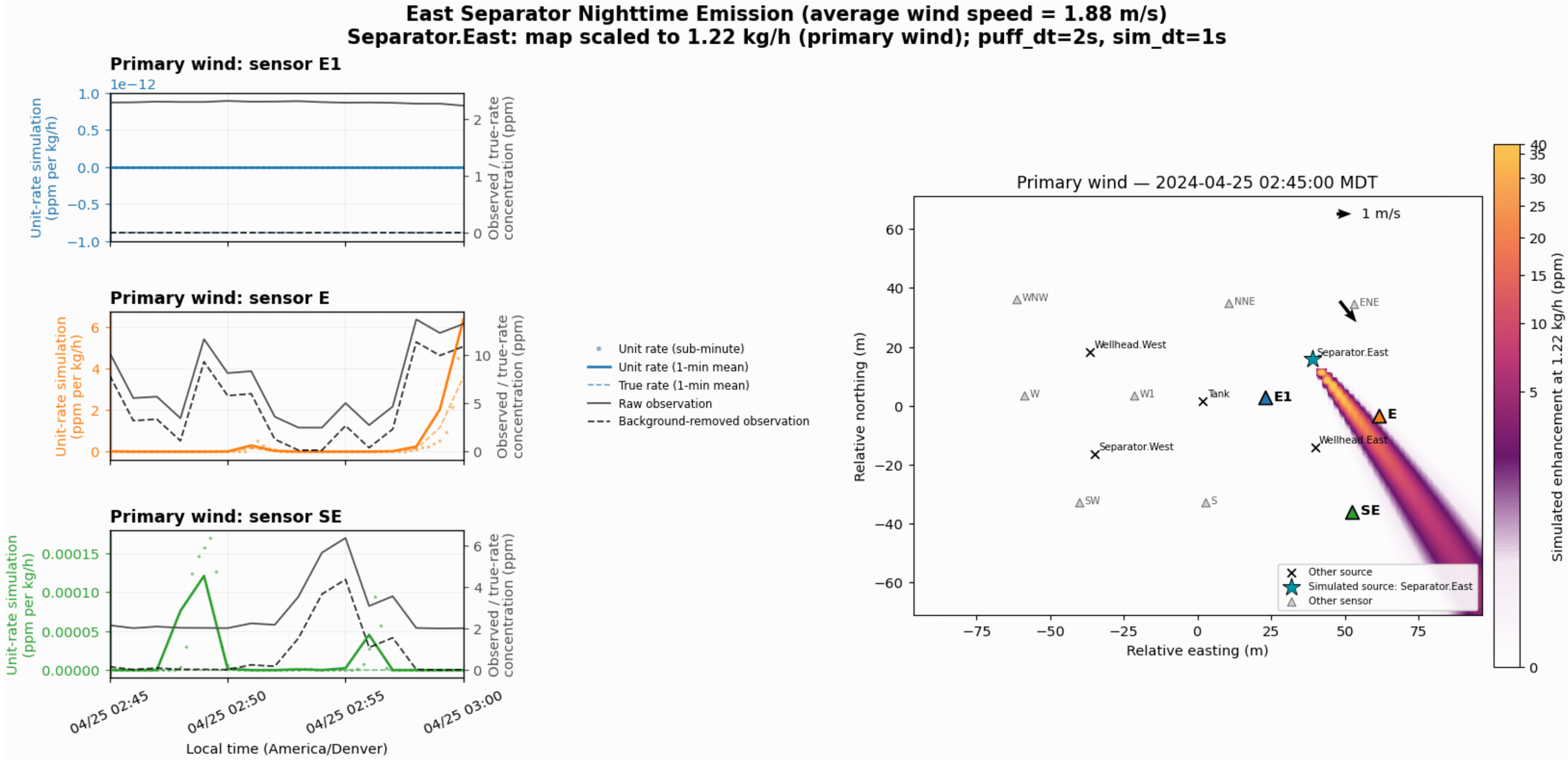
Motivating example: test the Gaussian puff in situations where we know what should be happening



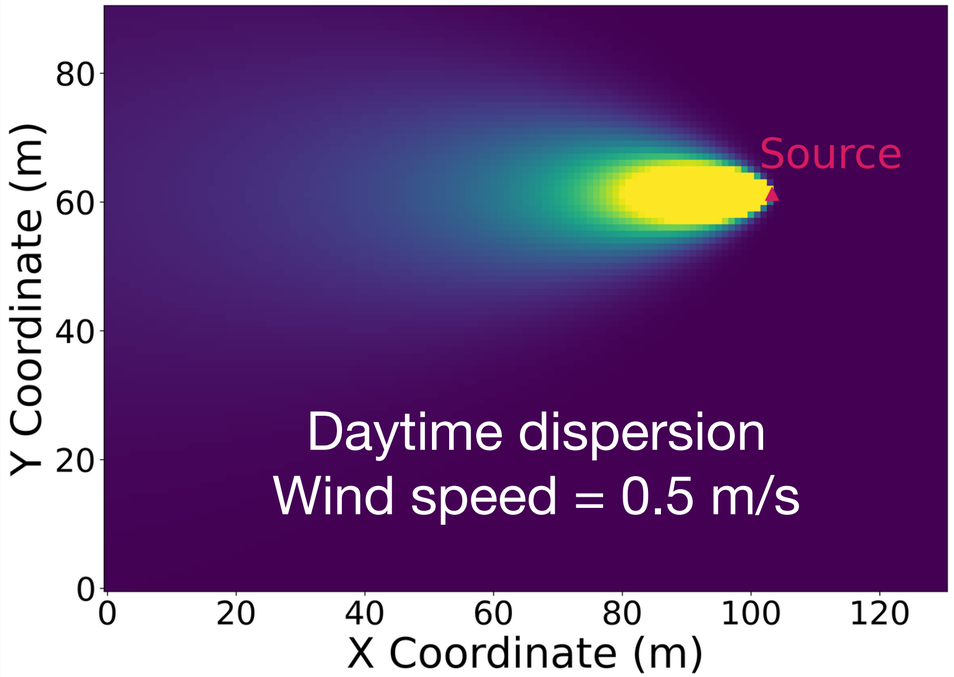
Methane Emissions Technology Evaluation Center (METEC)

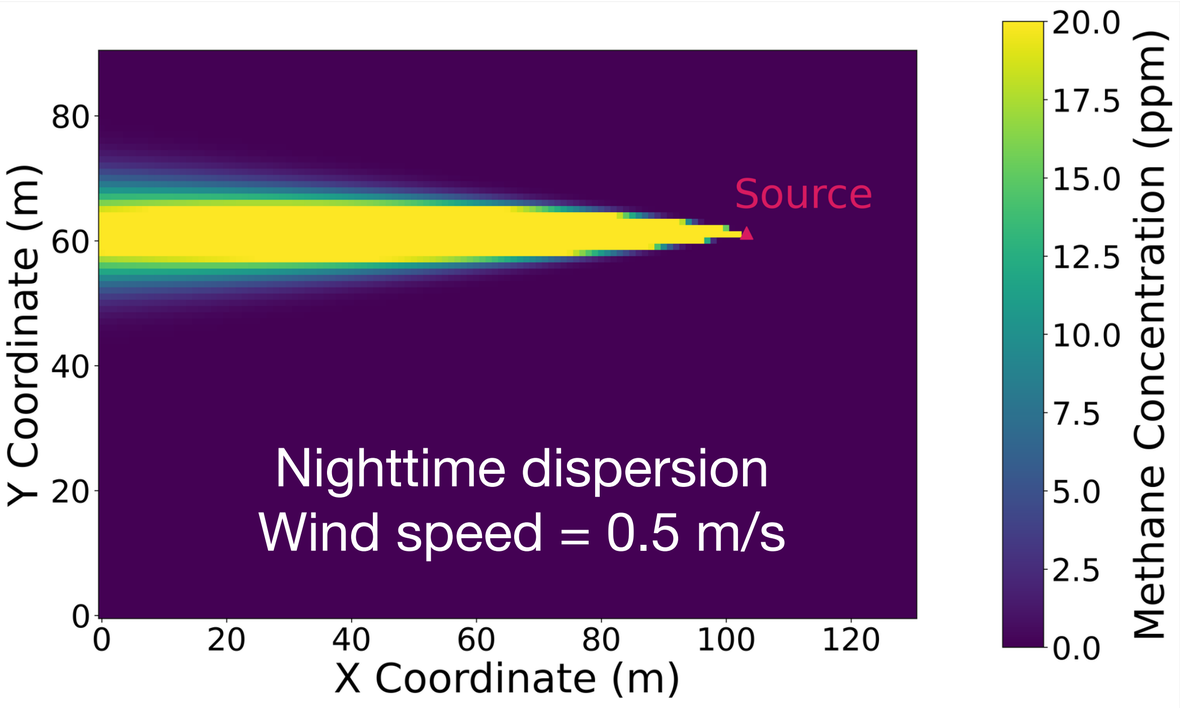
- Focus on controlled releases where just one source is emitting, sensors observed enhancements, but the Gaussian puff did not simulate enhancements

Direction #2: Better estimates of dispersion



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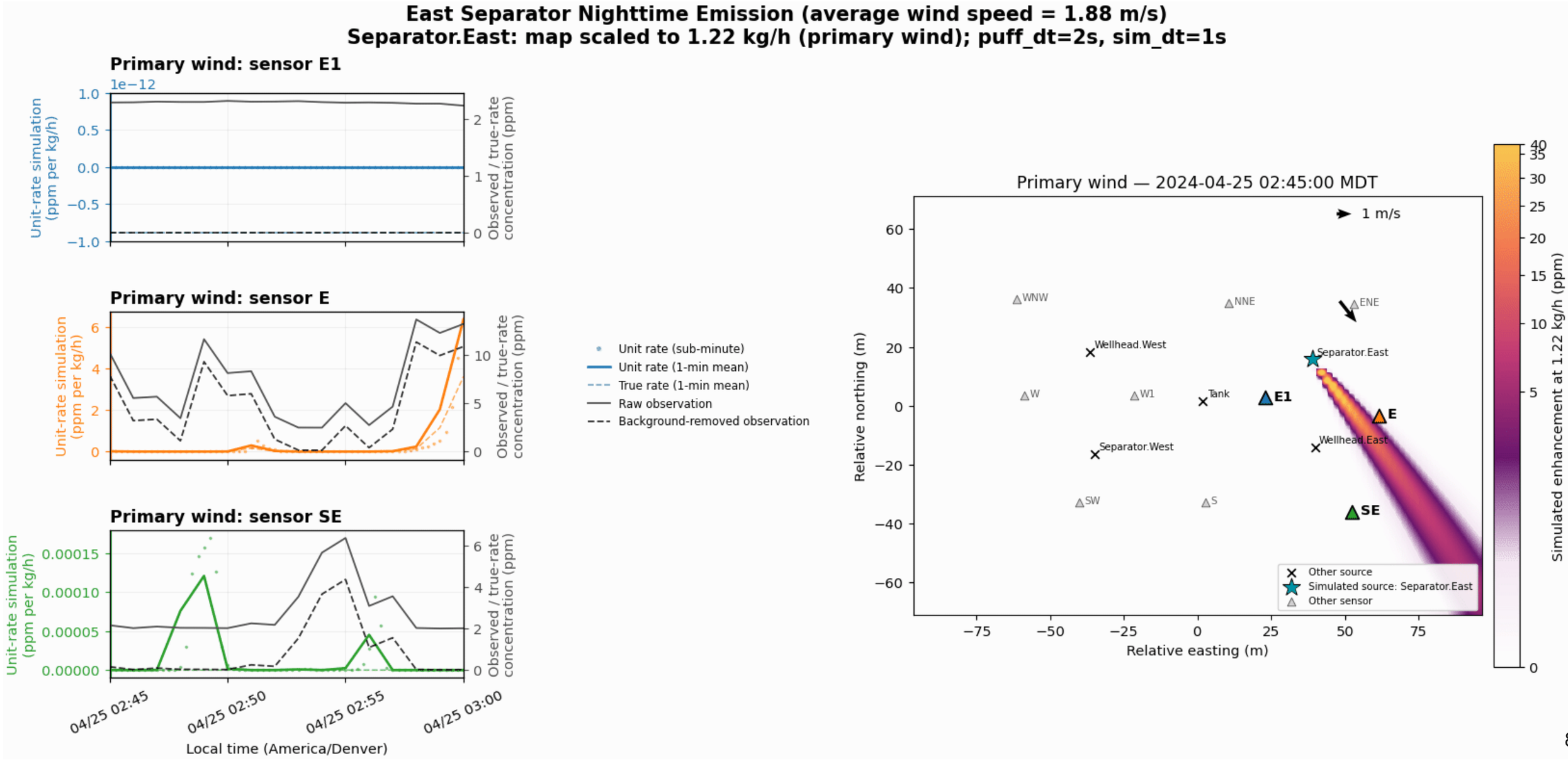




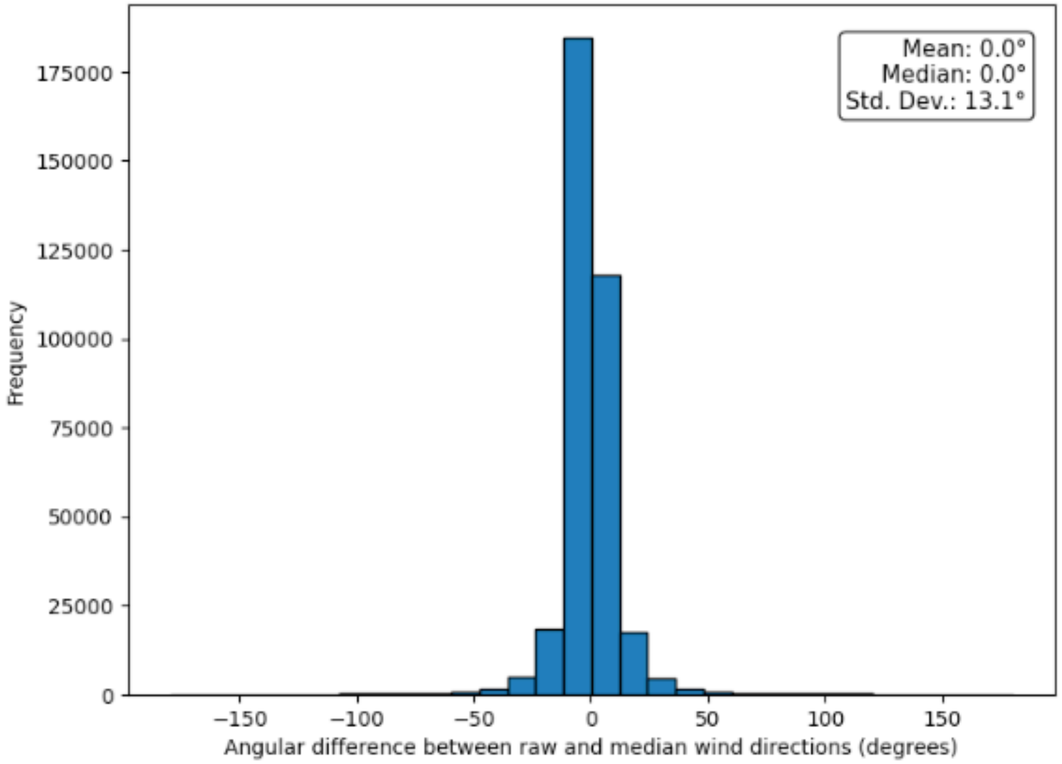
$$c_p(x, y, z, t, Q) = \frac{Q}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Can we optimize the dispersion coefficients based on the data?

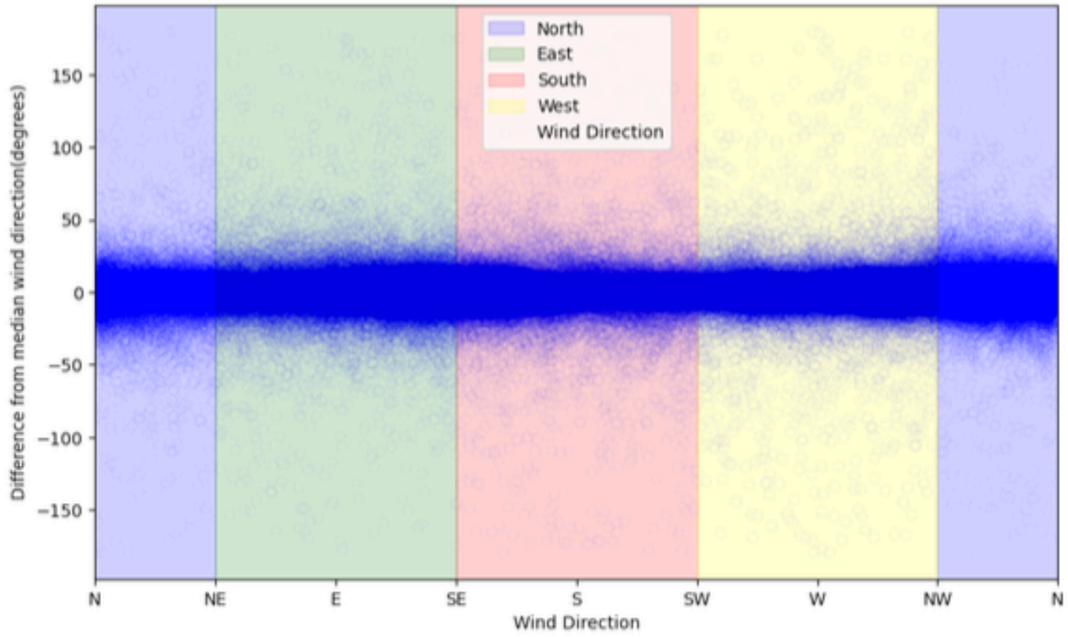
Direction #3: Better method for aggregating wind data



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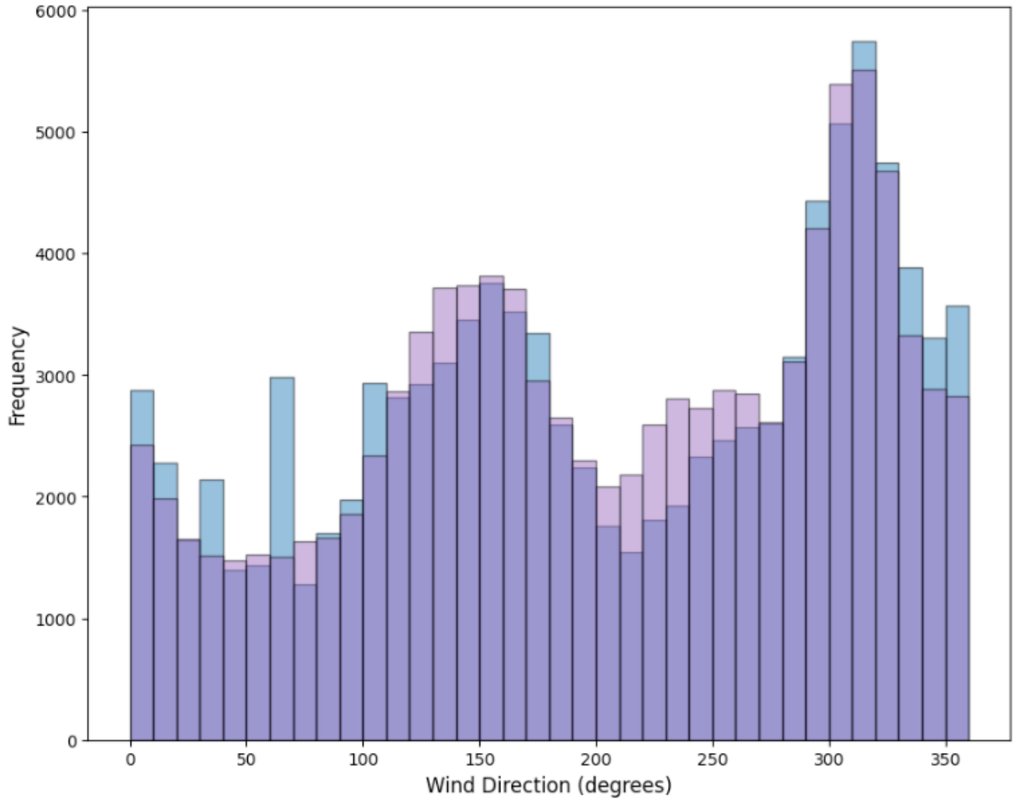


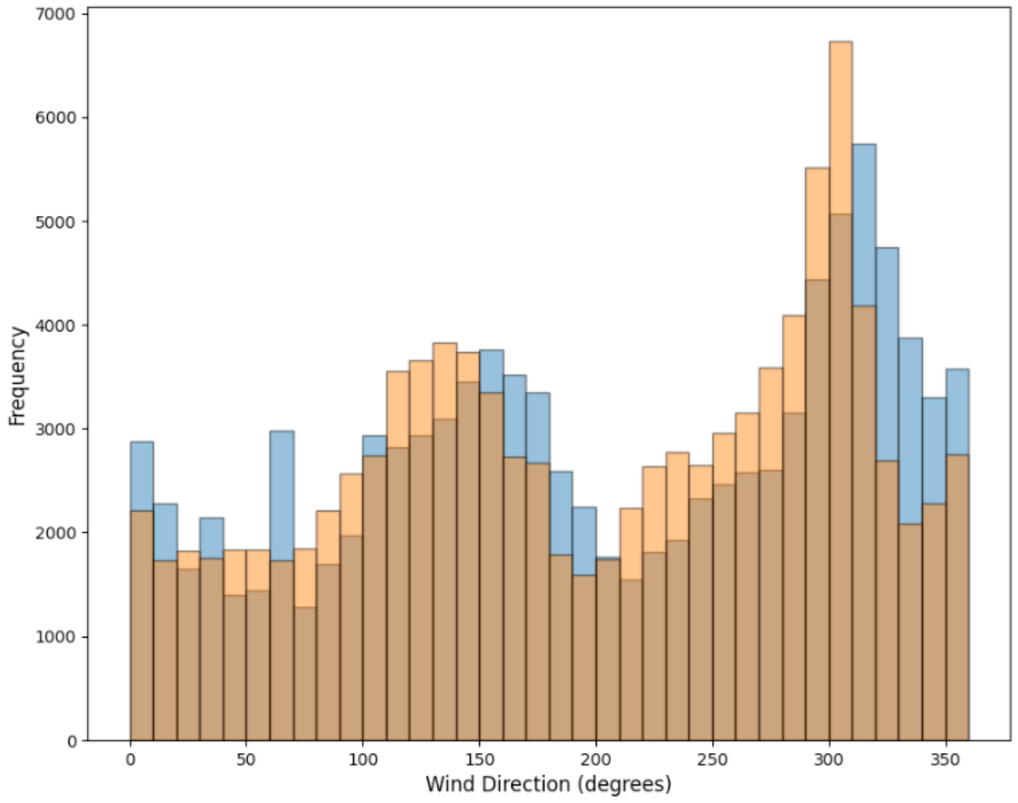
(a) Histogram of Angular difference between the median and raw wind directions.



(b) Plot of angular difference between the median and raw wind directions, broken out by raw wind direction.

Direction #3: Better method for aggregating wind data





Reference sensor

CMS anemometer #1

CMS anemometer #2

Improving the Gaussian puff model is an active area of research

1. Obstacle-aware dispersion model that runs fast **[Andres]**
2. Better estimates of dispersion during nighttime and low wind **[Michael]**
3. More accurate method for aggregating high frequency wind measurements **[Andy]**

Andres Pruet
Colorado School of Mines



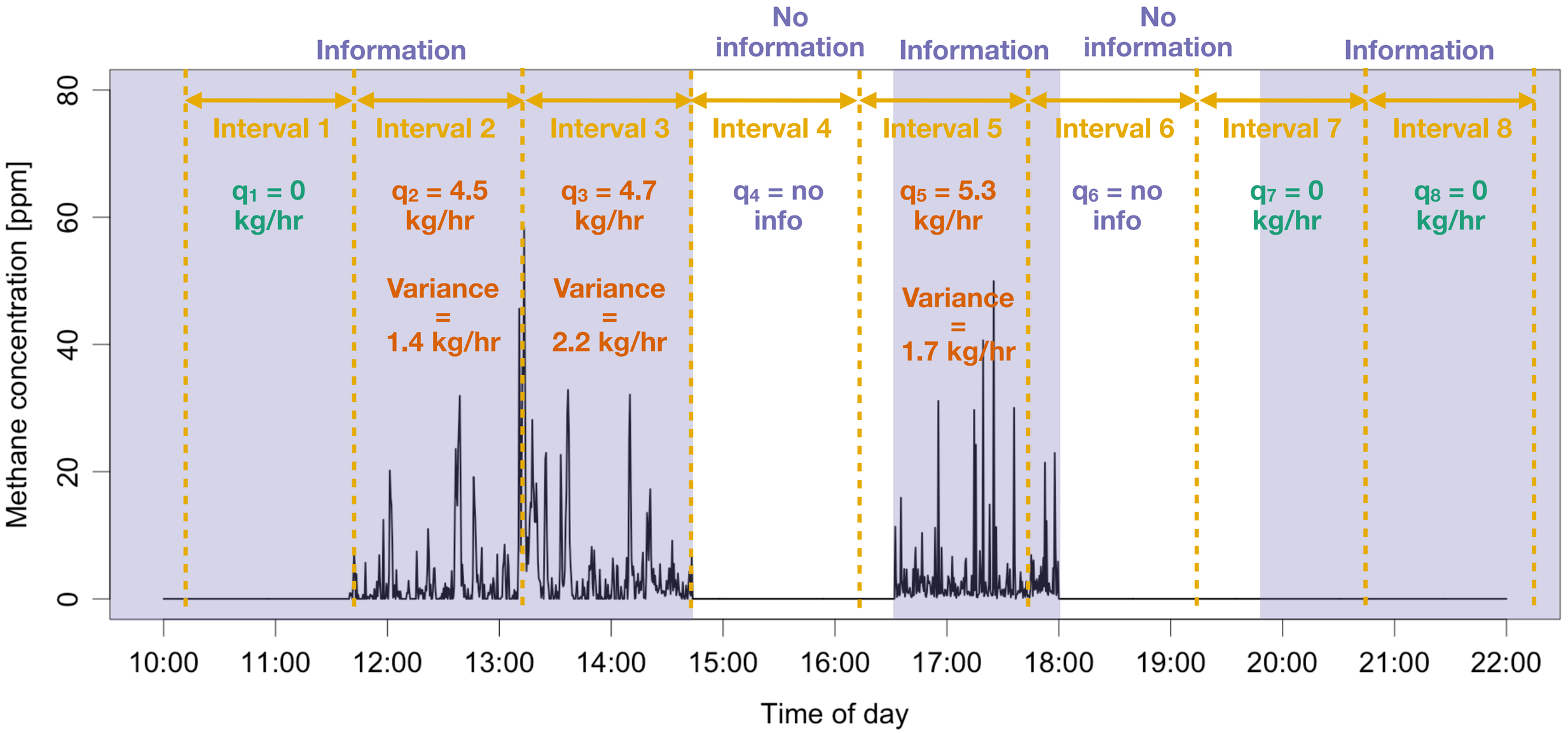
Michael Basanese
Colorado School of Mines



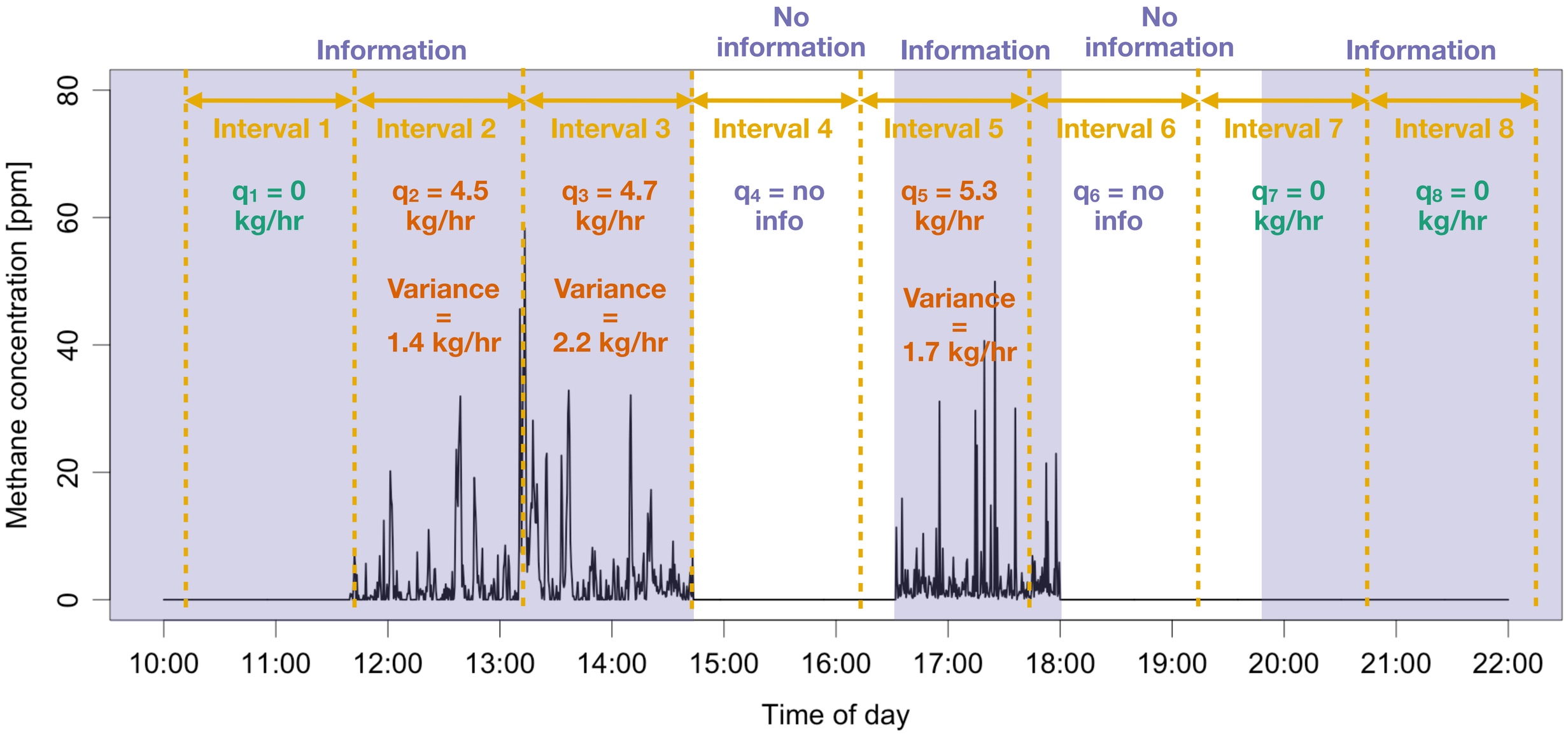
Andy Holmberg
Colorado School of Mines



Whether an interval is no information, zero emission rate, or non-zero emission rate depends on the data.



How do you turn these estimates into a measurement-derived inventory?



Thank you!
Questions?

Info vs. no info paper

Gaussian puff paper

Single source (DLQ) paper

Multisource (MDLQ) paper



Johns Hopkins University

EEMDL
Energy Emissions Modeling and Data Lab

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