Characterizing methane emissions on oil and gas sites

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Agenda

Introduction:

Methane emissions, oil and gas sites, measurement technologies

Chapter 1: Single-source emission detection, localization, and quantification

> Chapter 2: Reconciling aerial measurements and bottom-up inventories



Chapter 4: Multi-source emission detection, localization, and quantification

Chapter 5: Robust duration estimates



Methane is a potent greenhouse gas



GWP of 84x CO₂ over a 20 year period

GWP of 28x CO₂ over a 100 year period





Methane is a potent greenhouse gas

Global concentrations are increasing





GWP of 84x CO₂ over a 20 year period

GWP of 28x CO₂ over a 100 year period





Methane is a potent greenhouse gas

Global concentrations are increasing

Relatively short lifetime

CH4 lifetime = 9 years CO2 lifetime = **300-1000** years

> Effect of CH4 emission reductions will be felt within our lifetimes!



GWP of 84x CO₂ over a 20 year period

GWP of 28x CO₂ over a 100 year period





Methane is a potent greenhouse gas

- Global concentrations are increasing
- Relatively short lifetime
- Recent regulatory push

United States

H. R. 5376 (Inflation Reduction Act)

SEC. 136. (a) The Administrator shall impose and collect a fee from the owner or operator of **each applicable facility** that is required to report methane emissions ...

SEC. 136. (g)(2) ... calculation of fees under subsection (c) of this section, are based on **empirical data** and accurately reflect the total methane emissions from the applicable facilities.





Methane is a potent greenhouse gas

Global concentrations are increasing

Relatively short lifetime

Recent regulatory push

Amendments adopted by the European Parliament on 9 May 2023 on the proposal for a regulation of the European Parliament

... importers must provide a report with the following information for **each site** from which the import to the Union has taken place ...

... information specifying the
exporter's, or where relevant, the
producer's direct measurements of
site-level methane emissions,
conducted by independent service
provider ...

European Union



Methane is a potent greenhouse gas

Global concentrations are increasing

Relatively short lifetime

Recent regulatory push

The Oil & Gas Methane Partnership 2.0 (OGMP 2.0)

Level 5 — Emissions reported similarly to Level 4, but with the addition of **site-level measurements** (measurements that characterize site-level emissions distribution for a statistically representative population)

Global Initiatives











Example production oil and gas site



Example production oil and gas site



100 ft

Continuous monitoring system (CMS)



Example production oil and gas site



100 ft

Continuous monitoring system (CMS)

















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Chapter 1: Single-source emission detection, localization, and quantification







The continuous monitoring inverse problem





Separator



Tank













Separator









Wind direction







Open source framework for solving inverse problem







Methane Concentration [ppm]



























Open source framework for solving inverse problem







Gaussian puff atmospheric dispersion model







Gaussian puff atmospheric dispersion model















Repeat this for all other potential sources!







Open source framework for solving inverse problem

















Wind direction























Wind direction





















Simulation emission source

Wind direction












(((q))







Simulation emission source













Open source framework for solving inverse problem







Simulation is a linear function of emission rate

Volume of methane contained in puff *p*

$$c_p(x, y, z, t, Q) = Q \left[\frac{1}{(2\pi)^{3/2} \sigma_y^2 \sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right) \left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) \right] \left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) \right] \left[\exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{$$

contribution of puff *p*

$$c(x, y, z, t, Q) = \sum_{p=1}^{P} c_p(x, y, z, t, Q)$$

Total concentration
at (x, y, z, t)





Simulation is a linear function of emission rate

Volume of methane contained in puff *p*

$$c_p(x, y, z, t, Q) = Q \left[\frac{1}{(2\pi)^{3/2} \sigma_y^2 \sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right) \left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right) \right] \left[\exp\left(-\frac{(x-ut)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(x-ut)^2}{2\sigma_z^2}\right) \right] \left[\exp\left(-\frac{(x-ut)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(x-u$$

contribution of puff *p*

$$c(x, y, z, t, Q) = \sum_{p=1}^{P} c_p(x, y, z, t, Q)$$

Total concentration
at (x, y, z, t)























Open source framework for solving inverse problem







Evaluation on single-source controlled releases



Methane Emissions Technology Evaluation Center (METEC)

85 single-source controlled releases

Emission rates range from 0.2 to 6.4 kg/hr

Emission durations range from 0.5 to 8.25 hours





Evaluation on single-source controlled releases

Event-level false positive rate: 5.5%





Evaluation on single-source controlled releases





Chapter 1:

Single-source emission detection, localization, and quantification

Concluding thoughts:

Framework is already being used by some CMS technology vendors.

Detection, localization, and quantification of single-source methane emissions on oil and gas production sites using point-in-space continuous monitoring systems. William Daniels, Meng Jia, Dorit Hammerling. Elementa: Science of the Anthropocene, 12 (1), 00110, (2024).

Filling a critical need: a lightweight and fast Gaussian puff model implementation. Meng Jia, Ryker Fish, William Daniels, Brennan Sprinkle, Dorit Hammerling. Scientific Reports, in revision, (2024).



Chapter 2:

Reconciling aerial measurements and bottom-up inventories











CMS sensor

Bottom-up top-down reconciliation case study





CMS sensor





























Snapshot average Company emissions inventory (shown as bars)

Average snapshot measurement









 ∞

















- Company emissions inventory (shown as bars)
- CMS-based inventory estimate



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- Company emissions inventory (shown as bars)
- CMS-based inventory estimate



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CMS can explain the gap between bottom-up inventory and top-down aerial measurements





- Company emissions inventory (shown as bars)
- CMS-based inventory estimate



 \bigcirc







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Chapter 2:

Reconciling aerial measurements and bottom-up inventories

Concluding thoughts:

- Multi-scale measurements are complementary.

Towards multiscale measurement-informed methane inventories: reconciling bottom-up site-level inventories with top-down measurements using continuous monitoring systems. William Daniels, Jiayang (Lyra) Wang, Arvind Ravikumar, Matthew Harrison, Selina Roman-White, Fiji George, Dorit Hammerling. *Environmental Science and Technology*, 57(32), 11823-11833, (2023).

Multi-scale methane measurements at oil and gas facilities reveal necessary framework for improved emissions accounting.

Jiayang (Lyra) Wang, William Daniels, Dorit Hammerling, Matthew Harrison, Kaylyn Burmaster, Fiji George, Arvind Ravikumar. *Environmental Science and Technology,* 56(20), 14743-14752, (2022).

• Measurements at high temporal resolution are valuable, especially for site-level analysis.



Chapter 3: Intercomparison of CMS solutions













Wellhead







Sensor intercomparison setup







Finding #1: Raw concentration data different between co-located sensors





Finding #2: Quantification estimates vary dramatically at the 30-minute scale







Finding #2: Quantification estimates vary dramatically at the 30-minute scale









Finding #3: Quantification estimates begin to align at the month-scale



from the CMS vendors



Finding #3: Quantification estimates begin to align at the month-scale



from the CMS vendors



Emission rate estimates from the **DLQ algorithm**



Finding #4: Similar sites do not necessarily have similar emissions







Chapter 3:

Intercomparison of CMS solutions

Concluding thoughts:

- Important to assess CMA performance in the field in addition to controlled releases.
- scales.

Intercomparison of three continuous monitoring systems on operating oil and gas sites. William Daniels*, Spencer Kidd*, Lydia (Shuting) Yang, Shannon Stokes, Arvind Ravikumar, Dorit Hammerling. ACS ES&T Air, in press, (2024).

• Current CMS solutions may be more useful when data is aggregated at hourly or monthly



Chapter 4:

Multi-source emission detection, localization, and quantification





CMS sensor "Continuous monitoring system"





The multi-source continuous monitoring inverse problem















Model hierarchy

Assume a multiple linear regression model at the data level

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \dots, y_n\}, \beta \equiv \{\beta_1, \dots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$
Concentration
observations
from CMS sensors
$$Simulat
from for
each source$$

n = number of observations p = number of potential sources

$$= X\beta + \epsilon$$

ted concentrations orward model, with column assuming a different source





Model hierarchy

Assume a multiple linear regression model at the data level

$$y \equiv \{y_1, \dots, y_n\}, \beta \equiv \{\beta_1, \dots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

n = number of observations p = number of potential sources

 $v = X\beta + \epsilon$

 $\epsilon \sim N(0,\sigma^2 R)$



Model hierarchy

Assume a multiple linear regression model at the data level

$$y \equiv \{y_1, \dots, y_n\}, \beta \equiv \{\beta_1, \dots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

Let the errors follow an AR(1) process such that



n = number of observations p = number of potential sources

 $v = X\beta + \epsilon$




Assume a multiple linear regression model at the data level

$$y \equiv \{y_1, \dots, y_n\}, \beta \equiv \{\beta_1, \dots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

 $\epsilon \sim N(0,\sigma^2 R)$

Let the errors follow an AR(1) process such that

This gives us: $y \sim N(X\beta, \sigma^2 R)$

n = number of observations p = number of potential sources

 $v = X\beta + \epsilon$

 $\epsilon_i = r\epsilon_{i-1} + w$





Given an AR(1) process for ϵ , the correlation matrix is



n = number of observations p = number of potential sources







Given an AR(1) process for ϵ , the correlation matrix is

$$R = \begin{bmatrix} 1 \\ r \\ r^2 \\ \vdots \\ r^{n-1} \end{bmatrix}$$

which has closed form expressions for the inverse and determinant:

$$R^{-1} = \frac{1}{(1 - r^2)} \begin{bmatrix} 1 & -r & 0 & \dots & 0 \\ -r & 1 + r^2 & -r & \dots & \vdots \\ 0 & -r & 1 + r^2 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 1 \end{bmatrix}$$

n = number of observations p = number of potential sources



and
$$|R| = (1 - r^2)^{n-1}$$





Data-level:

$$y = X\beta + \epsilon$$

$$\epsilon \sim N(0, \sigma^2 R)$$

The remainder of the hierarchy takes the following form

Spike-and-slab prior allows samples to be identically zero

Proportion of samples where $z_i = 1$ gives posterior probability that source *i* is emitting

 $z_i \sim \text{Bernoulli}(\theta_i)$ $\theta_i \sim \text{Beta}(a_i, b_i) \blacktriangleleft$ n = number of observations p = number of potential sources

 $\mathbf{P}_{i} \sim \begin{cases} 0, & z_{i} = 0 \\ \operatorname{Exp}(\tau_{i}^{2}\sigma^{2}), & z_{i} = 1 \end{cases}$ "Slab" component is non-negative a_i, b_i, c_i, d_i can $\tau_i^2 \sim \text{Inv-Gamma}(c_i, d_i) \blacktriangleleft$ contain $\sigma^2 \sim \text{Inv-Gamma}(\nu/2, \nu/2)$ operator insight $\nu \sim \text{Inv-Gamma}(\alpha_1, \alpha_2)$ $r \sim \text{Uniform}(0, 1)$



$$\beta_i \sim \begin{cases} 0, & z_i = 0\\ \operatorname{Exp}(\tau_i^2 \sigma^2), & z_i = 1 \end{cases}$$
$$z_i \sim \operatorname{Bernoulli}(\theta_i)$$
$$\theta_i \sim \operatorname{Beta}(a_i, b_i)$$
$$\tau_i^2 \sim \operatorname{Inv-Gamma}(c_i, d_i)$$
$$\sigma^2 \sim \operatorname{Inv-Gamma}(\nu/2, \nu/2)$$
$$\nu \sim \operatorname{Inv-Gamma}(\alpha_1, \alpha_2)$$
$$r \sim \operatorname{Uniform}(0, 1)$$





Sampling from the posterior

We can derive Gibbs updates for all parameters except ν .

$$\begin{split} \theta_{i} | \xi \sim & \text{Beta}(z_{i} + a_{i}, 1 - z_{i} + b_{i}) \\ \sigma^{2} | \xi \sim & \text{Inv-Gamma} \left(\frac{\nu}{2} + \frac{n}{2}, \frac{\nu}{2} + \frac{1}{2} (y - X\beta)^{T} R^{-1} (y - X\beta) \right) \\ r | \xi \sim & \left\{ \mathcal{N}(X\beta, \sigma^{2}R) & 0 < r < 1 \\ 0 & \text{otherwise} \end{array} \right. \\ \tau_{i}^{2} | \xi \sim & \text{Inv-Gamma} \left(z_{i} + c_{i}, \frac{\beta_{i}}{\sigma^{2}} + d_{i} \right) \\ \beta_{i} | \xi \sim & \left\{ \begin{array}{c} 0 & z_{i} = 0 \\ \mathcal{N} \left(\left(\frac{X^{T} R^{-1} X}{\sigma^{2}} \right)^{-1} \left(\frac{X^{T} R^{-1} y}{\sigma^{2}} - \frac{e_{i}}{\tau_{i}^{2} \sigma^{2}} \right), \left(\frac{X^{T} R^{-1} X}{\sigma^{2}} \right)^{-1} \right) \\ z_{i} = 1 \end{array} \right. \\ z_{i} | \xi \sim & \text{Bernoulli} \left(1 - \frac{1 - \theta_{i}}{\left(1 - \theta_{i} \right) + \theta_{i} \left(\frac{1}{\tau_{i}^{2} \sigma^{2}} \right) \exp \left(\frac{\left(\frac{\sum_{j=1}^{n} (w_{j} X_{j,i}^{*} + w_{j}^{*} X_{j,i}) - \frac{2}{\tau_{i}^{2}} \right)^{2}}{4\sigma^{2} \sum_{j=1}^{n} X_{j,i} X_{j,i}^{*}} \right) \left(\frac{2\sigma^{2}\pi}{\sum_{j=1}^{n} X_{j,i} X_{j,i}^{*}} \right)^{1/2} \left(\frac{1}{2} \right) \\ \mathcal{N} \left(\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{2\sigma^{2}\pi}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{2\sigma^{2}\pi}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{2\sigma^{2}\pi}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{2\sigma^{2}\pi}{2} \right)^{1/2} \left(\frac{1}{2} \right) \\ \mathcal{N} \left(\frac{1}{2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{2\sigma^{2}\pi}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{2\sigma^{2}\pi}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right) \right$$

 $\nu | \xi \sim ?$ (Use a Metropolis–Hastings step)

es from each al gives you m the joint rior!





Methane Emissions Technology Evaluation Center (METEC)

337 controlled releases:

- 99 (29%) single-source
- 238 (71%) multi-source

Emission rates range from **0.08** to **7.2** kg/hr

Emission durations range from **0.5** to **8** hours







For each controlled release, replace actual concentration observations with

Vary the degree of autocorrelation

where β_T are the true emission rates and are errors that follow an AR(1) process.

$$\tilde{y} = X\beta_T + \tilde{\epsilon}$$





Vary the degree of autocorrelation

where β_T are the true emission rates and are errors that follow an AR(1) process.



For each controlled release, replace actual concentration observations with

$$\tilde{y} = X\beta_T + \tilde{\epsilon}$$





Vary the degree of autocorrelation

where β_T are the true emission rates and are errors that follow an AR(1) process.



For each controlled release, replace actual concentration observations with

$$\tilde{y} = X\beta_T + \tilde{\epsilon}$$















estimates per inversion window





Vary the degree of spike misalignment

20 Observations Concentration [ppm] Background-removed observations 15 Gaussian puff 10 S 20 40 0

For each controlled release, replace actual concentration observations with

$$\tilde{y} = X\beta_T + \tilde{\epsilon}$$

but move a given percent of the spikes in the fake observations to a different time during the release.





Vary the degree of spike misalignment



For each controlled release, replace actual concentration observations with

$$\tilde{y} = X\beta_T + \tilde{\epsilon}$$

but move a given percent of the spikes in the fake observations to a different time during the release.



Vary the degree of spike misalignment



For each controlled release, replace actual concentration observations with

$$\tilde{y} = X\beta_T + \tilde{\epsilon}$$

but move a given percent of the spikes in the fake observations to a different time during the release.





Chapter 4: Multi-source emission detection, localization, and quantification

To do before submission:

• Finish comparison to the other methods

A Bayesian hierarchical model for methane emission source apportionment. William Daniels, Douglas Nychka, Dorit Hammerling. *Journal of the American Statistical Association,* in preparation, (2024).



Chapter 5: **Robust duration estimates**



A policy driven research project



ENVIRONMENTAL PROTECTION AGENCY

40 CFR Part 98

[EPA-HQ-OAR-2023-0234; FRL-10246-01-OAR]

RIN 2060–AV83

Greenhouse Gas Reporting Rule: **Revisions and Confidentiality Determinations for Petroleum and** Natural Gas Systems

AGENCY: Environmental Protection Agency (EPA).

ACTION: Proposed rule.

SUMMARY: The Environmental Protection Agency (EPA) is proposing to amend requirements that apply to the petroleum and natural gas systems source category of the Greenhouse Gas Reporting Rule to ensure that reporting is based on empirical data, accurately reflects total methane emissions and waste emissions from applicable facilities, and allows owners and operators of applicable facilities to submit empirical emissions data that appropriately demonstrate the extent to which a charge is owed. The EPA is also proposing changes to requirements that

Federal eRulemaking Portal. EPA may publish any comment received to its public docket. Do not submit to www.regulations.gov (our preferred the EPA's docket at www.regulations.gov any information Mail: U.S. Environmental Protection you consider to be confidential business Agency, EPA Docket Center, Air and information (CBI), proprietary business Radiation Docket, Mail Code 28221T, information (PBI), or other information whose disclosure is restricted by statute. 1200 Pennsylvania Avenue NW, Multimedia submissions (audio, video, Washington, DC 20460. Hand Delivery or Courier (by etc.) must be accompanied by a written comment. The written comment is scheduled appointment only): EPA considered the official comment and Docket Center, WJC West Building, Room 3334, 1301 Constitution Avenue should include discussion of all points you wish to make. The EPA will NW, Washington, DC 20004. The Docket generally not consider comments or Center's hours of operations are 8:30 comment contents located outside of the a.m.–4:30 p.m., Monday-Friday (except primary submission (*i.e.*, on the web, Federal holidays). Instructions: All submissions received cloud, or other file sharing system). Commenters who would like the EPA to further consider in this rulemaking any relevant comments that they provided received may be posted without change on the 2022 Proposed Rule regarding to *www.regulations.gov/*, including any proposed revisions at issue in this personal information provided. For proposal must resubmit those comments to the EPA during this proposal's comments and additional information comment period. Please visit on the rulemaking process, see the www.epa.gov/dockets/commenting-epa-"Public Participation" heading of the dockets for additional submission SUPPLEMENTARY INFORMATION section of methods; the full EPA public comment this document. policy; information about CBI, PBI, or The virtual hearing, if requested, will multimedia submissions, and general be held using an online meeting guidance on making effective platform, and the EPA will provide comments.

method). Follow the online instructions for submitting comments. must include the Docket Id. No. for this proposed rulemaking. Comments detailed instructions on sending information on its website

Federal Register/Vol. 88, No. 146/Tuesday, August 1, 2023/Proposed Rules



A policy driven research project

40 CFR Part 98: Proposed updates to the EPA's Greenhouse Gas Reporting Program (GHGRP) to take effect January 2025

... also proposing a 100 kg/hr CH₄ emission threshold to align with the super-emitter response program proposed in the NSPS 0000b. These emissions are generally intermittent, with widely varying durations ...

... also proposing that reporters would provide the start date and time of the release, duration of the release, and the method used to determine the start date and time ... Oil and gas operators required to report all methane emissions > 100 kg/hr

For each of these emissions, the operator must estimate an emission duration



Methane Concentration [ppm]



























One problem... incomplete sensor coverage





One problem... incomplete sensor coverage





One problem... incomplete sensor coverage



CMS do not provide emission information when the wind blows between sensors





However, we can estimate when this happens!





Downwind region does not overlap with CMS sensors = period of "no information"

However, we can estimate when this happens!





Downwind region does overlap with CMS sensors = period of "information"

Probabilistic Duration Model Step 1: Identify naive events



12:00

13:00

14:00

10:00

11:00



Time of day





Probabilistic Duration Model Step 1: Identify naive events



12:00

10:00

11:00

13:00

14:00

Example: we want a duration estimate for naive event 1



Time of day









Probabilistic Duration Model

Step 2: Identify periods of information







Time of day







Probabilistic Duration Model

Step 4: Sample start and end times



$$\mathbb{P}_{i,j} = 1 - \frac{|q_i - q_j|}{P_{95}(q) - P_5(q)}$$

$$\mathbb{P}_{1,2} = 0.85$$
Information
No
rmation
No
rmation
Range of
possible
end times for
e event 1
Range of
possible
end times for
naive event 2
16:00 17:00 18:00 19:00 20:00 21:00 22:00

Time of day





Probabilistic Duration Model

Step 5: Compute distribution of durations



Emission event duration [hours]

Max of possible durations (8.2 hours)


Probabilistic Duration Model Mixture model of uniform distributions

We want the distribution of durations for naive event k.





Probabilistic Duration Model Mixture model of uniform distributions

We want the distribution of durations for naive event k.

First, consider the simplest case where there is zero probability of combining with neighboring events.

 $S_k \sim \text{Unif}(\cdot, \cdot)$ and $E_k \sim \text{Unif}(\cdot, \cdot)$

Here the durations are simply: $D_k = E_k - S_k \sim \text{Trap}(\cdot, \cdot, \cdot, \cdot)$.



Probabilistic Duration Model Mixture model of uniform distributions

We want the distribution of durations for naive event k.

First, consider the simplest case where there is zero probability of combining with neighboring events.

$$S_k \sim \text{Unif}(\cdot, \cdot)$$
 and $E_k \sim \text{Unif}(\cdot, \cdot)$

Here the durations are simply: $D_k = E_k - S_k \sim \text{Trap}(\cdot, \cdot, \cdot, \cdot)$.

Next, consider the situation with *n* preceding events and *m* subsequent events:

$$S_k \sim \sum_{i=1}^n \mathbb{P}_{k,i} S_i$$
 and $E_k \sim \sum_{j=1}^m \mathbb{P}_{k,j} E_j$

Again the durations are: $D_k = E_k - S_k \sim ?$



Case study: Bounding the duration of an aerial measurement

Aerial technology detects separator emission of 9.6 kg/hr





methane

Throw back to the multi-source evaluation



estimates per inversion window





Throw back to the multi-source evaluation - information filtered



estimates per inversion window





Chapter 5:

Robust duration estimates

Concluding thoughts:

- EPA interested in this methodology.
- Note sure if the WEC will survive, but Europe might implement something similar.

Estimating methane emission durations using continuous monitoring systems. William Daniels, Meng Jia, Dorit Hammerling. Environmental Science and Technology Letters, 11(11), 1187-1192, (2024).











