Advanced monitoring methods:

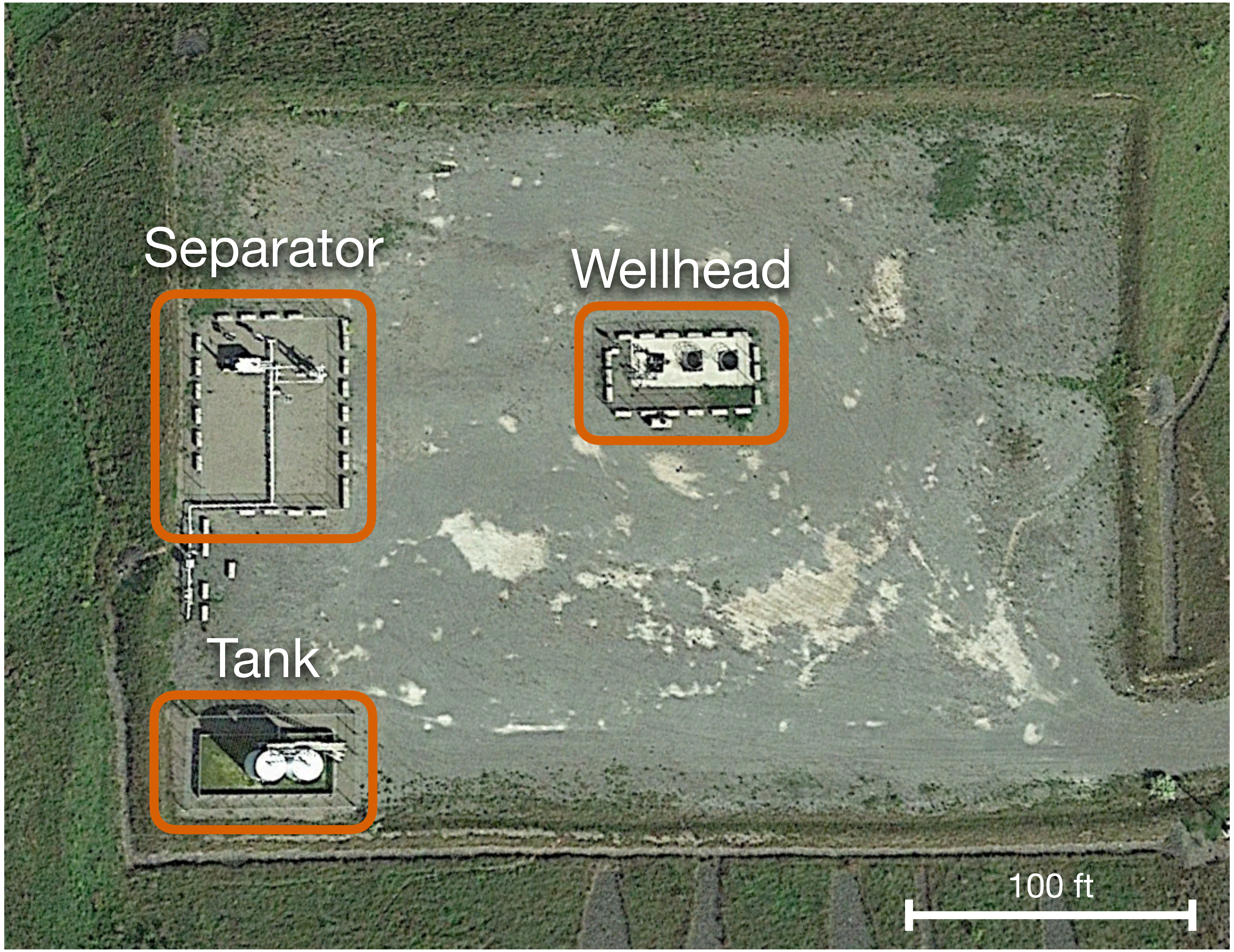
Estimating methane emission source and rate with continuous monitoring systems

William Daniels and the
Colorado School of Mines Team

Department of Applied Mathematics and Statistics

COLORADO SCHOOL OF MINES

Example production oil and gas site



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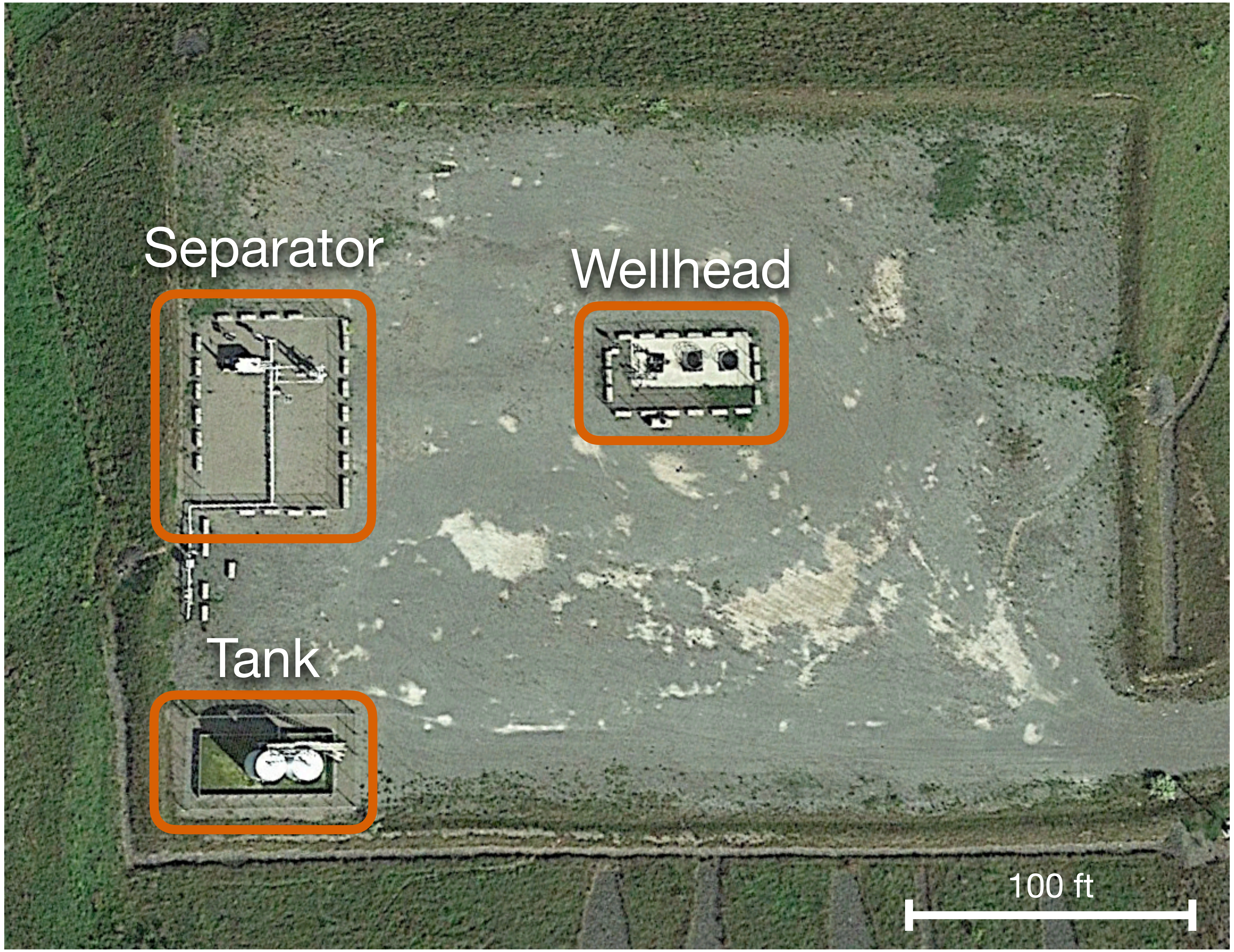
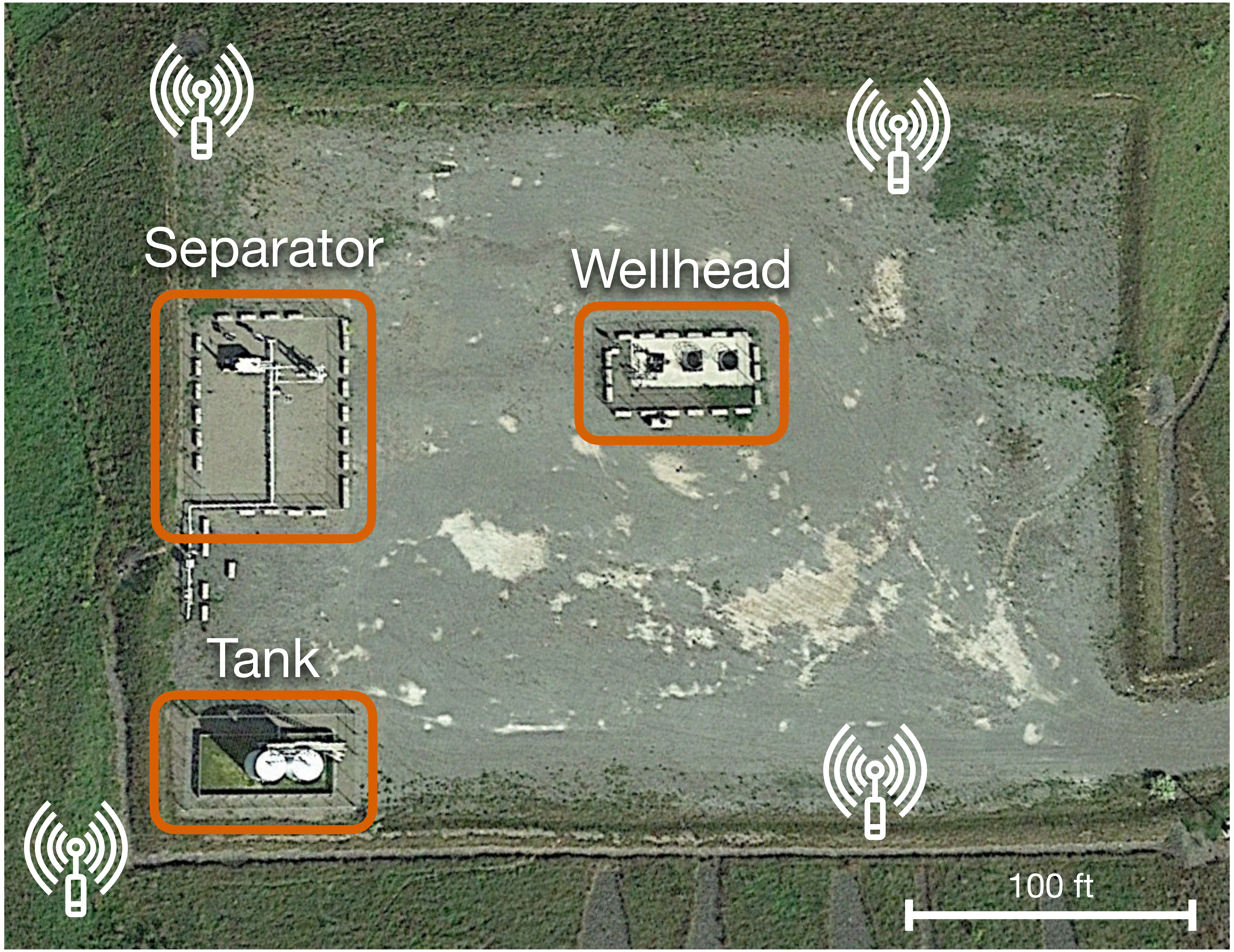


Continuous monitoring system (CMS)

Example production oil and gas site



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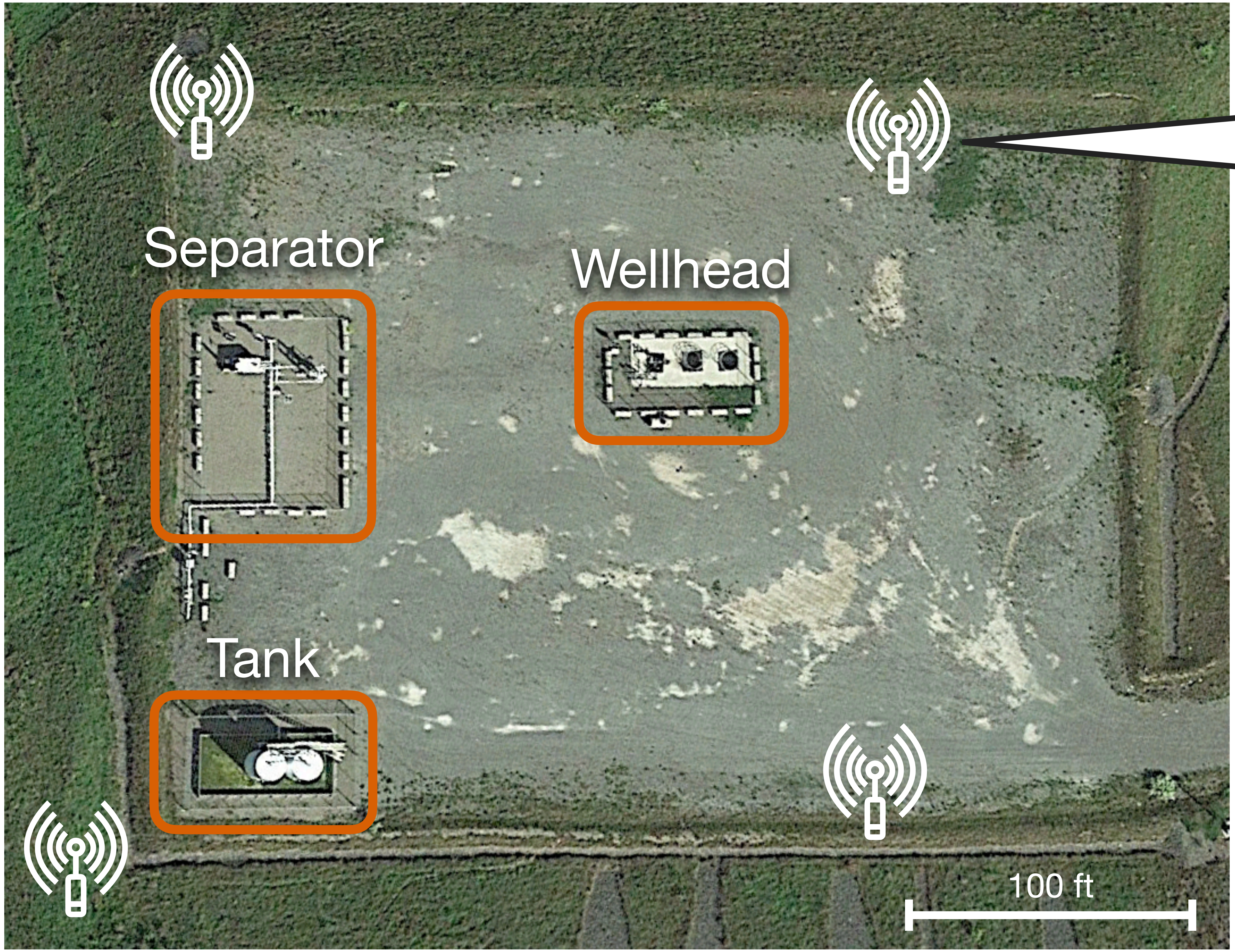
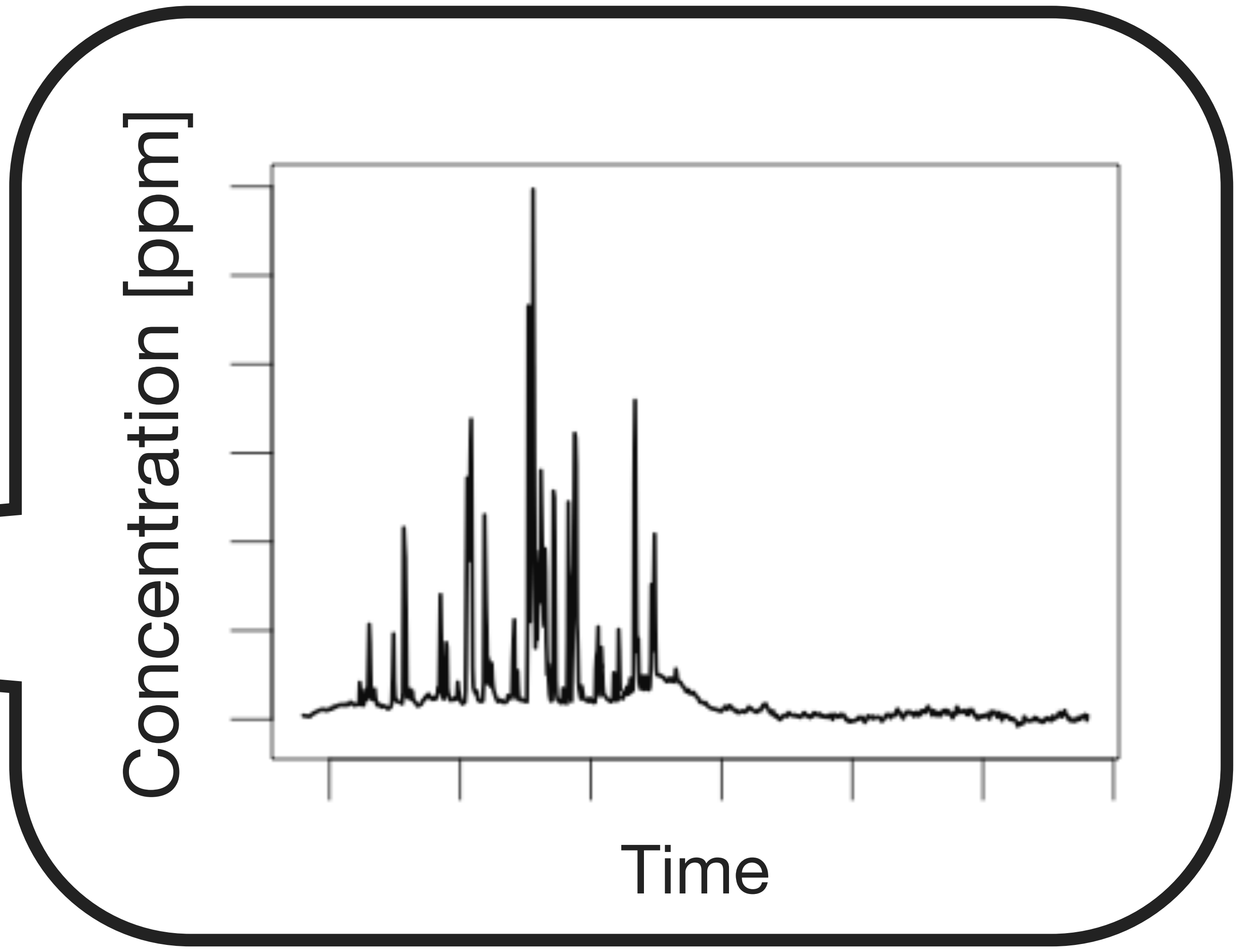
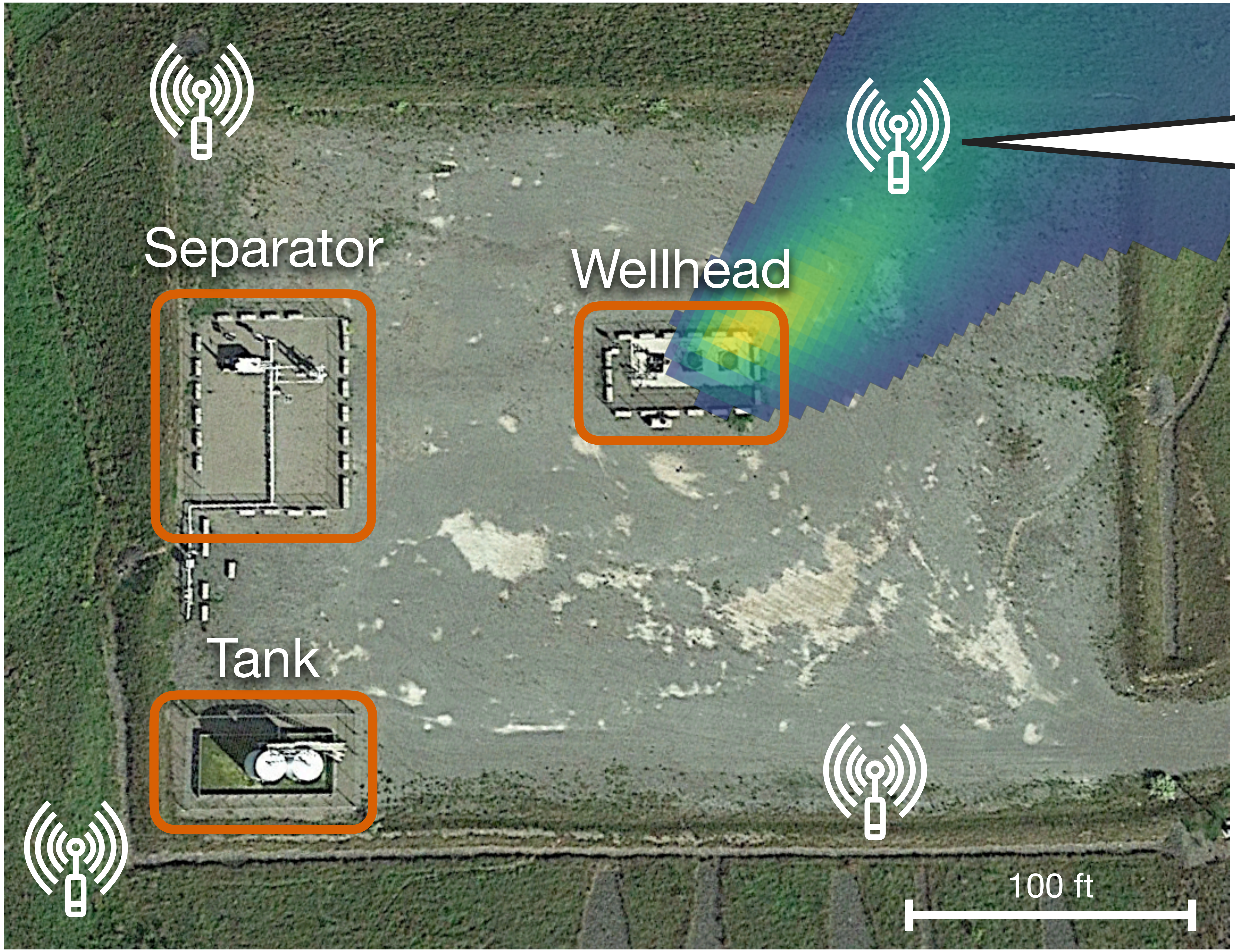
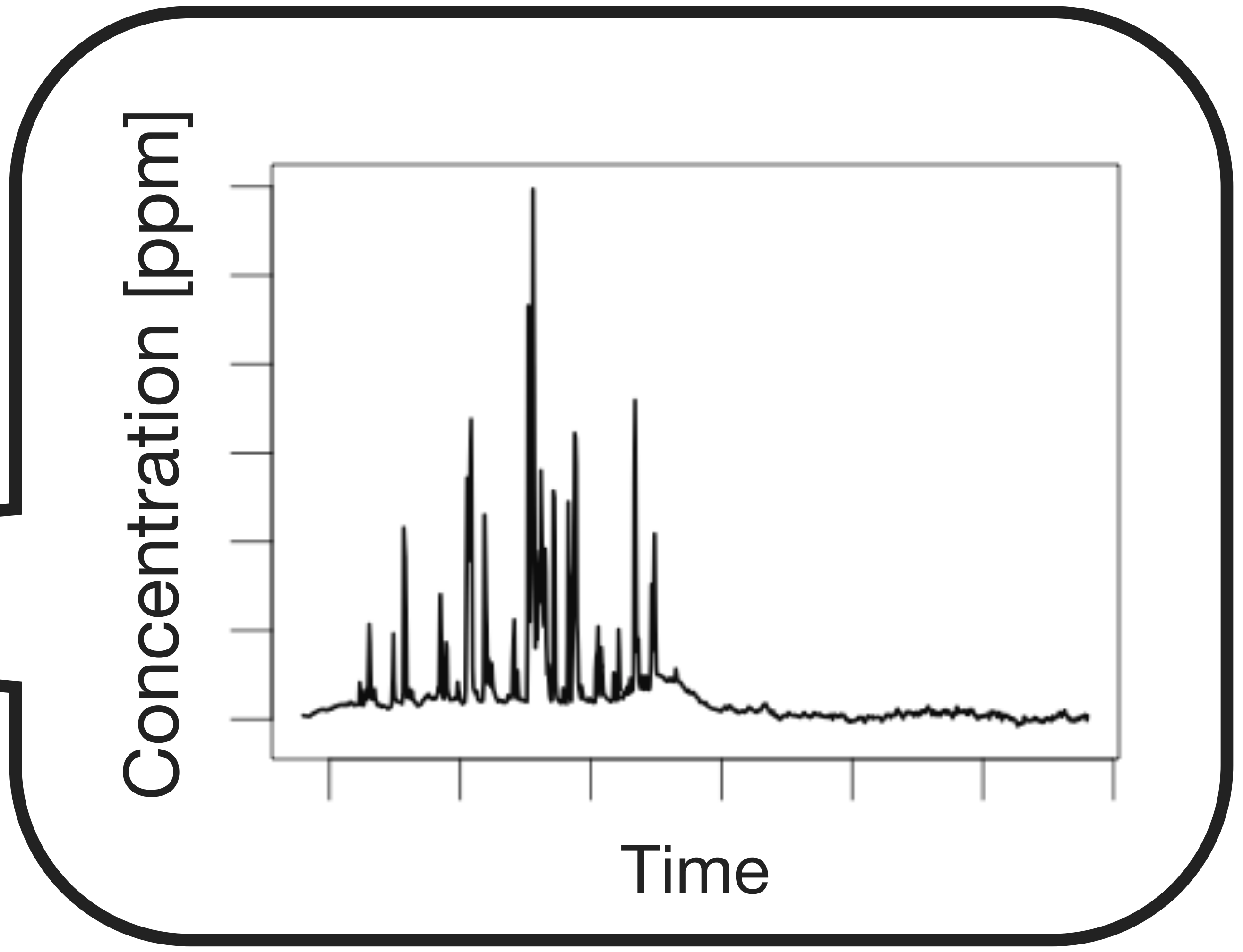
Example production oil and gas site





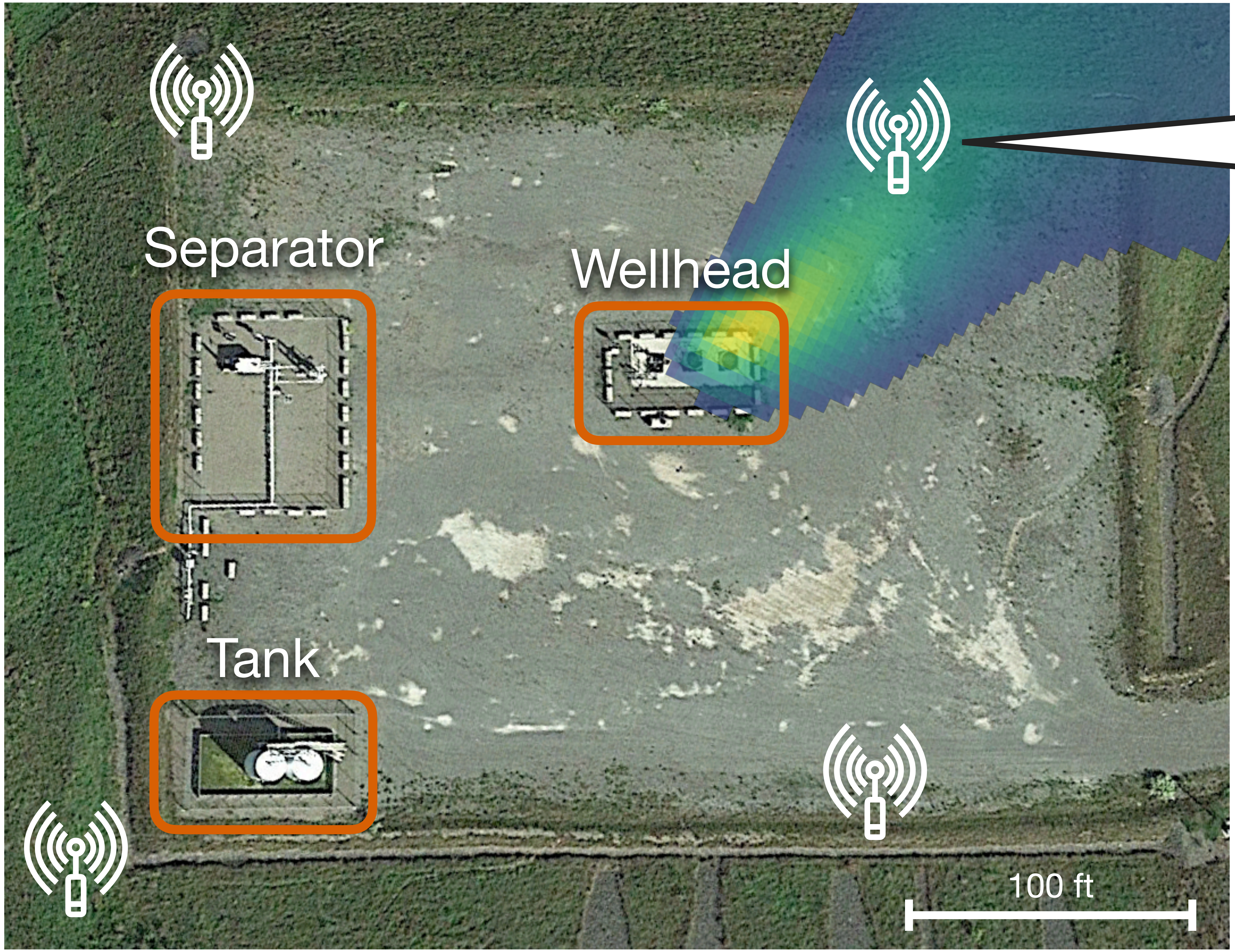
Example production oil and gas site

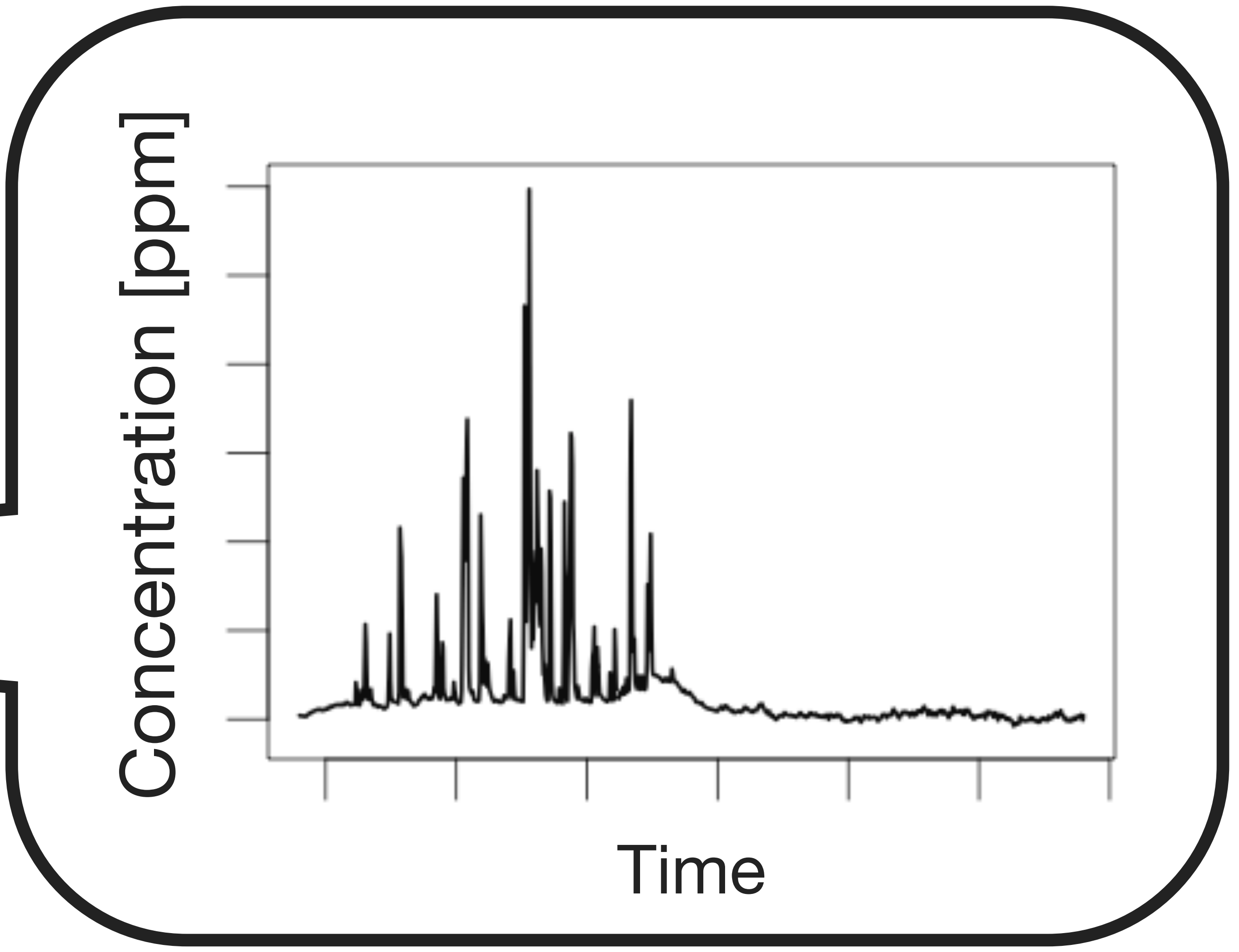




Aerial measurement technology

Example production oil and gas site

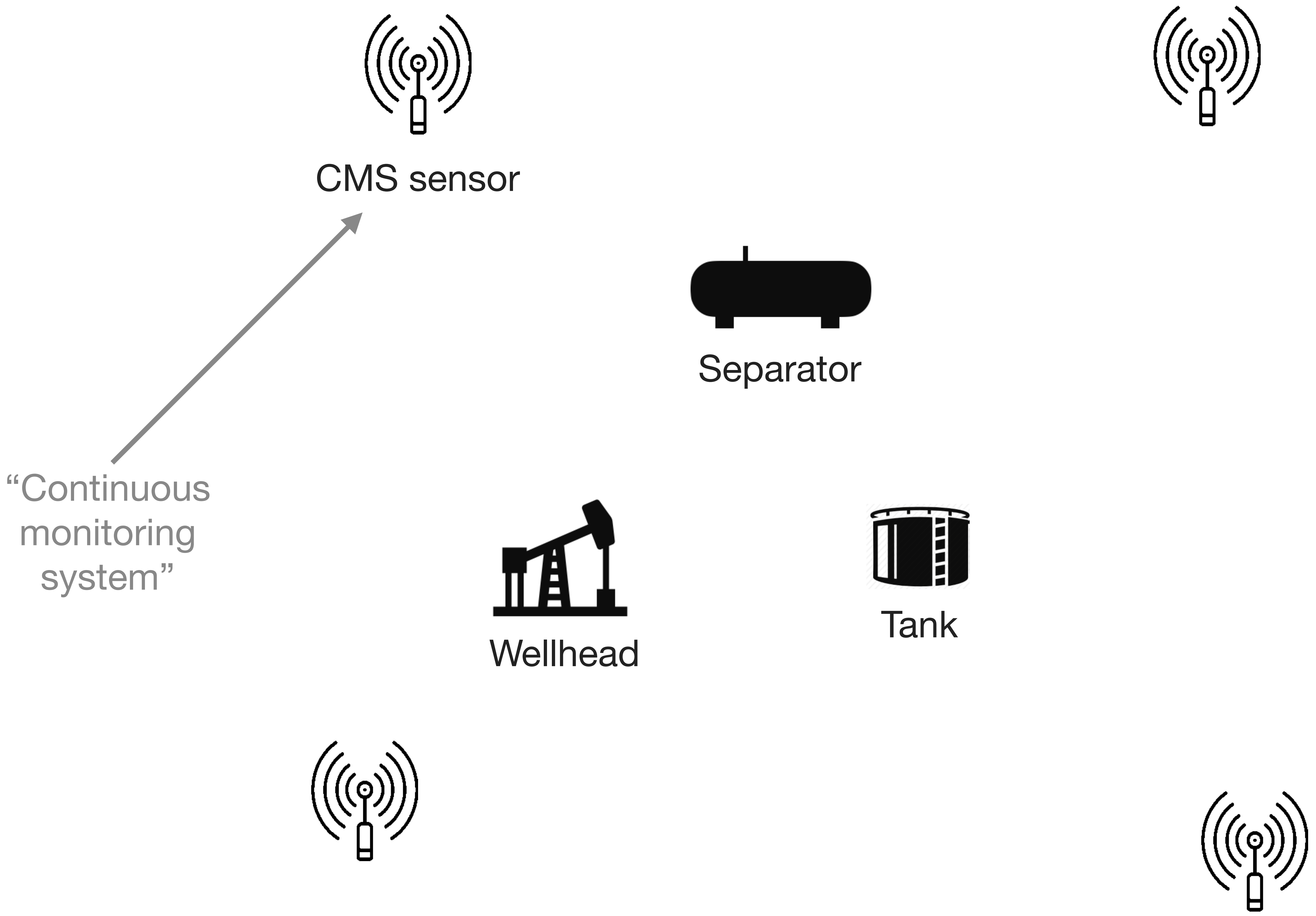




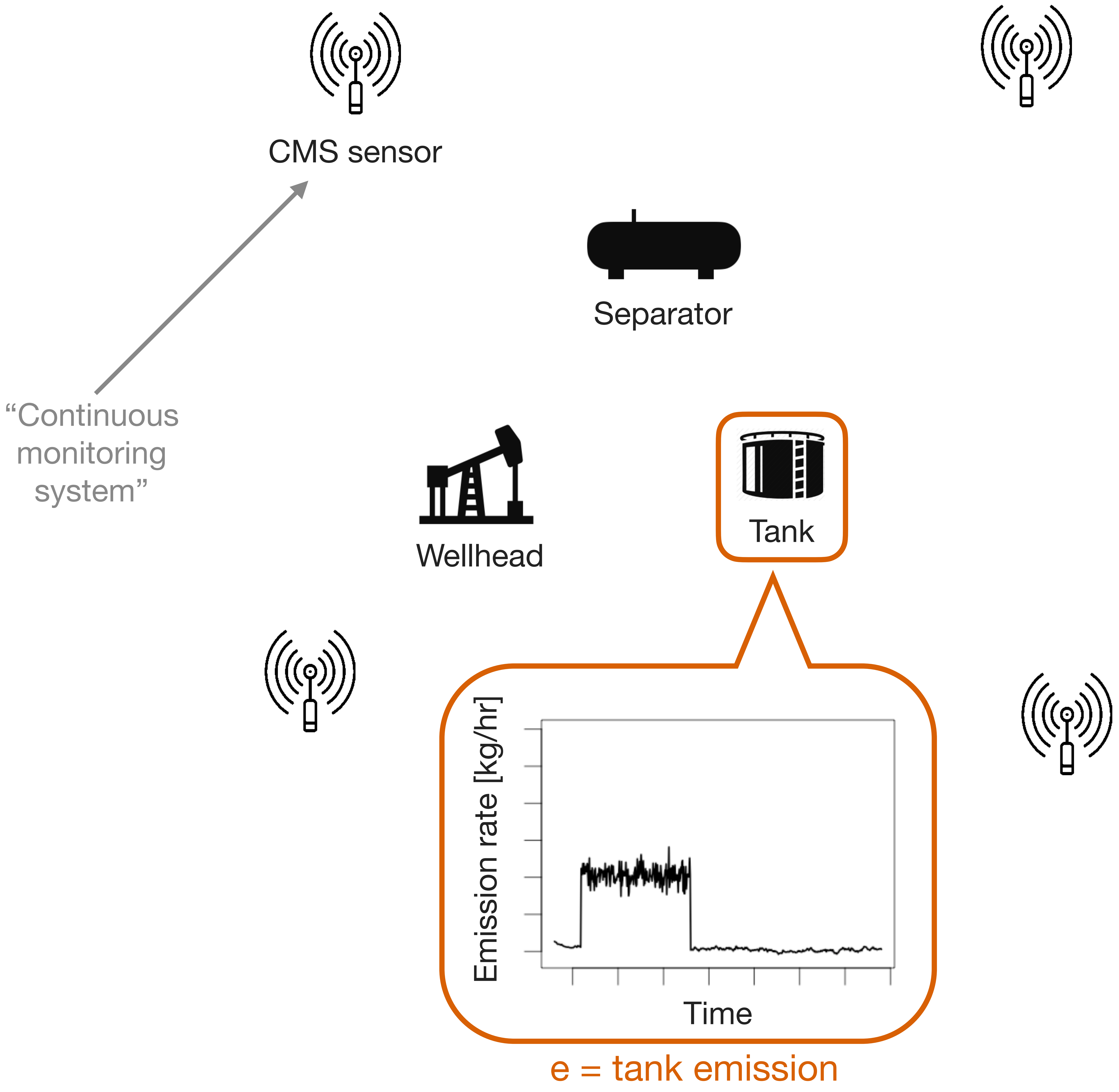
Event detection:
When is an emission happening?

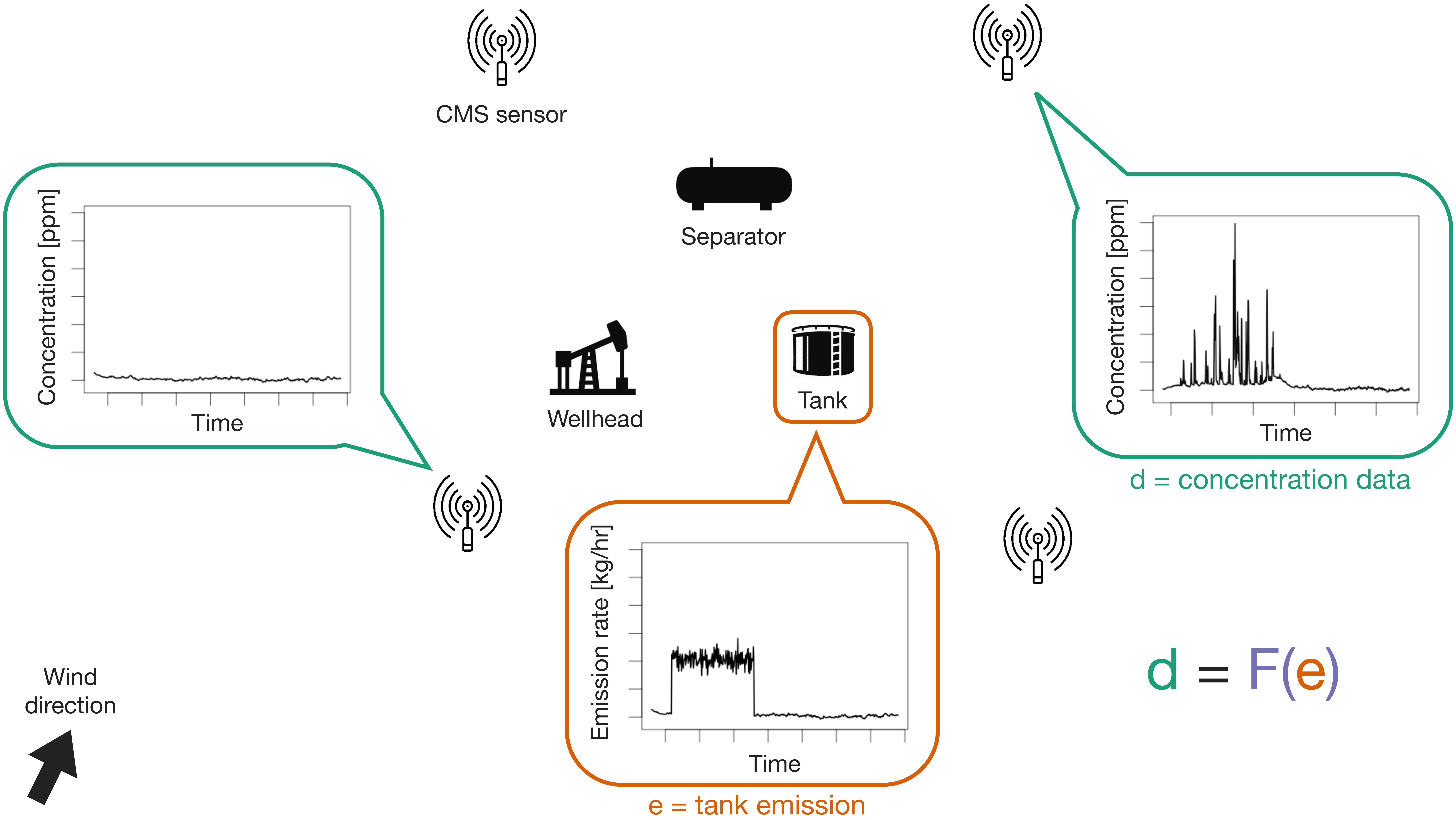
Localization:
Where is the emission coming from?

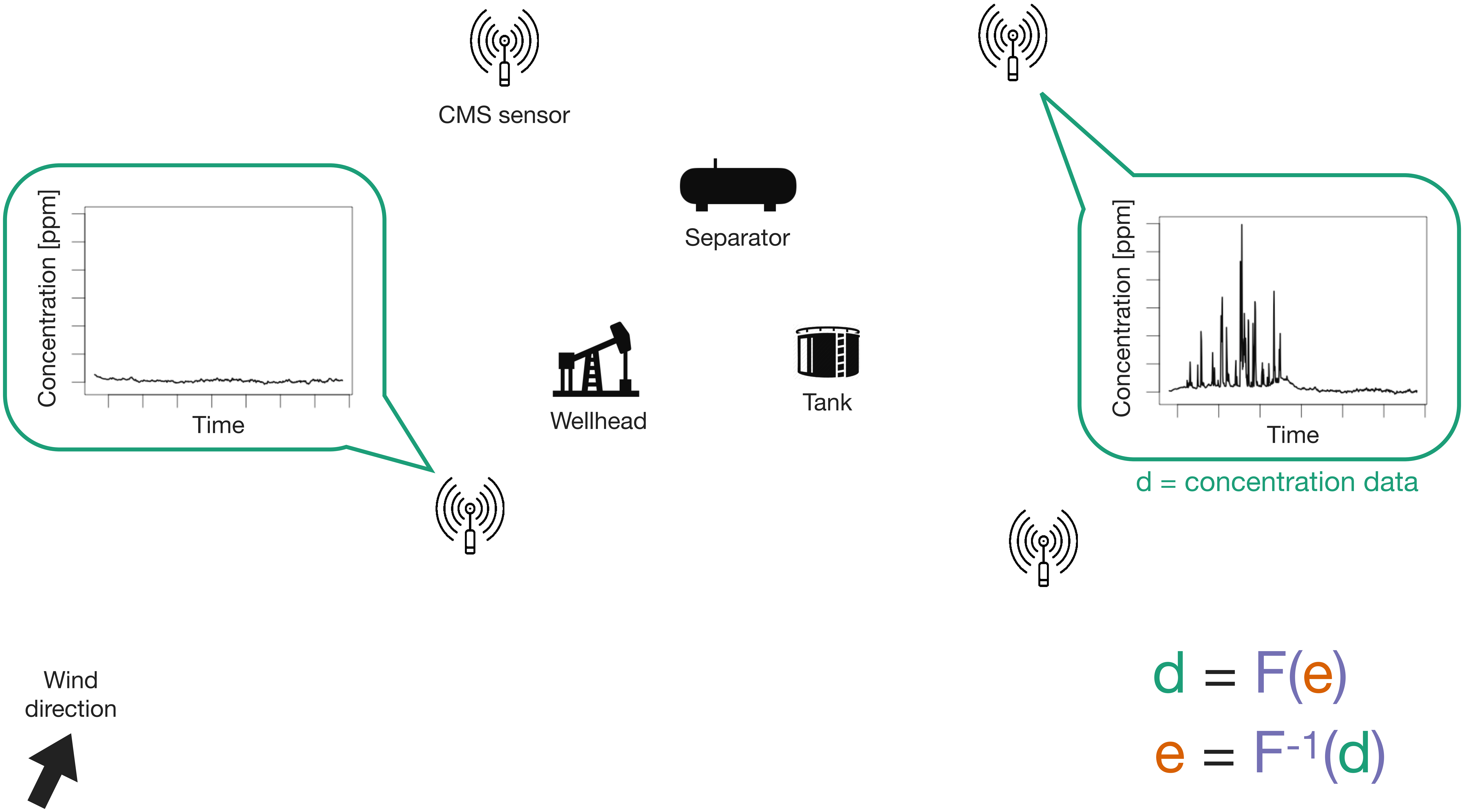
Quantification:
How much is being emitted?



The single-source continuous monitoring inverse problem


CMS sensor
Separator
"Continuous monitoring system"
Wellhead
Tank
Emission rate [kg/hr]
Time
e = tank emission


CMS sensor
Separator
Concentration [ppm]
Time
Wellhead
Tank
Concentration [ppm]
Time
d = concentration data
Emission rate [kg/hr]
Time
e = tank emission
Wind
direction
d = F(e)



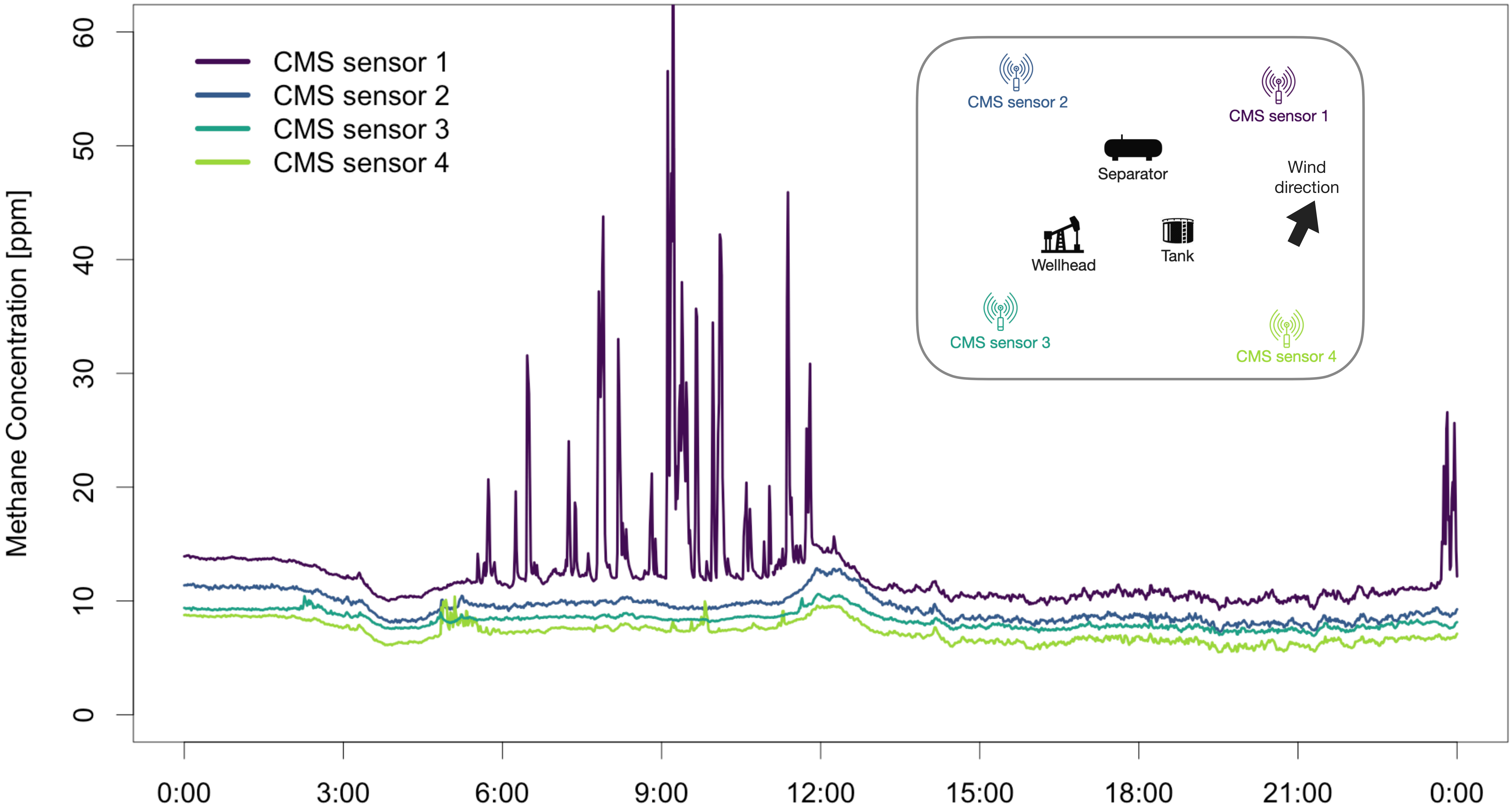
CMS sensor
Separator
Wellhead
Tank
Concentration [ppm]
Time
Concentration [ppm]
Time
d = concentration data
Wind
direction

$$d = F(e)$$

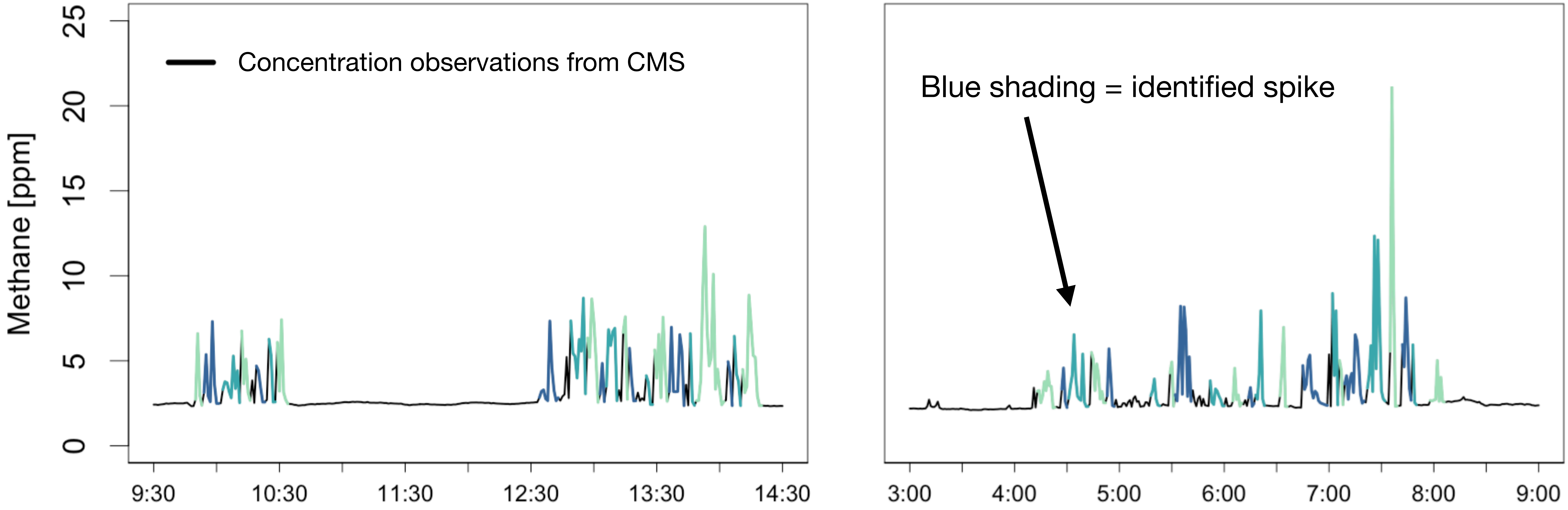
$$e = F^{-1}(d)$$

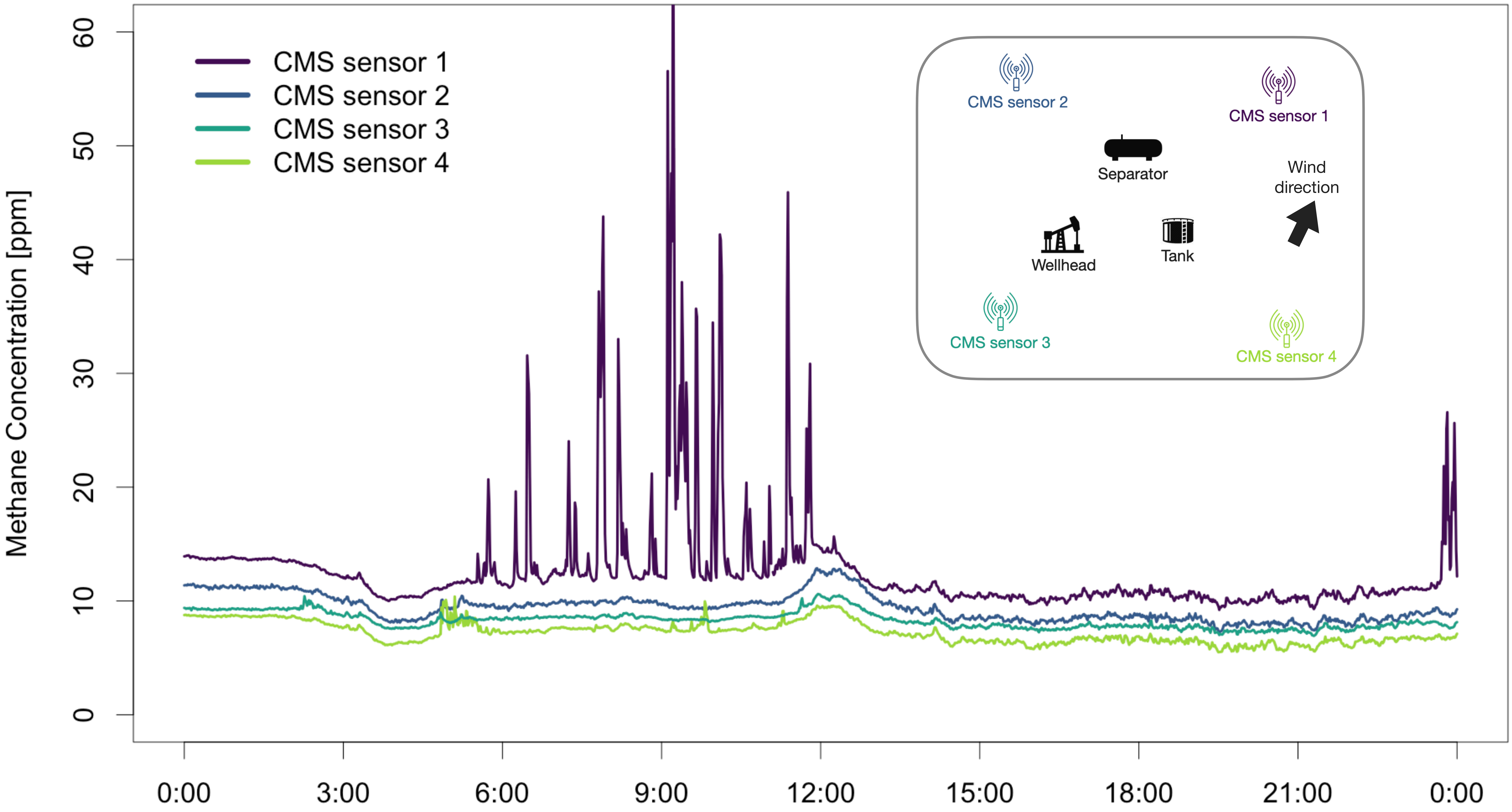
Open source framework for solving inverse problem

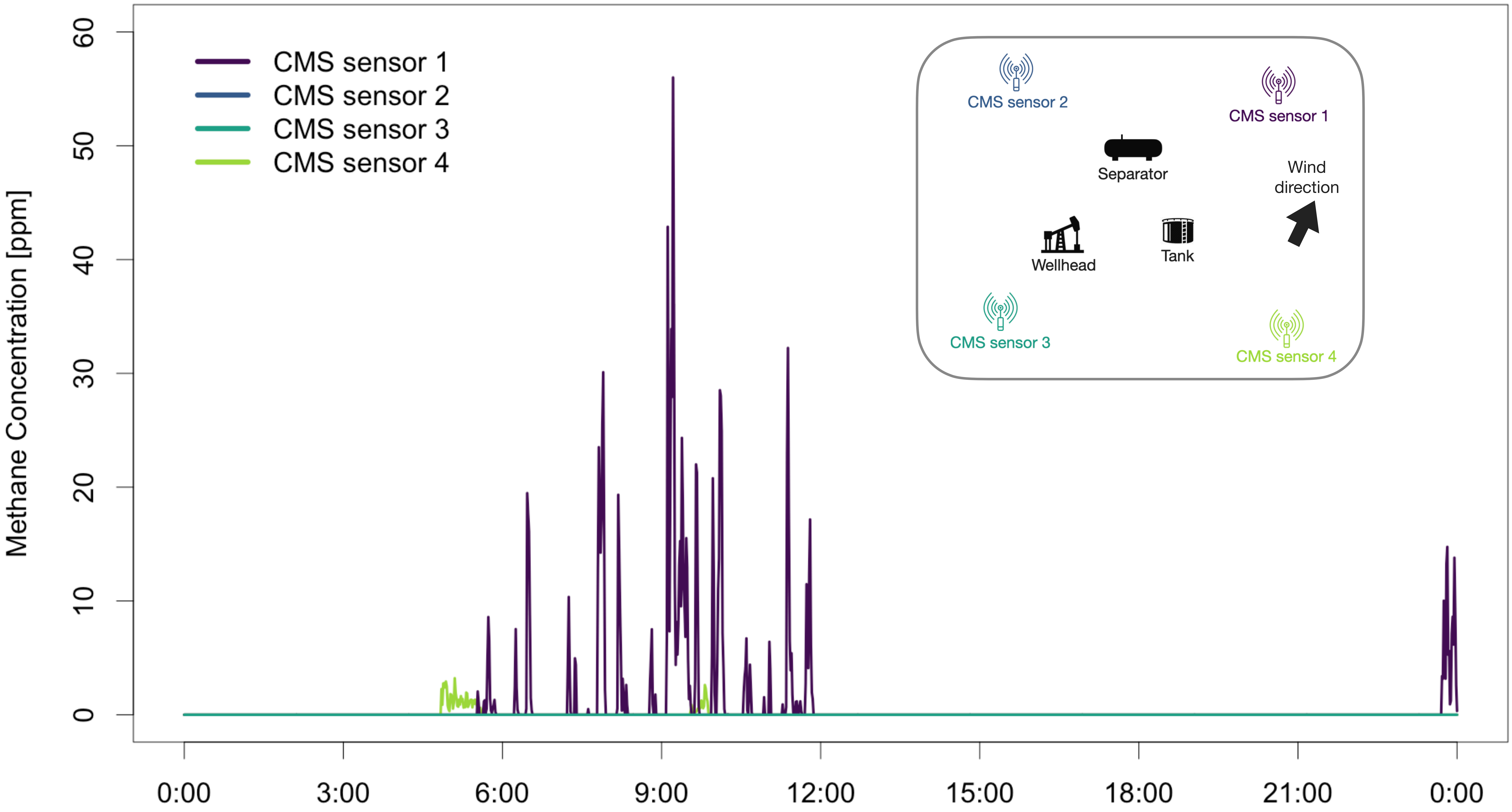
STEP 1: Background removal and event detection → STEP 2: Simulation → STEP 3: Localization → STEP 4: Quantification

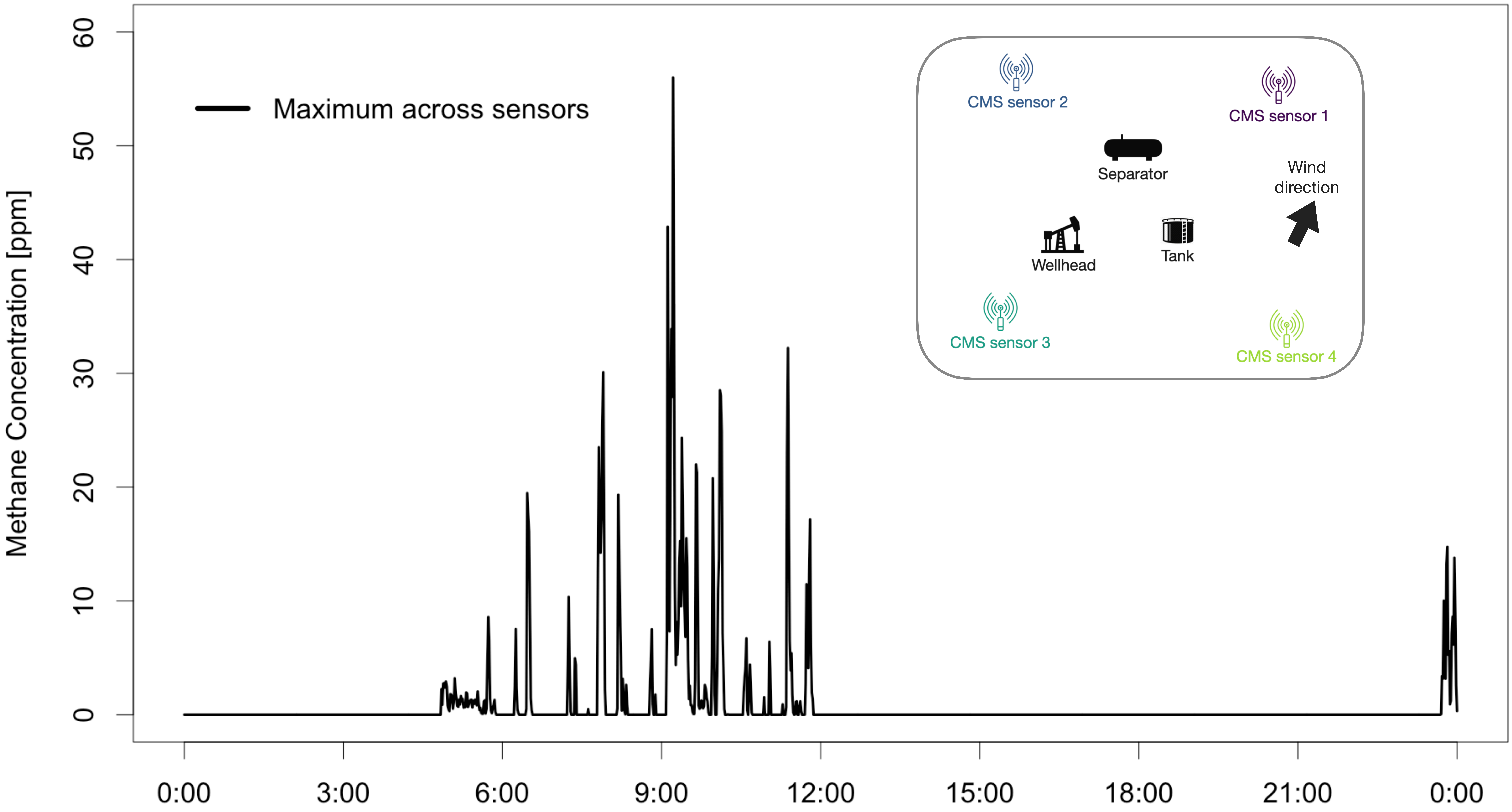

Methane Concentration [ppm]
0
10
20
30
40
50
60
CMS sensor 1
CMS sensor 2
CMS sensor 3
CMS sensor 4
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00

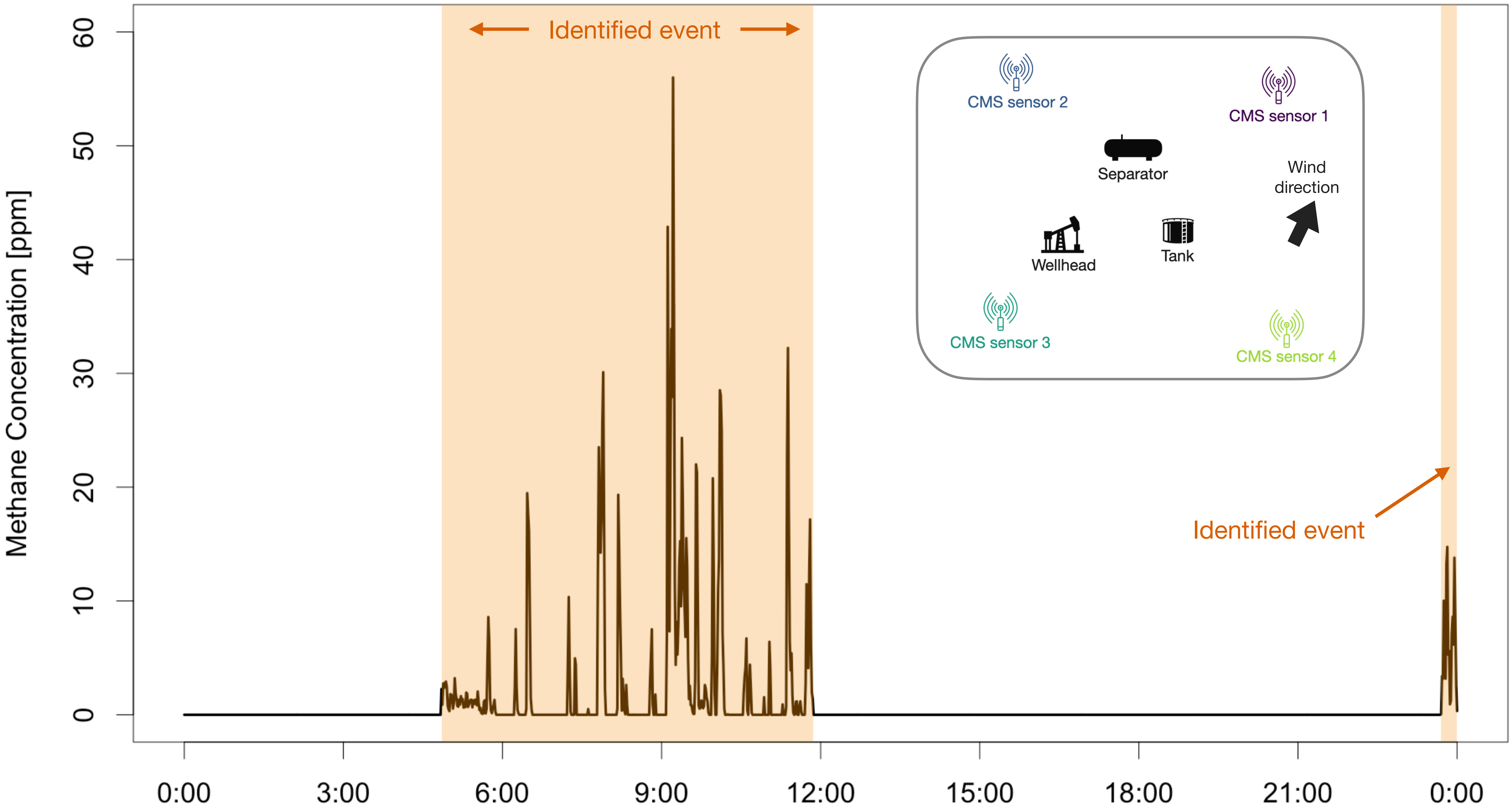
Spike detection algorithm examples

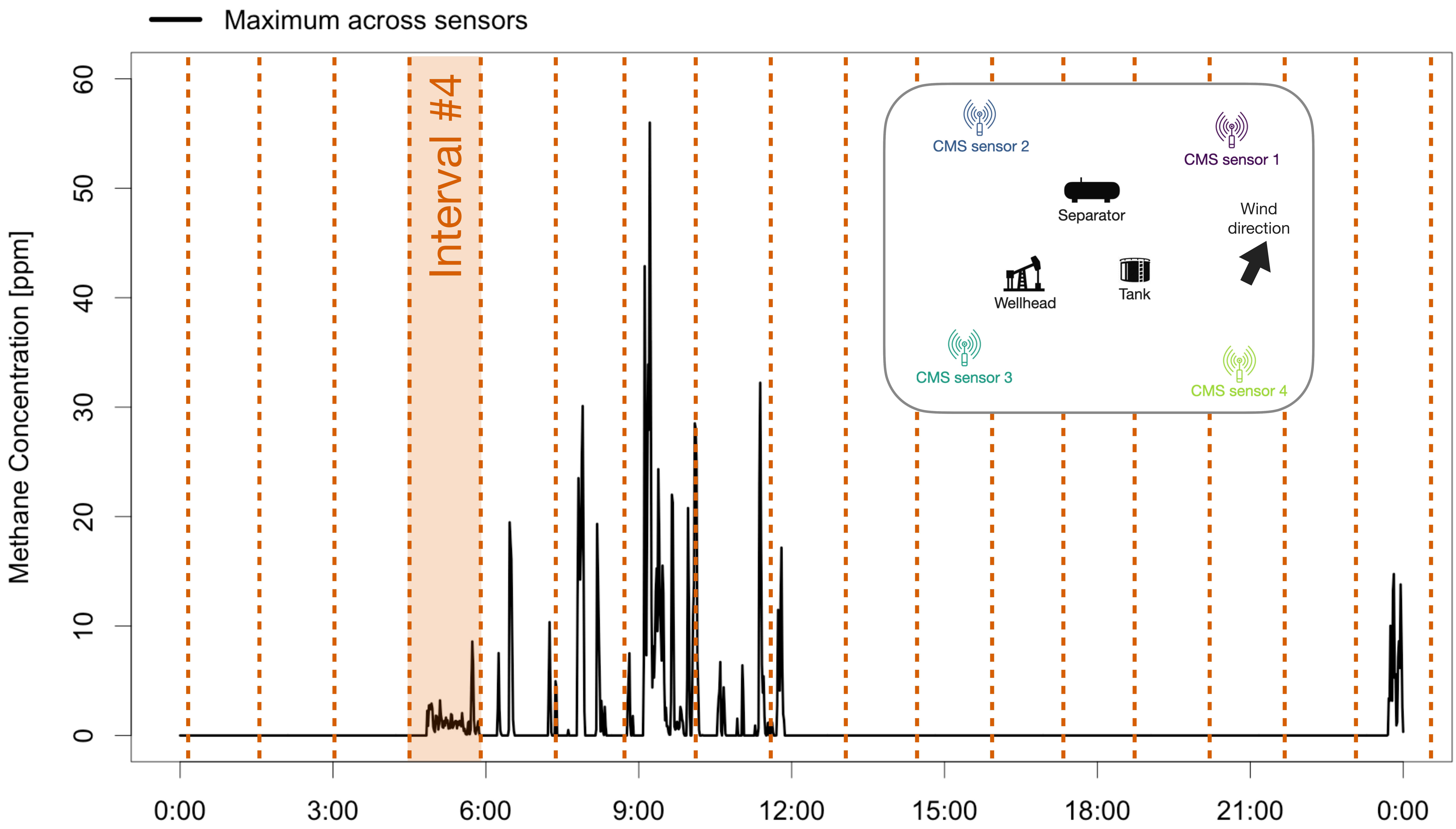



Methane Concentration [ppm]
0
10
20
30
40
50
60
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00
CMS sensor 1
CMS sensor 2
CMS sensor 3
CMS sensor 4
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4


CMS sensor 1
CMS sensor 2
CMS sensor 3
CMS sensor 4
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4
Methane Concentration [ppm]
0
10
20
30
40
50
60
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00


Maximum across sensors
Methane Concentration [ppm]
0
10
20
30
40
50
60
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00
CMS sensor 2
CMS sensor 1
Separator
Wind
direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4


Identified event
CMS sensor 2
CMS sensor 1
Separator
Wind direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4
Identified event
Methane Concentration [ppm]
0
10
20
30
40
50
60
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00


Maximum across sensors
Methane Concentration [ppm]
0
10
20
30
40
50
60
Interval #4
CMS sensor 2
CMS sensor 1
Separator
Wind
direction
Wellhead
Tank
CMS sensor 3
CMS sensor 4
0:00
3:00
6:00
9:00
12:00
15:00
18:00
21:00
0:00

Open source framework for solving inverse problem

STEP 1:
Background removal
and event detection

→

STEP 2:
Simulation

→

STEP 3:
Localization

→

STEP 4:
Quantification

Gaussian plume model:

models the transport of methane by assuming that everything is steady state

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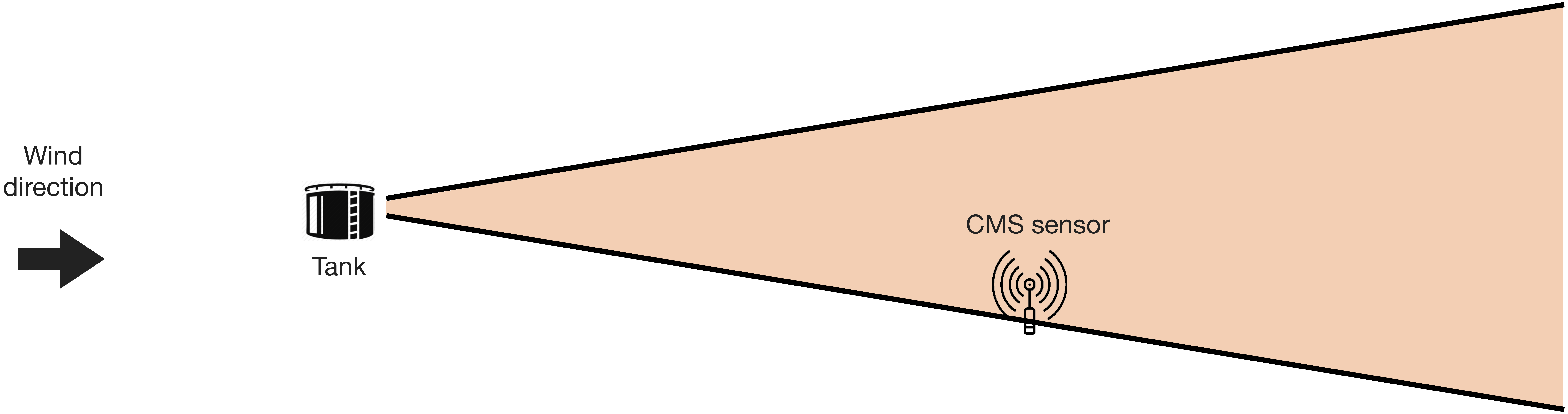
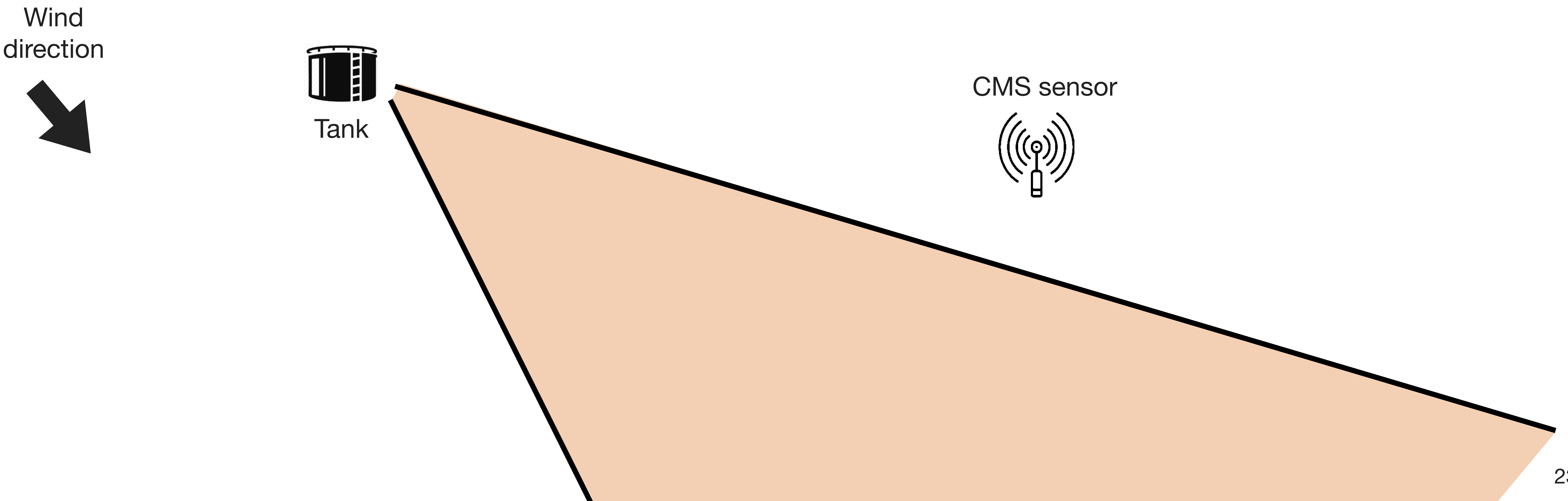
models the transport of methane by assuming that everything is steady state



Gaussian plume model:

models the transport of methane by assuming that everything is steady state



Gaussian puff model:

approximates a continuous release of methane as a sum of many small “puffs”

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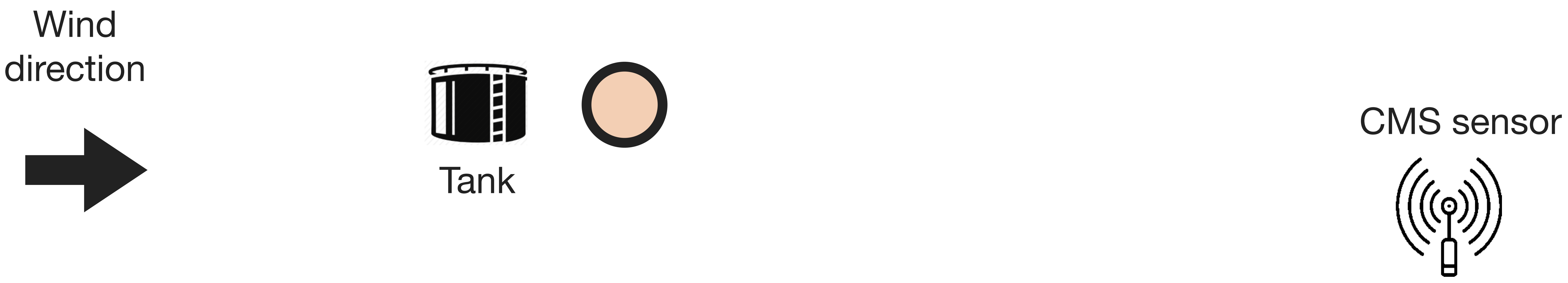

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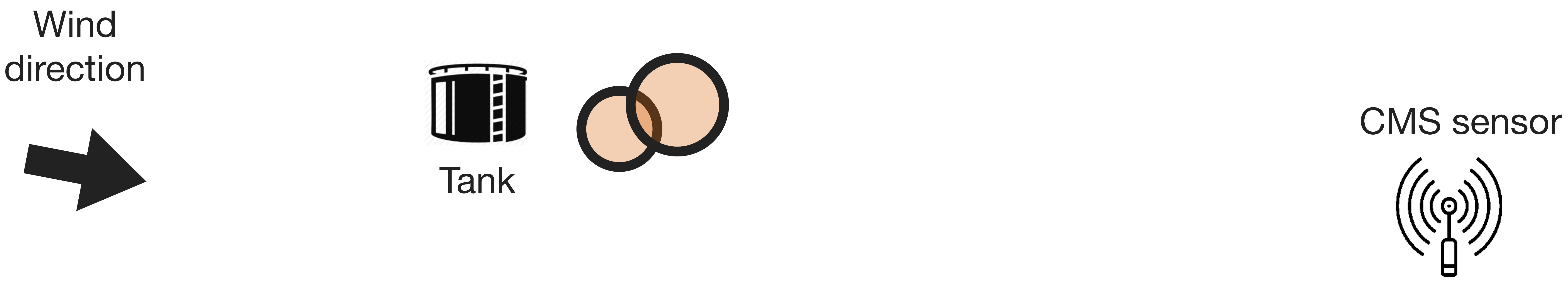

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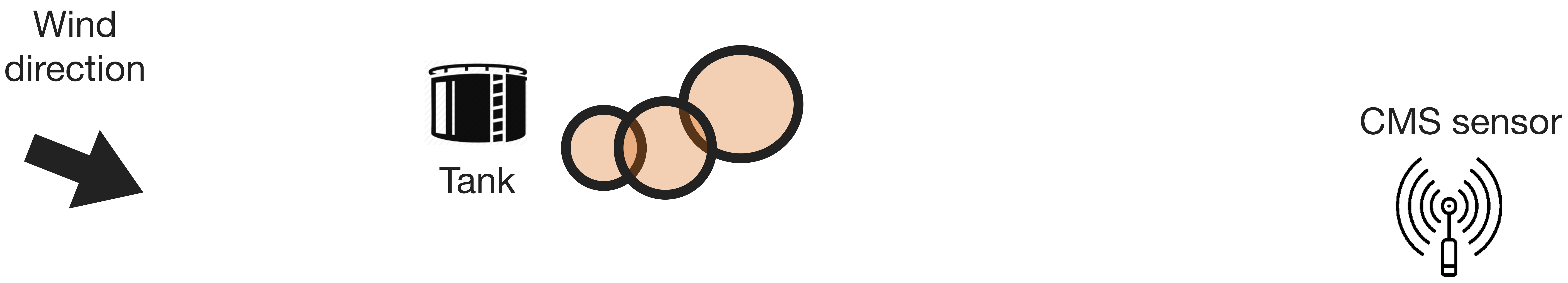

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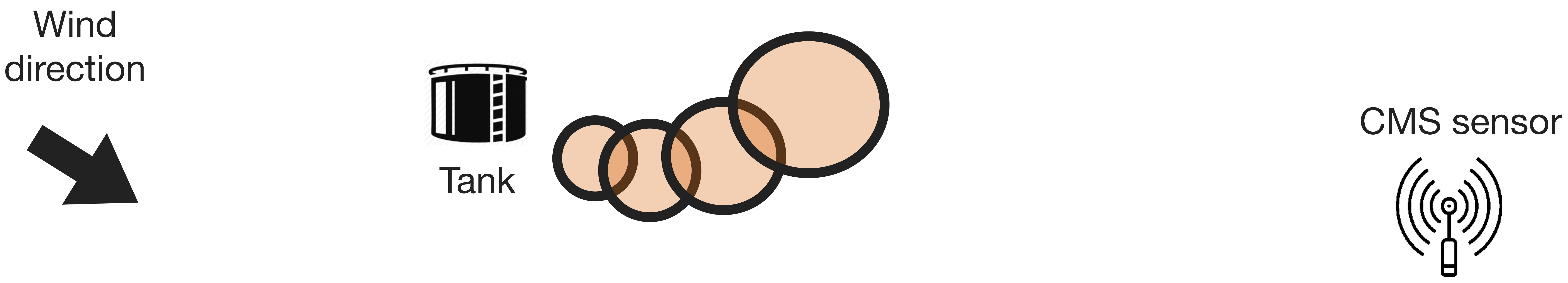
approximates a continuous release of methane as a sum of many small “puffs”



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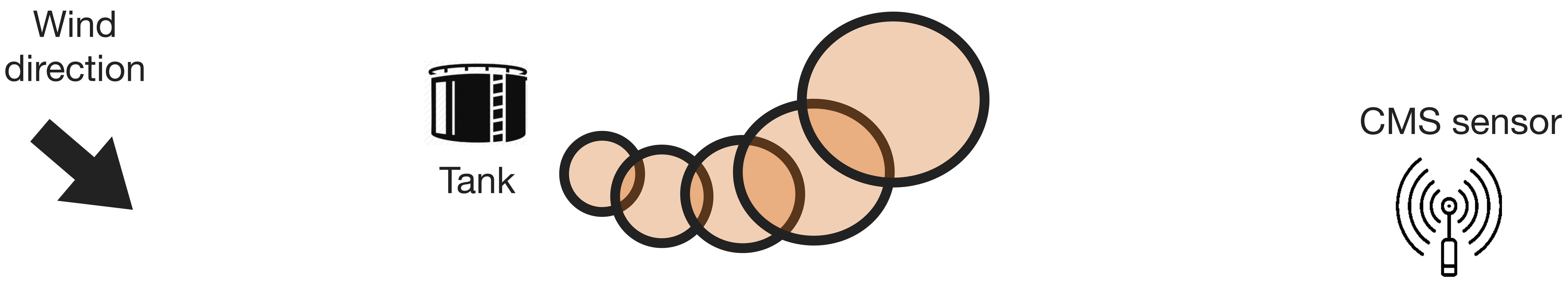

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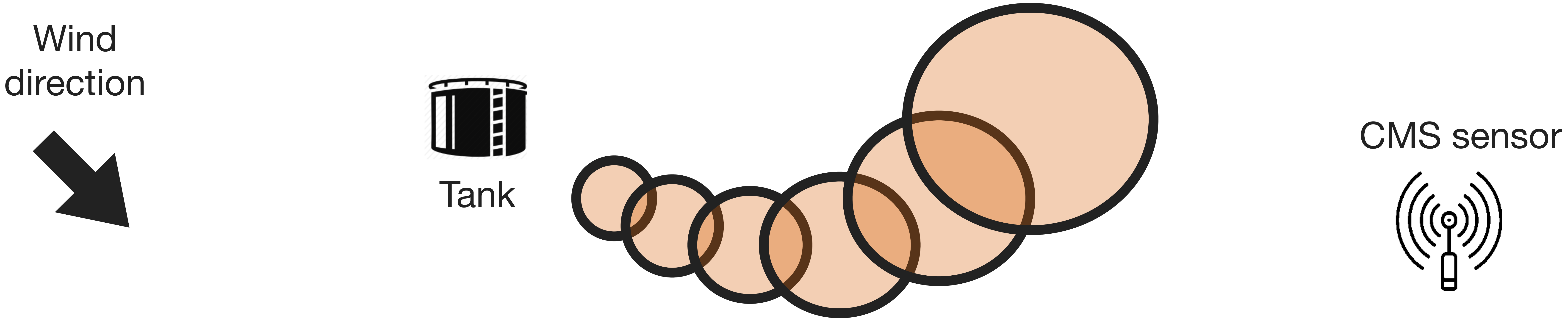

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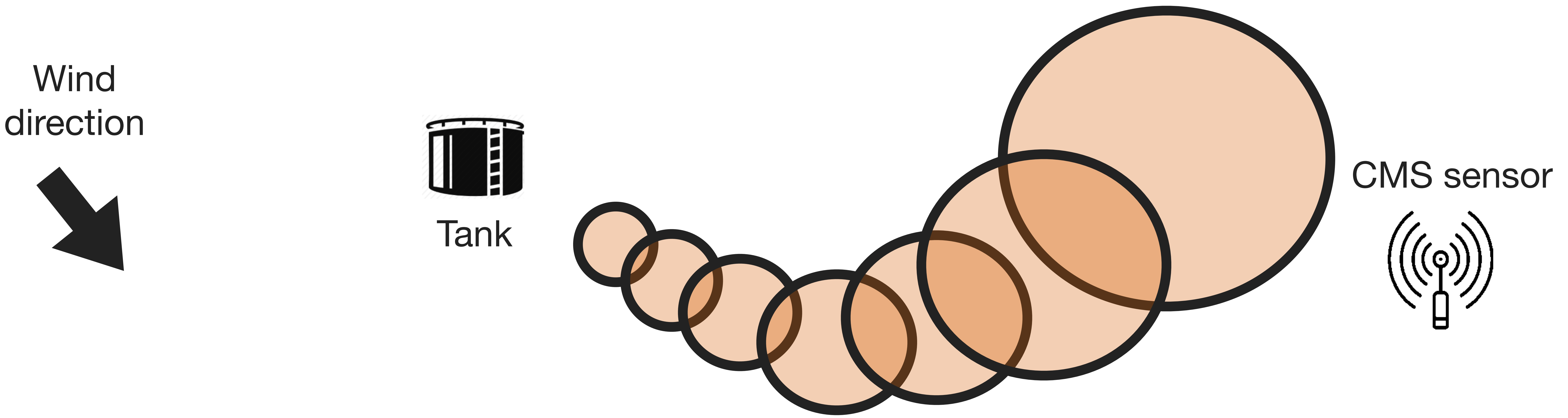
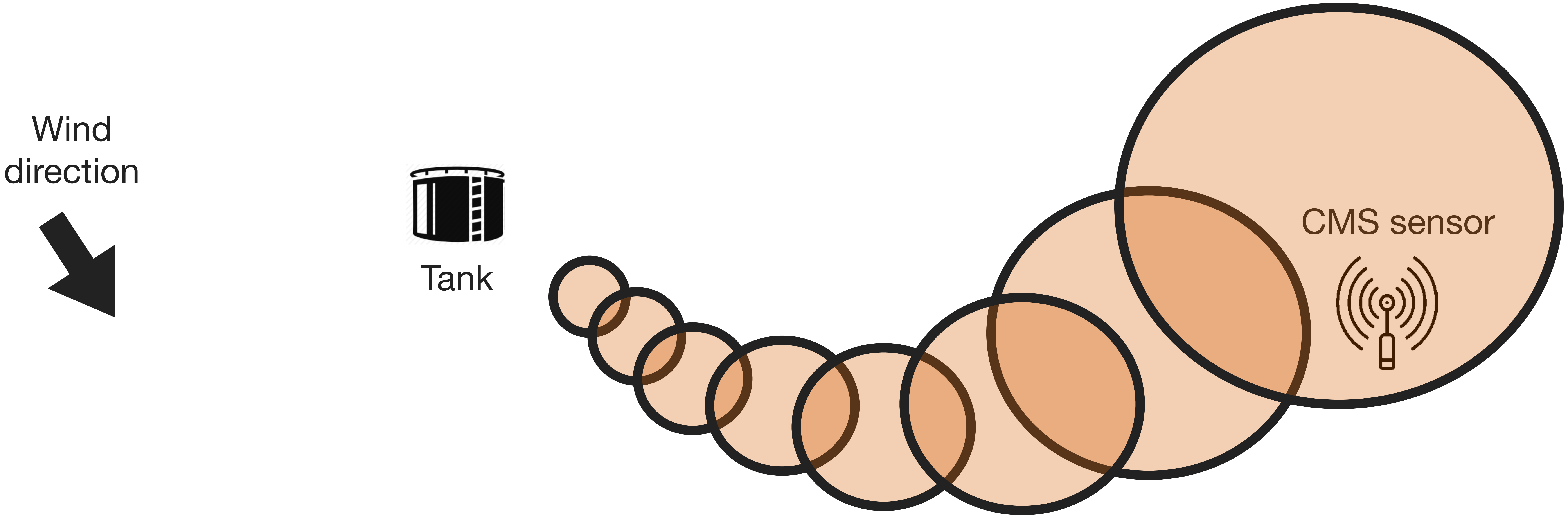
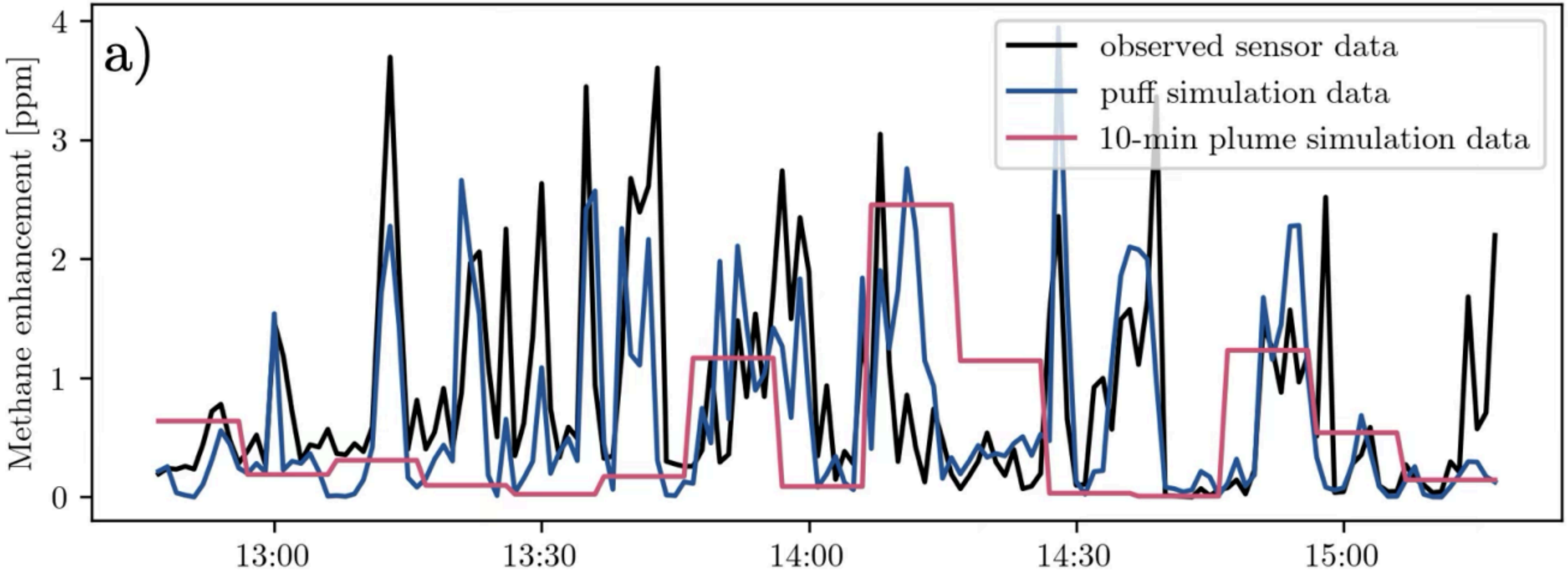


Gaussian puff model:

approximates a continuous release of methane as a sum of many small "puffs"



The Gaussian puff can leverage high frequency wind data, while the Gaussian plume requires a temporal average.



Gaussian plume model: mathematical definition

Set up coordinate system so that source is at (0,0,H) and positive x-axis aligns with downwind vector

Emission rate

Exponential decay as you move away from the downwind vector in the vertical dimension

$$c(x,y,z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Predicted methane concentration at sensor location (x,y,z)

Exponential decay as you move away from the downwind vector in the horizontal plane

Gaussian puff model: mathematical definition

Set up coordinate system so that source is at (0,0,H) and positive x-axis aligns with downwind vector

Total volume of methane contained in puff p

$$c_p(x, y, z, t, Q) = \frac{Q}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Predicted methane concentration at sensor location (x,y,z) and time t from puff p

Exponential decay in concentration in horizontal plane (x, y)

Exponential decay in concentration in vertical dimension (z)

Gaussian puff model: mathematical definition

Set up coordinate system so that source is at (0,0,H) and positive x-axis aligns with downwind vector

Total concentration at (x, y, z, t)

$$c(x, y, z, t, Q) = \sum_{p=1}^{P} c_p(x, y, z, t, Q)$$

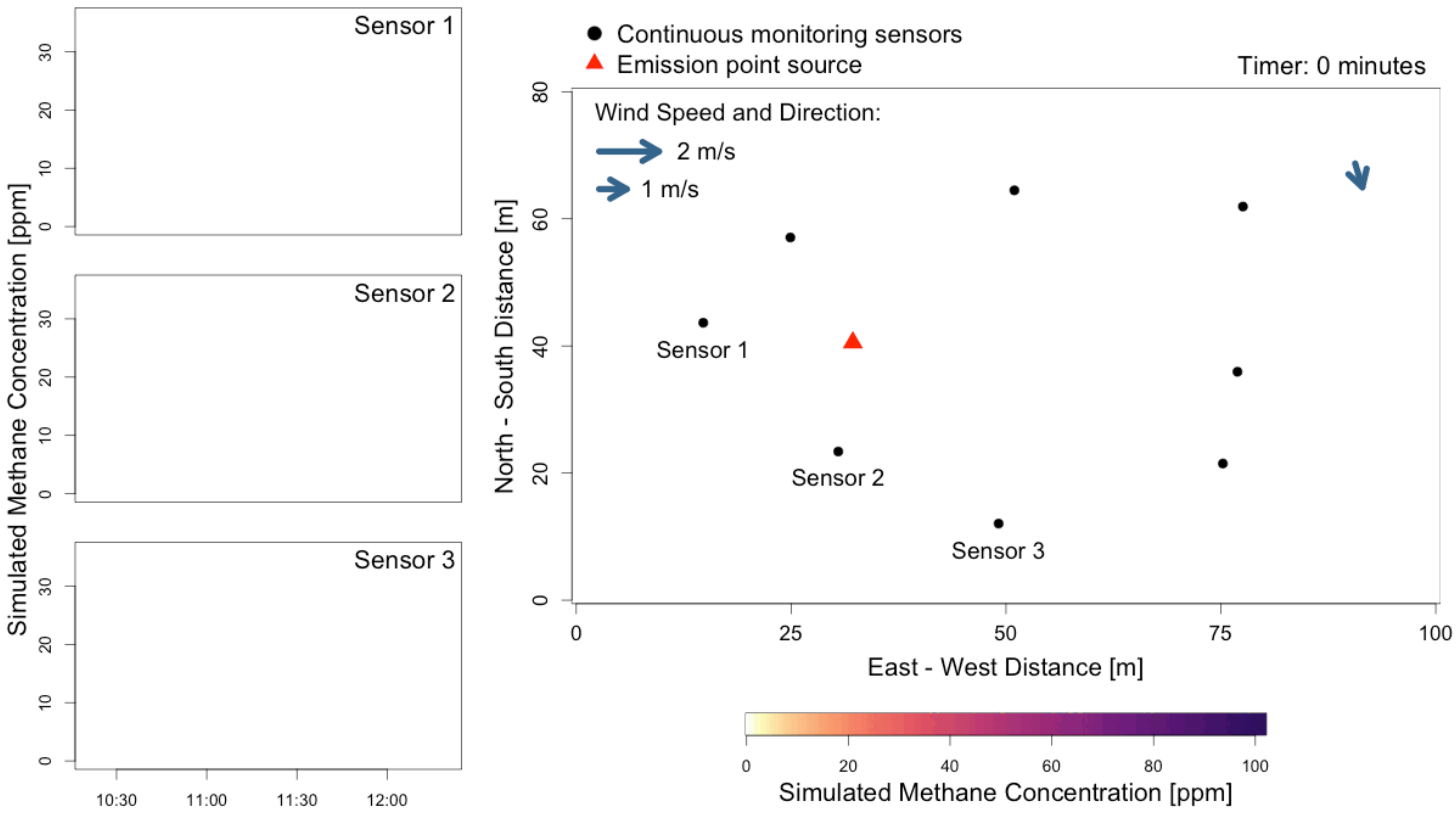
Total volume of methane contained in puff p

$$c_p(x, y, z, t, Q) = \frac{Q}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Predicted methane concentration at sensor location (x,y,z) and time t from puff p

Exponential decay in concentration in horizontal plane (x, y)

Exponential decay in concentration in vertical dimension (z)



Sensor 1
Sensor 2
Sensor 3
Simulated Methane Concentration [ppm]
0
10
20
30
10:30
11:00
11:30
12:00
Continuous monitoring sensors
Emission point source
Timer: 0 minutes
Wind Speed and Direction:
2 m/s
1 m/s
North - South Distance [m]
East - West Distance [m]
0
25
50
75
100
0
20
40
60
80
100
Simulated Methane Concentration [ppm]

Repeat this for all other potential sources!

Improving the Gaussian puff model is an active area of research

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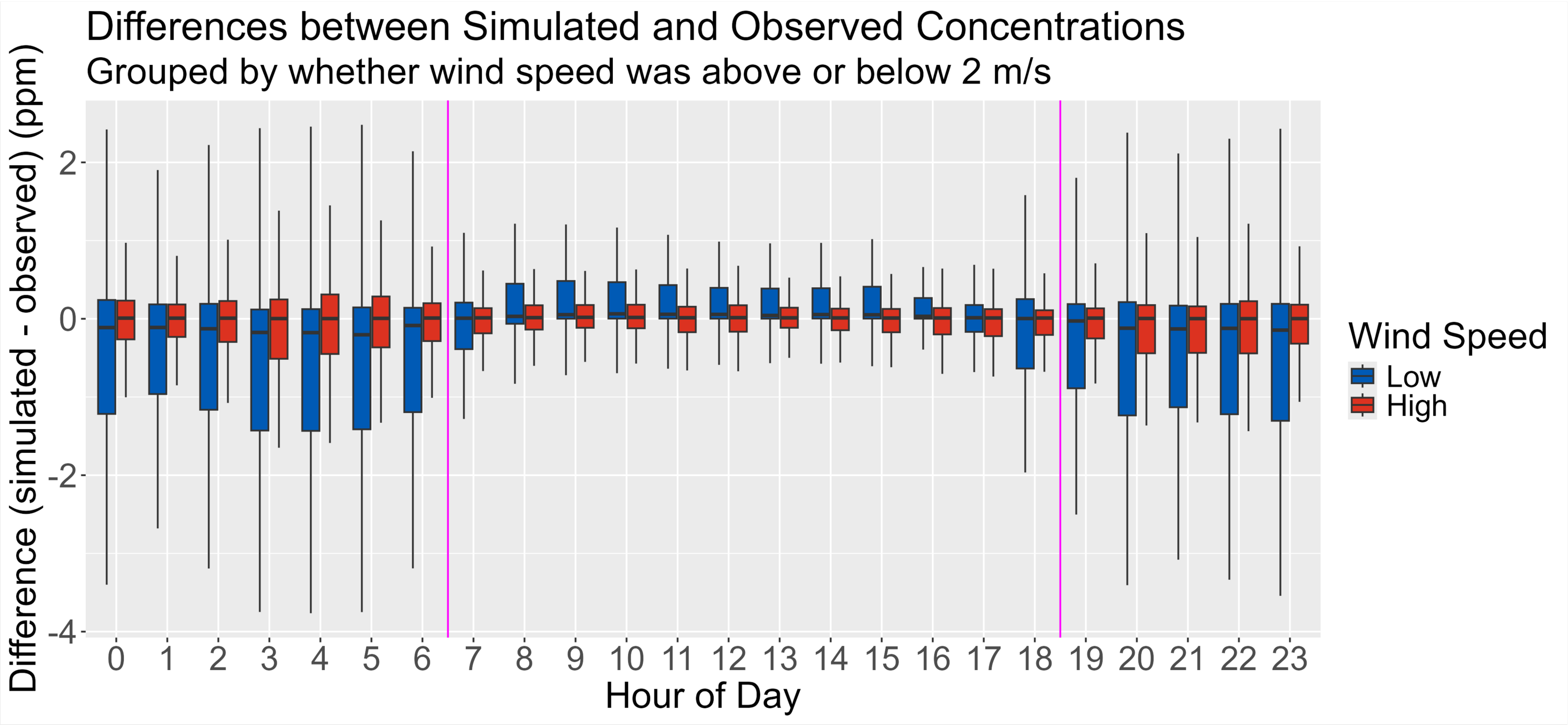
Direction #1: Better fidelity at low wind speeds



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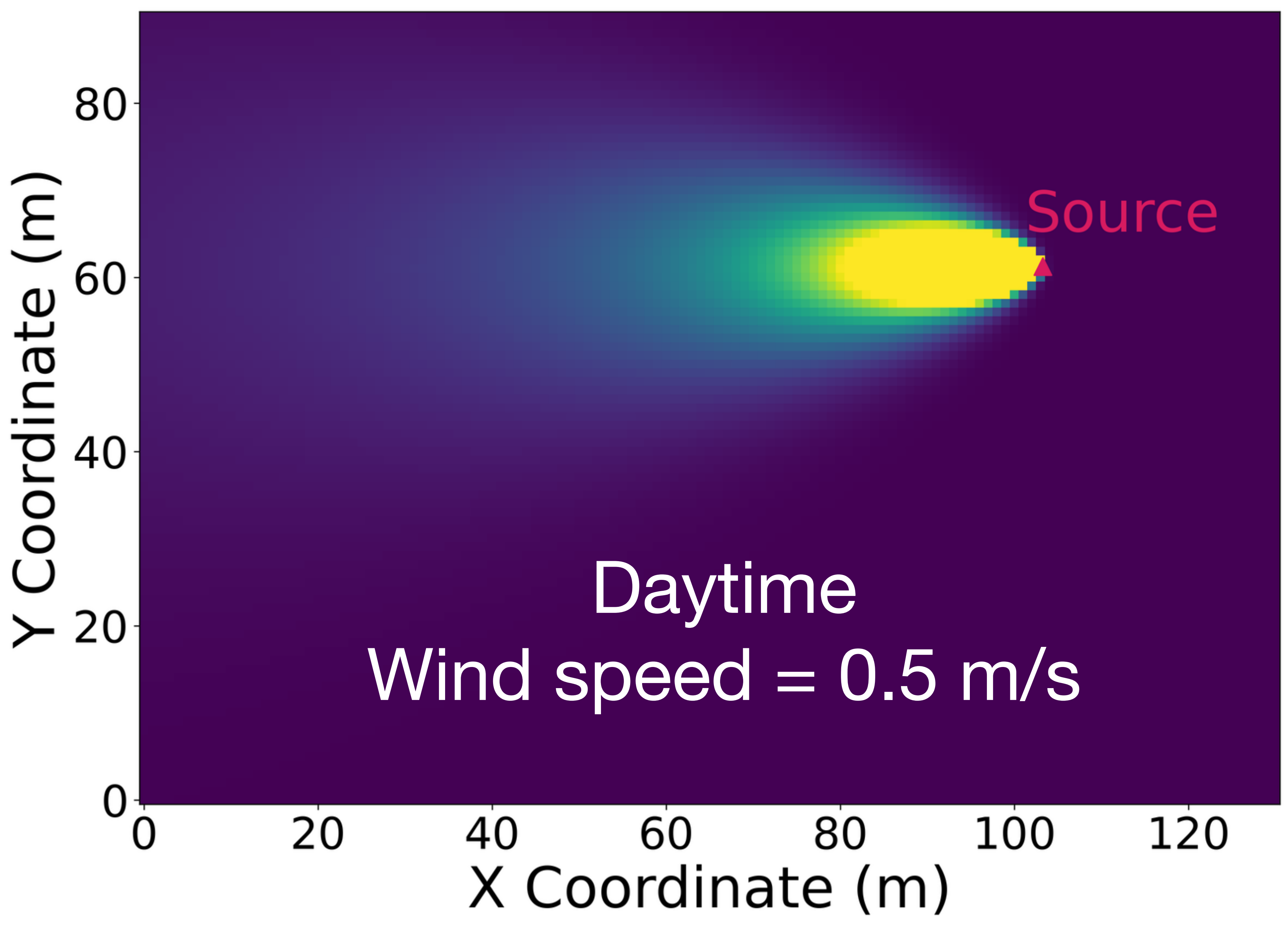

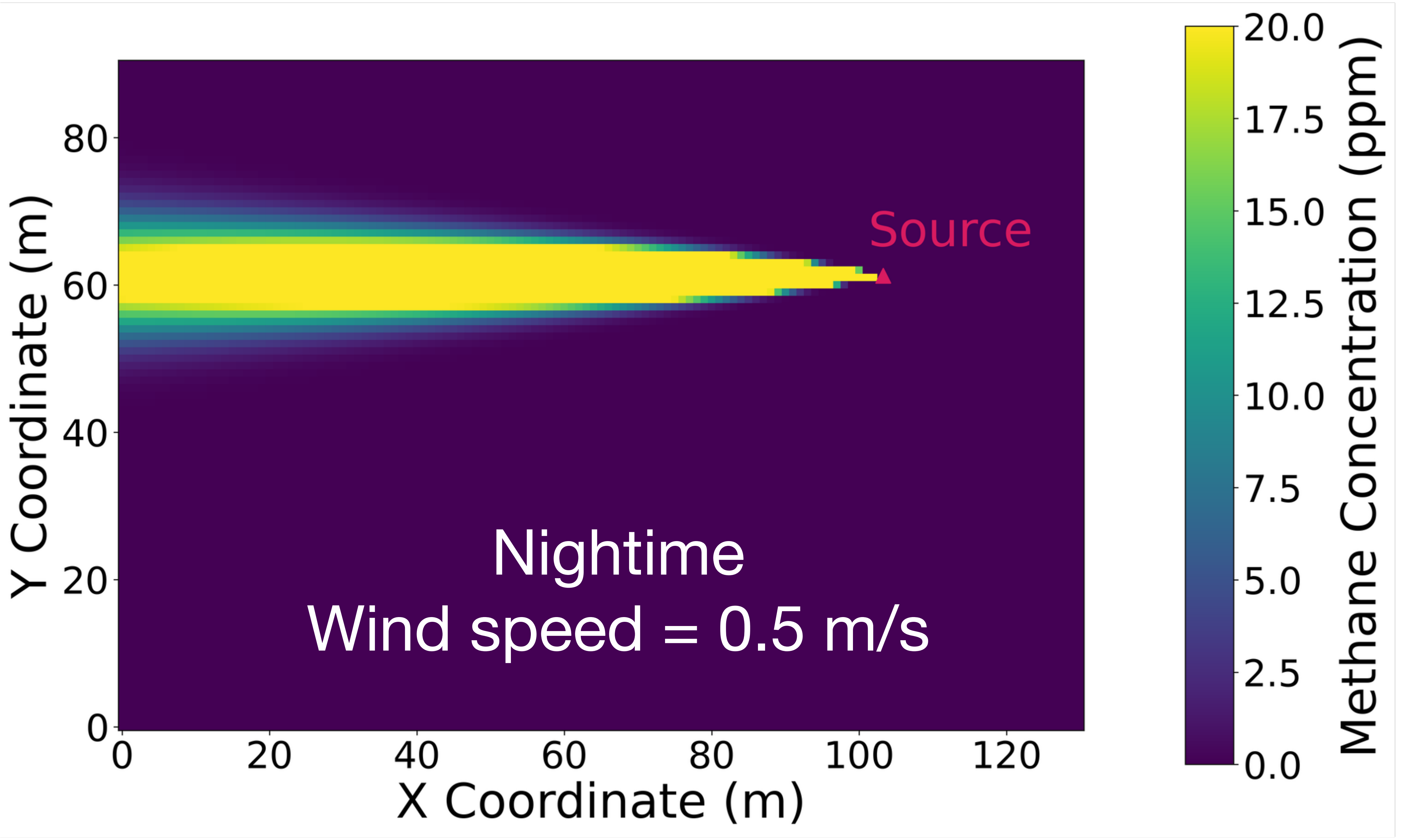

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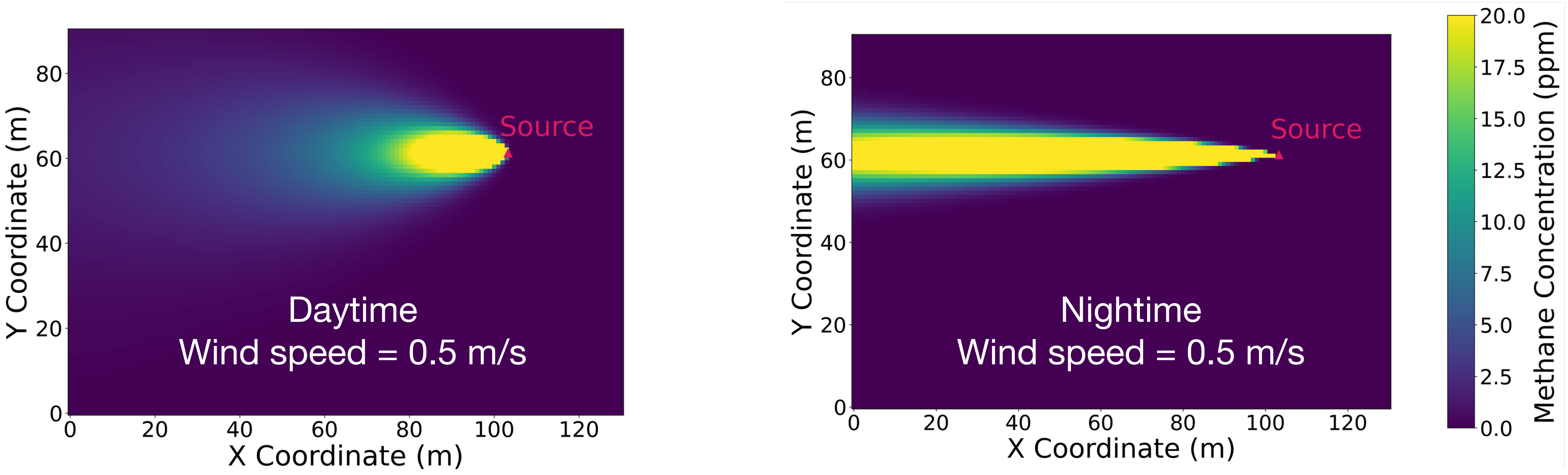
Direction #1: Better fidelity at low wind speeds

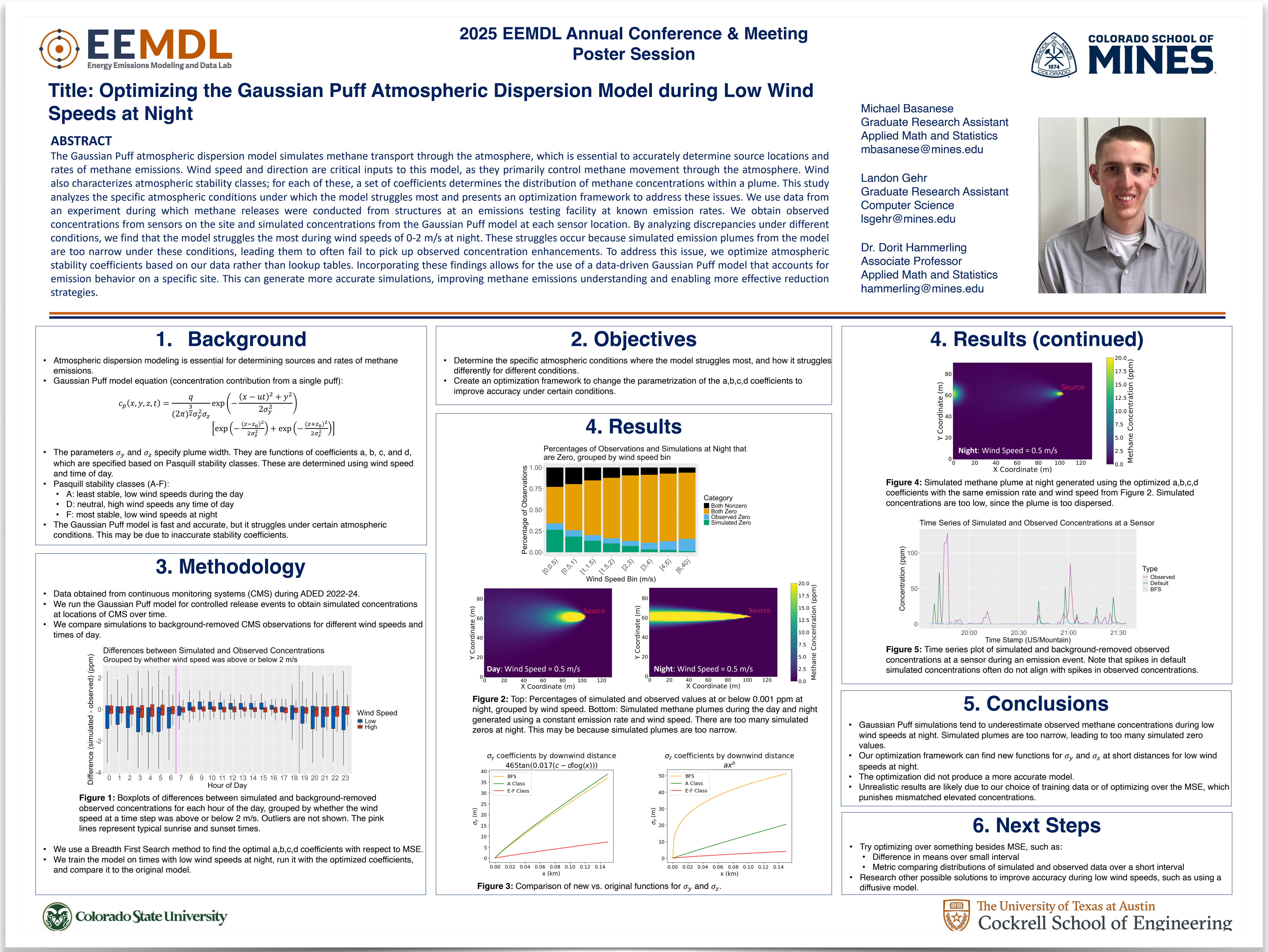


$$c_p(x, y, z, t, Q) = \frac{Q}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Can we optimize the dispersion coefficients based on the data?

Improving the Gaussian puff model is an active area of research

Direction #1: Better fidelity at low wind speeds



See Michael Basanese's poster for more details!

"Optimizing the Gaussian Puff Atmospheric Dispersion Model during Low Wind Speeds at Night"

Improving the Gaussian puff model is an active area of research

Direction #2: Account for obstacles

- The current Gaussian puff model is not aware of obstacles, like tanks or buildings.

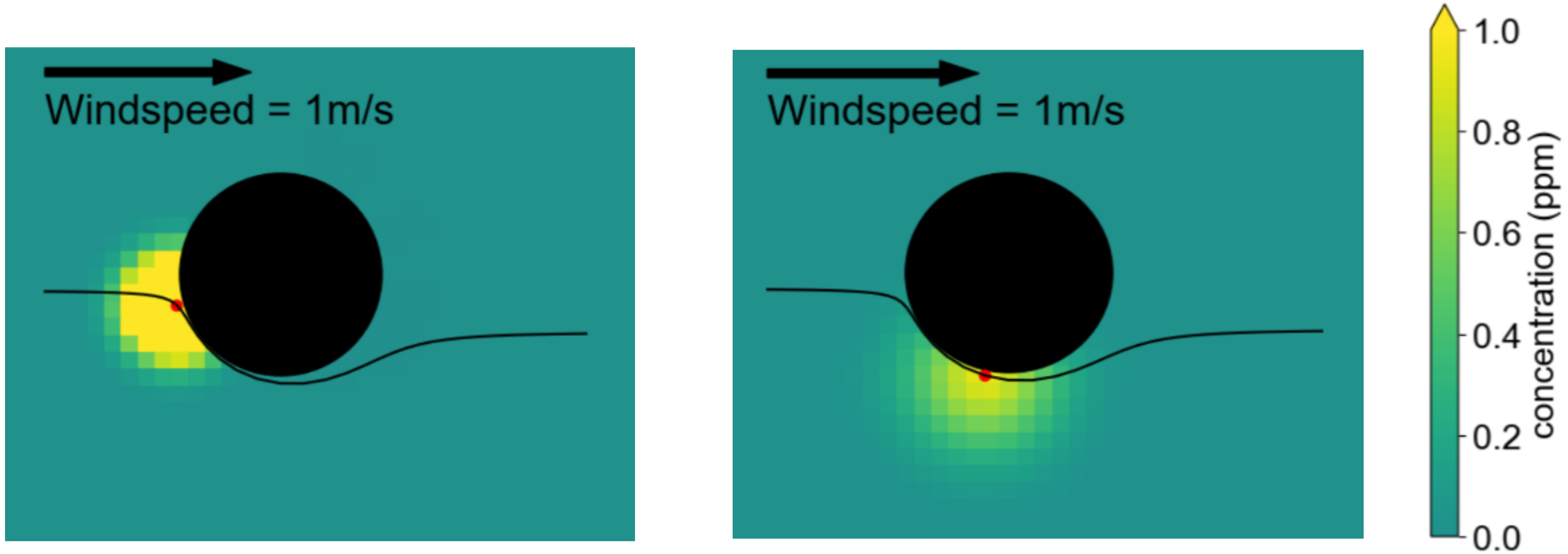
Improving the Gaussian puff model is an active area of research

Direction #2: Account for obstacles

- The current Gaussian puff model is not aware of obstacles, like tanks or buildings.
- Use the Method of Fundamental Solutions (MFS) to approximate the wind field, $\mu(x)$, around an obstacle

$$u(x) = \nabla \phi(x) + u_\infty$$

$$\phi(x) = \sum_{i=1}^{N_1} \alpha_i G_{x_i}(x)$$



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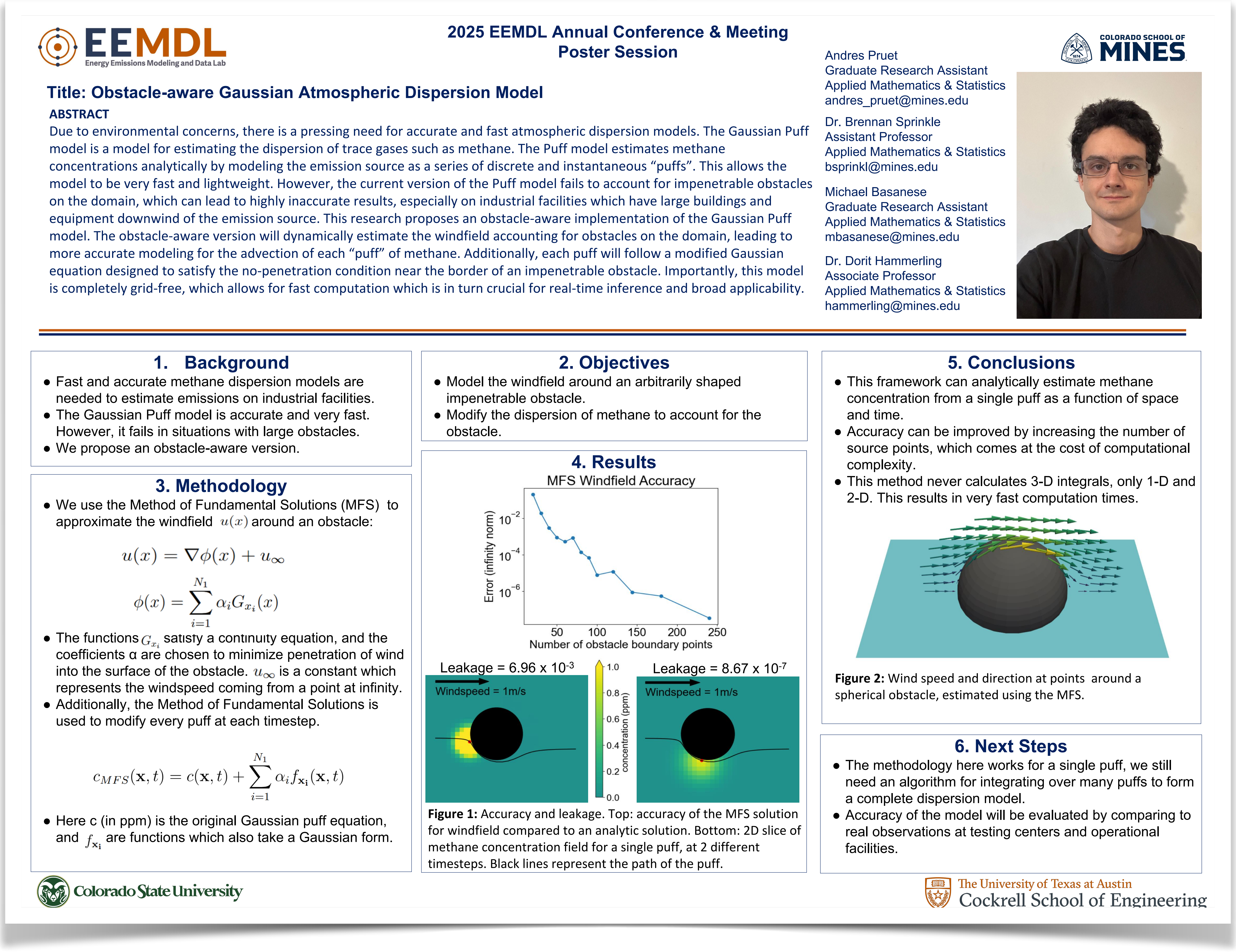
Direction #2: Account for obstacles



EEMDL Energy Emissions Modeling and Data Lab

2025 EEMDL Annual Conference & Meeting
Poster Session

COLORADO SCHOOL OF MINES

Title: Obstacle-aware Gaussian Atmospheric Dispersion Model

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Applied Mathematics & Statistics
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ABSTRACT

Due to environmental concerns, there is a pressing need for accurate and fast atmospheric dispersion models. The Gaussian Puff model is a model for estimating the dispersion of trace gases such as methane. The Puff model estimates methane concentrations analytically by modeling the emission source as a series of discrete and instantaneous "puffs". This allows the model to be very fast and lightweight. However, the current version of the Puff model fails to account for impenetrable obstacles on the domain, which can lead to highly inaccurate results, especially on industrial facilities which have large buildings and equipment downwind of the emission source. This research proposes an obstacle-aware implementation of the Gaussian Puff model. The obstacle-aware version will dynamically estimate the windfield accounting for obstacles on the domain, leading to more accurate modeling for the advection of each "puff" of methane. Additionally, each puff will follow a modified Gaussian equation designed to satisfy the no-penetration condition near the border of an impenetrable obstacle. Importantly, this model is completely grid-free, which allows for fast computation which is in turn crucial for real-time inference and broad applicability.

1. Background

- Fast and accurate methane dispersion models are needed to estimate emissions on industrial facilities.
- The Gaussian Puff model is accurate and very fast. However, it fails in situations with large obstacles.
- We propose an obstacle-aware version.

2. Objectives

- Model the windfield around an arbitrarily shaped impenetrable obstacle.
- Modify the dispersion of methane to account for the obstacle.

3. Methodology

- We use the Method of Fundamental Solutions (MFS) to approximate the windfield $u(x)$ around an obstacle:

$$u(x) = \nabla\phi(x) + u_\infty$$

$$\phi(x) = \sum_{i=1}^{N_1} \alpha_i G_{x_i}(x)$$

- The functions G_{x_i} satisfy a continuity equation, and the coefficients α are chosen to minimize penetration of wind into the surface of the obstacle. u_∞ is a constant which represents the windspeed coming from a point at infinity.
- Additionally, the Method of Fundamental Solutions is used to modify every puff at each timestep.

$$c_{MFS}(\mathbf{x}, t) = c(\mathbf{x}, t) + \sum_{i=1}^{N_1} \alpha_i f_{\mathbf{x_i}}(\mathbf{x}, t)$$

- Here c (in ppm) is the original Gaussian puff equation, and $f_{\mathbf{x_i}}$ are functions which also take a Gaussian form.

4. Results

MFS Windfield Accuracy

Error (infinity norm)

Number of obstacle boundary points

Leakage = 6.96 x 10^{-3}

Windspeed = 1m/s

Leakage = 8.67 x 10^{-7}

Windspeed = 1m/s

concentration (ppm)

Figure 1: Accuracy and leakage. Top: accuracy of the MFS solution for windfield compared to an analytic solution. Bottom: 2D slice of methane concentration field for a single puff, at 2 different timesteps. Black lines represent the path of the puff.

5. Conclusions

- This framework can analytically estimate methane concentration from a single puff as a function of space and time.
- Accuracy can be improved by increasing the number of source points, which comes at the cost of computational complexity.
- This method never calculates 3-D integrals, only 1-D and 2-D. This results in very fast computation times.

Figure 2: Wind speed and direction at points around a spherical obstacle, estimated using the MFS.

6. Next Steps

- The methodology here works for a single puff, we still need an algorithm for integrating over many puffs to form a complete dispersion model.
- Accuracy of the model will be evaluated by comparing to real observations at testing centers and operational facilities.

Colorado State University

The University of Texas at Austin Cockrell School of Engineering

See Andres Pruet's poster for more details! (presented by Michael)

"Obstacle-aware Gaussian Atmospheric Dispersion Model"

Open source framework for solving inverse problem

STEP 1:
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and event detection

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Simulation

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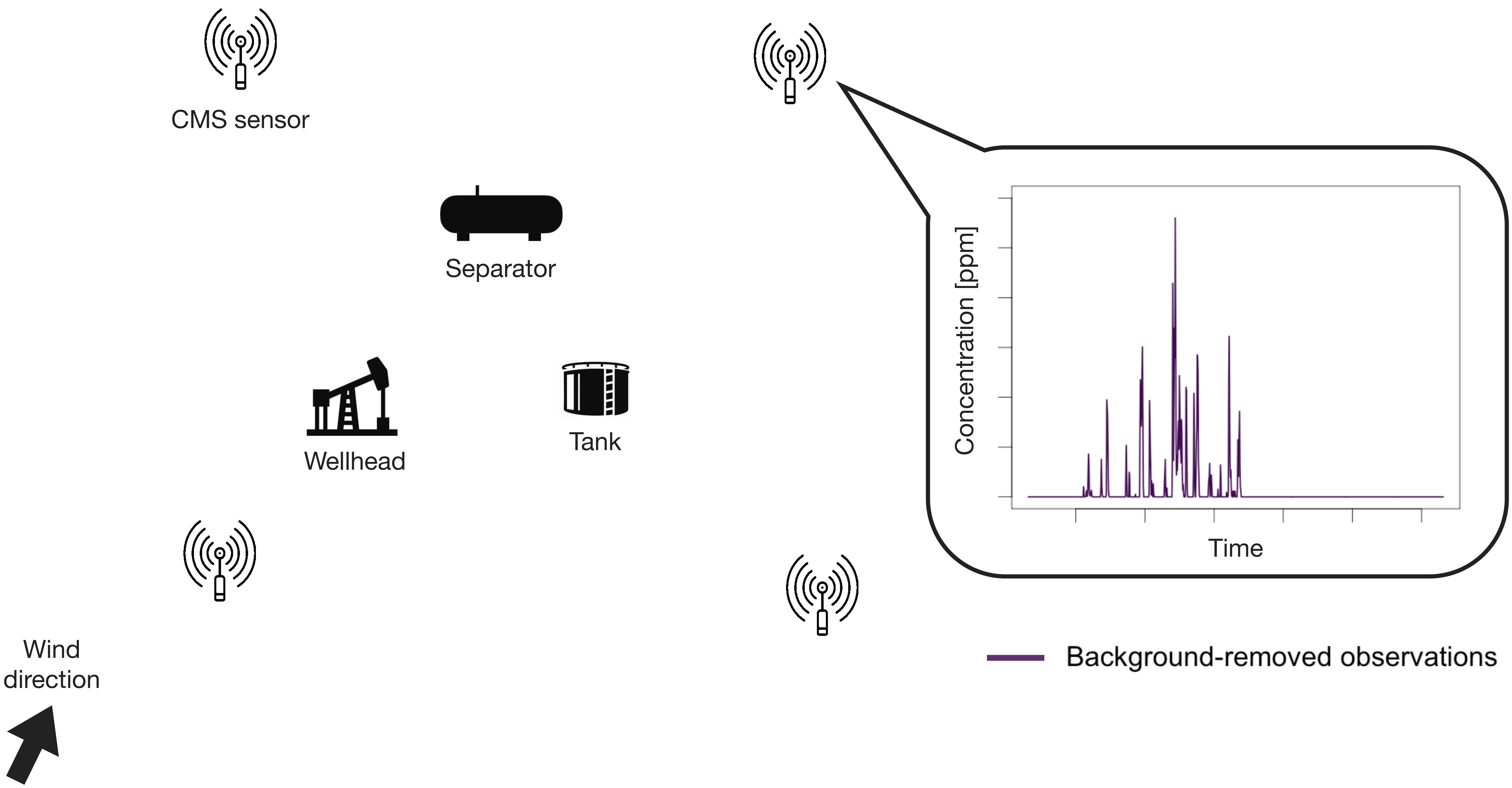
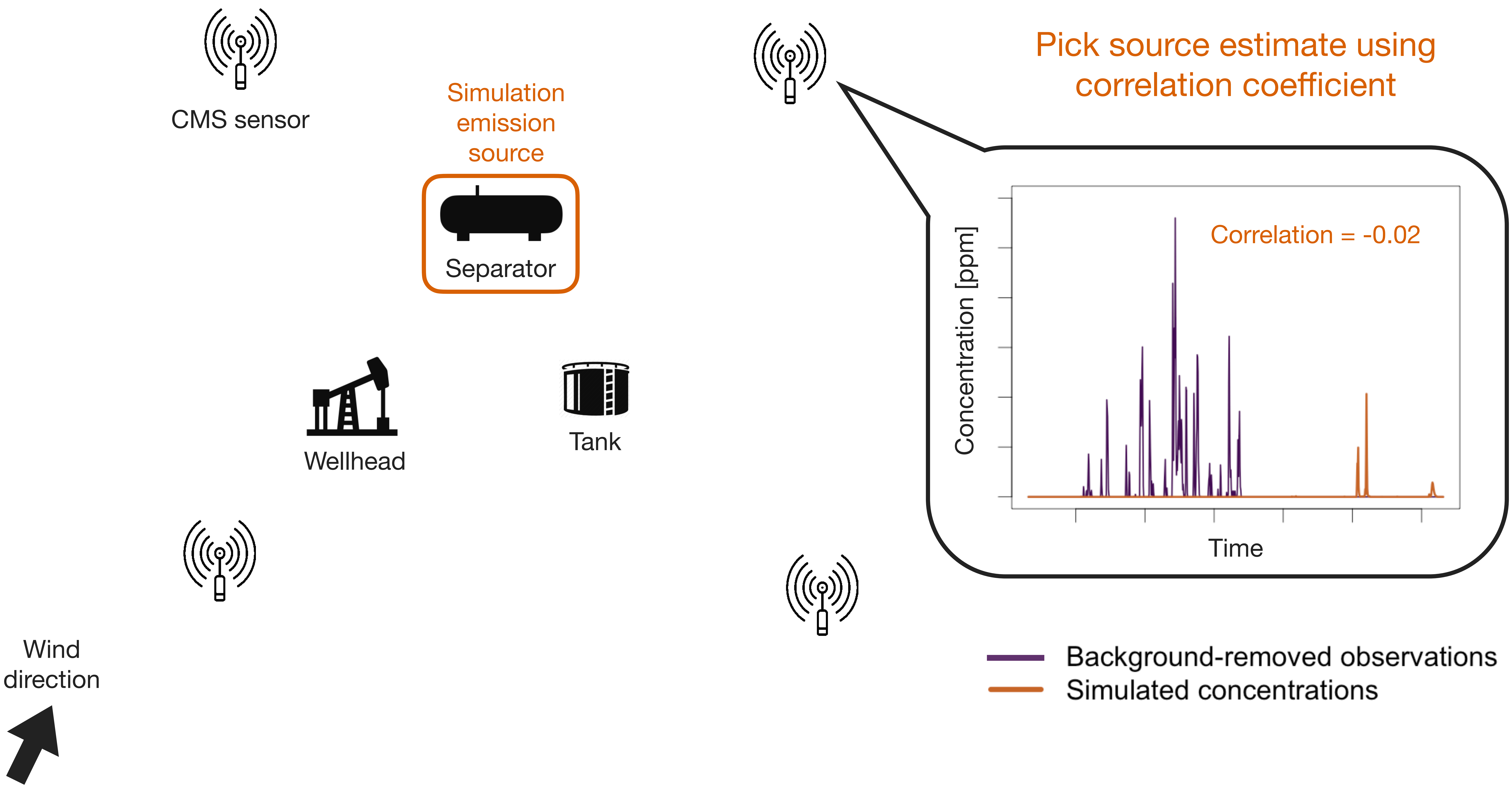
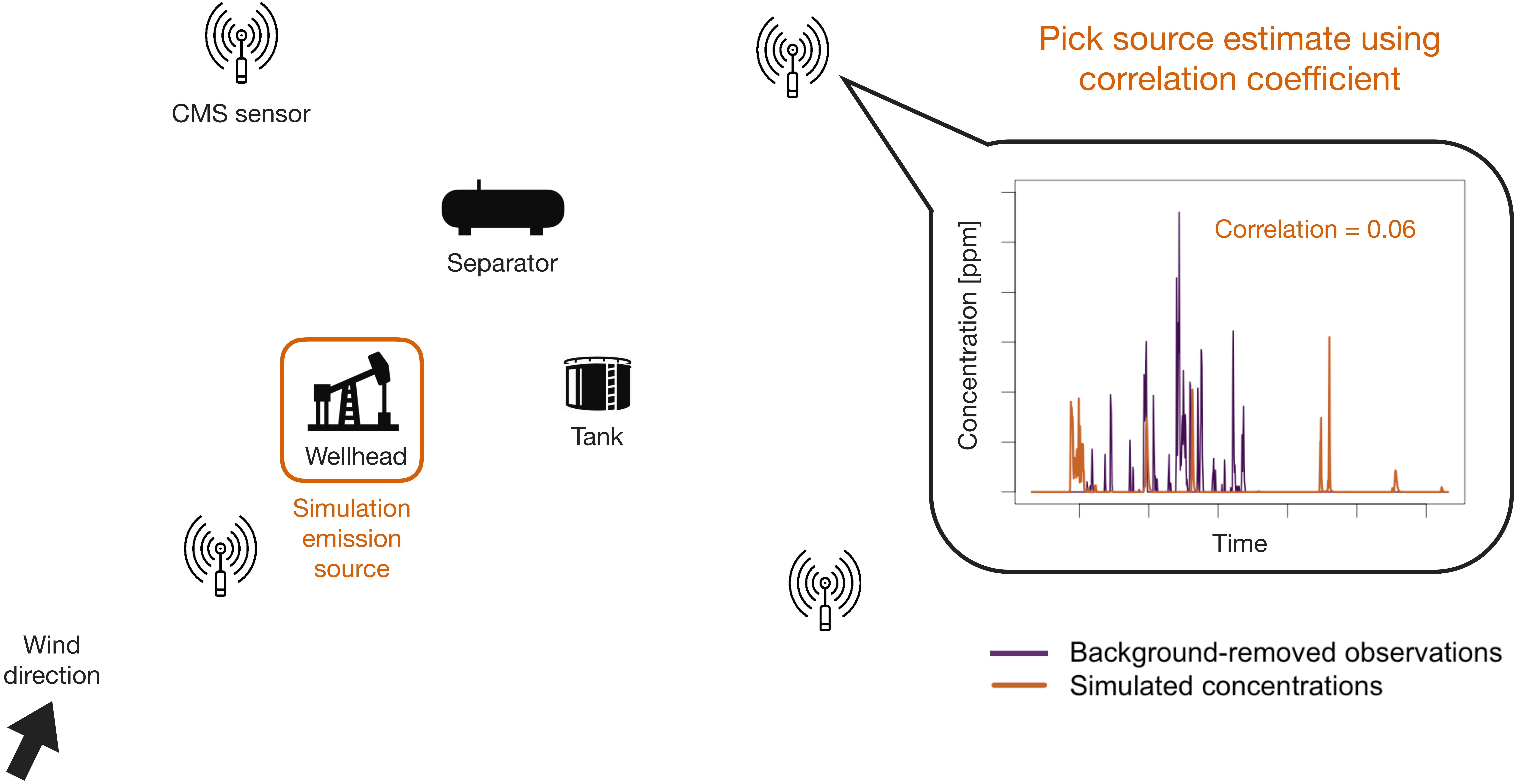
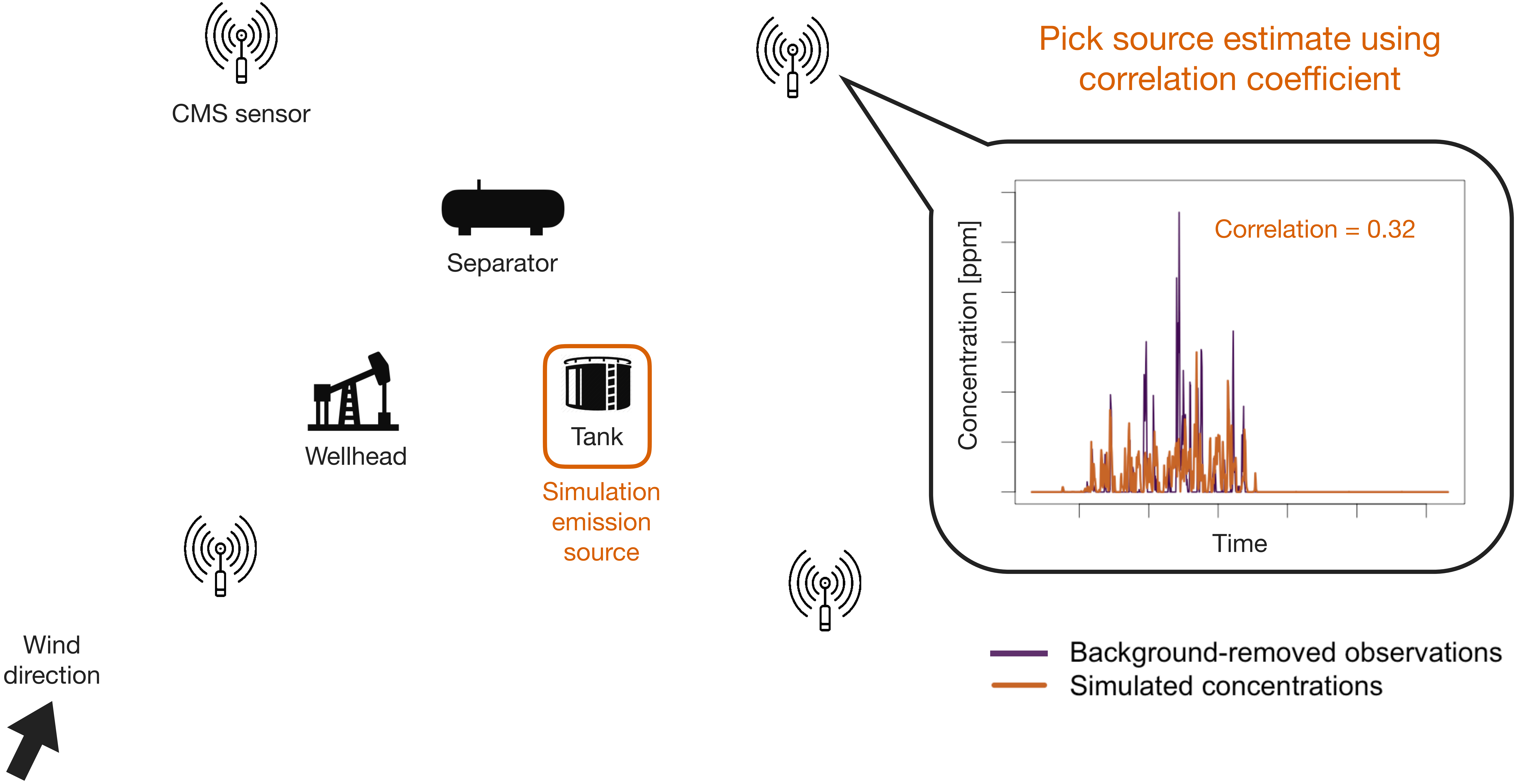
STEP 4:
Quantification


CMS sensor
Separator
Wellhead
Tank
Wind
direction
Concentration [ppm]
Time
Background-removed observations


CMS sensor
Simulation emission source
Separator
Wellhead
Tank
Wind direction
Pick source estimate using correlation coefficient
Correlation = -0.02
Concentration [ppm]
Time
Background-removed observations
Simulated concentrations



CMS sensor
Separator
Wellhead
Tank
Simulation emission source
Wind direction
Pick source estimate using correlation coefficient
Correlation = 0.06
Concentration [ppm]
Time
Background-removed observations
Simulated concentrations


CMS sensor
Separator
Wellhead
Tank
Simulation
emission
source
Wind
direction
Pick source estimate using
correlation coefficient
Correlation = 0.32
Concentration [ppm]
Time
Background-removed observations
Simulated concentrations

Open source framework for solving inverse problem

STEP 1:
Background removal
and event detection

→

STEP 2:
Simulation

→

STEP 3:
Localization

→

STEP 4:
Quantification

Simulation is a linear function of emission rate

Volume of methane contained in puff p

$$c_p(x,y,z,t,Q) = Q\,\frac{1}{(2\pi)^{3/2}\sigma_y^2\sigma_z}\exp\left(-\frac{(x-ut)^2+y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right)+\exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Concentration contribution of puff p

$$c(x,y,z,t,Q) = \sum_{p=1}^{P} c_p(x,y,z,t,Q)$$

Total concentration at (x,y,z,t)

Simulation is a linear function of emission rate

Volume of methane contained in puff p

$$c_p(x, y, z, t, Q) = Q \frac{1}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x-ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right]$$

Concentration contribution of puff p

$$c(x, y, z, t, Q) = \sum_{p=1}^{P} c_p(x, y, z, t, Q)$$

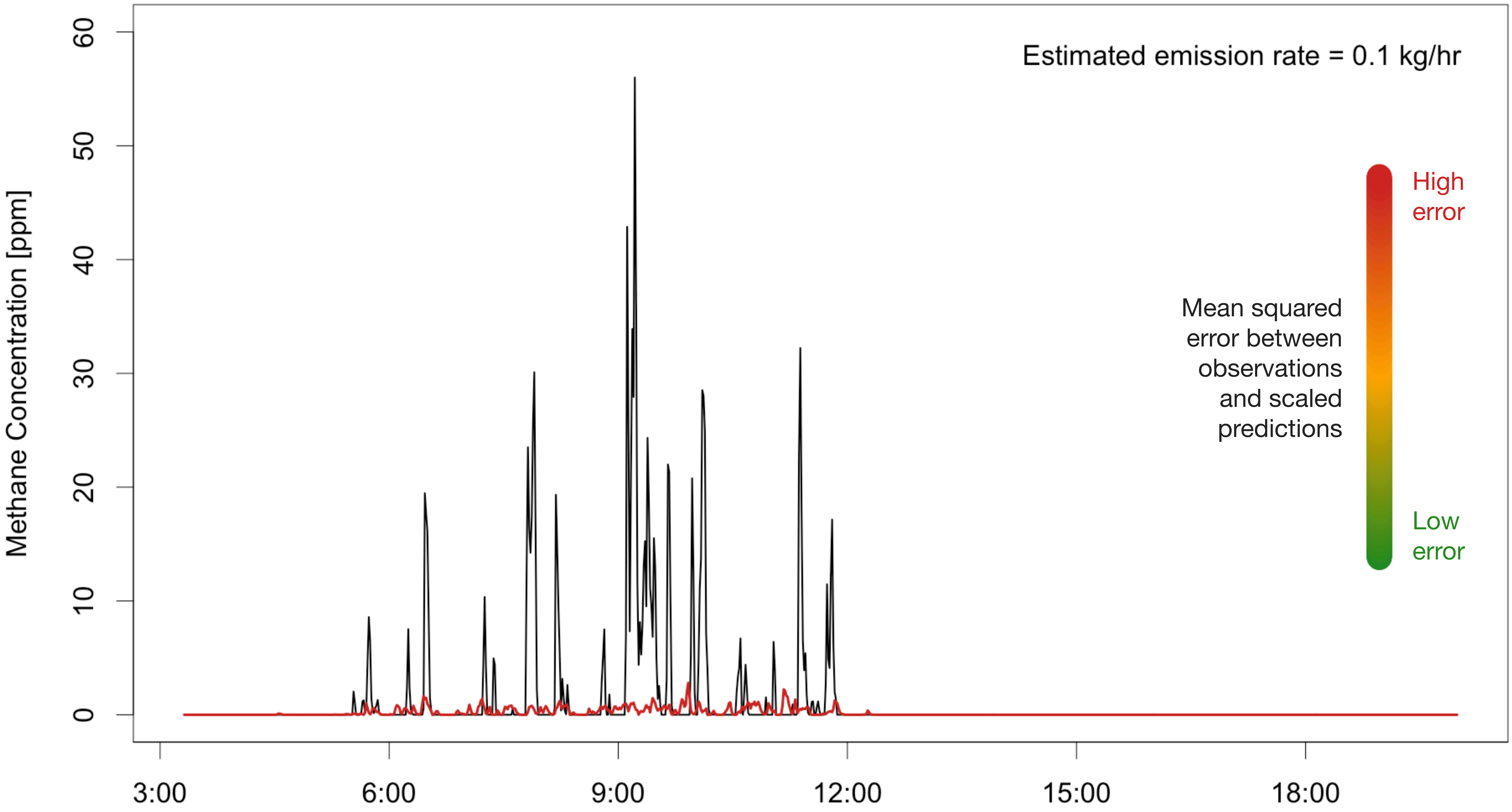
Total concentration at (x, y, z, t)

Concentration data

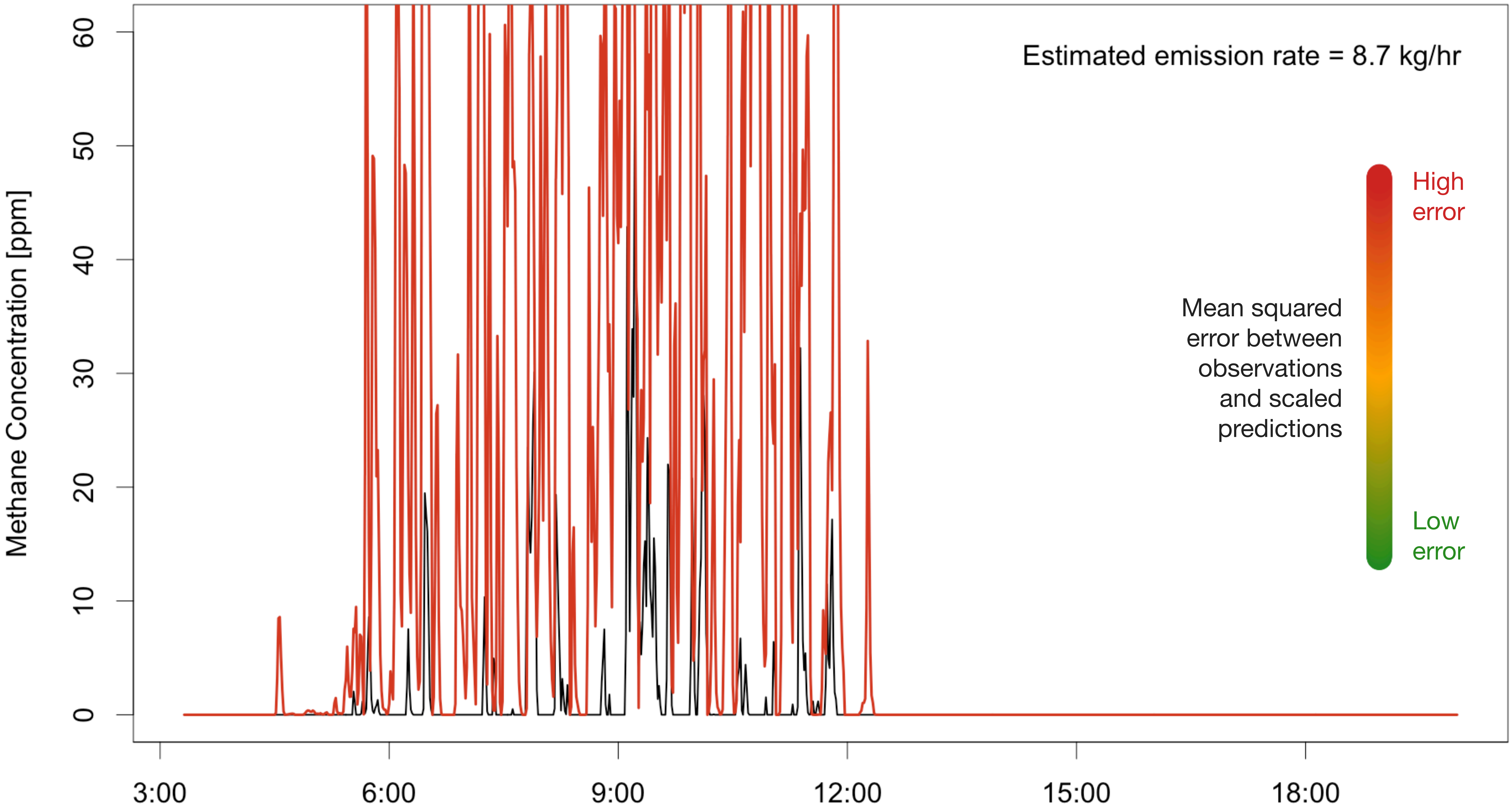
$$\hat{Q} = \underset{Q}{\operatorname{argmin}} \left\{ \frac{1}{n} \sum_{t=1}^{n} \left(d(x, y, z, t) - c(x, y, z, t, Q) \right)^2 \right\}$$

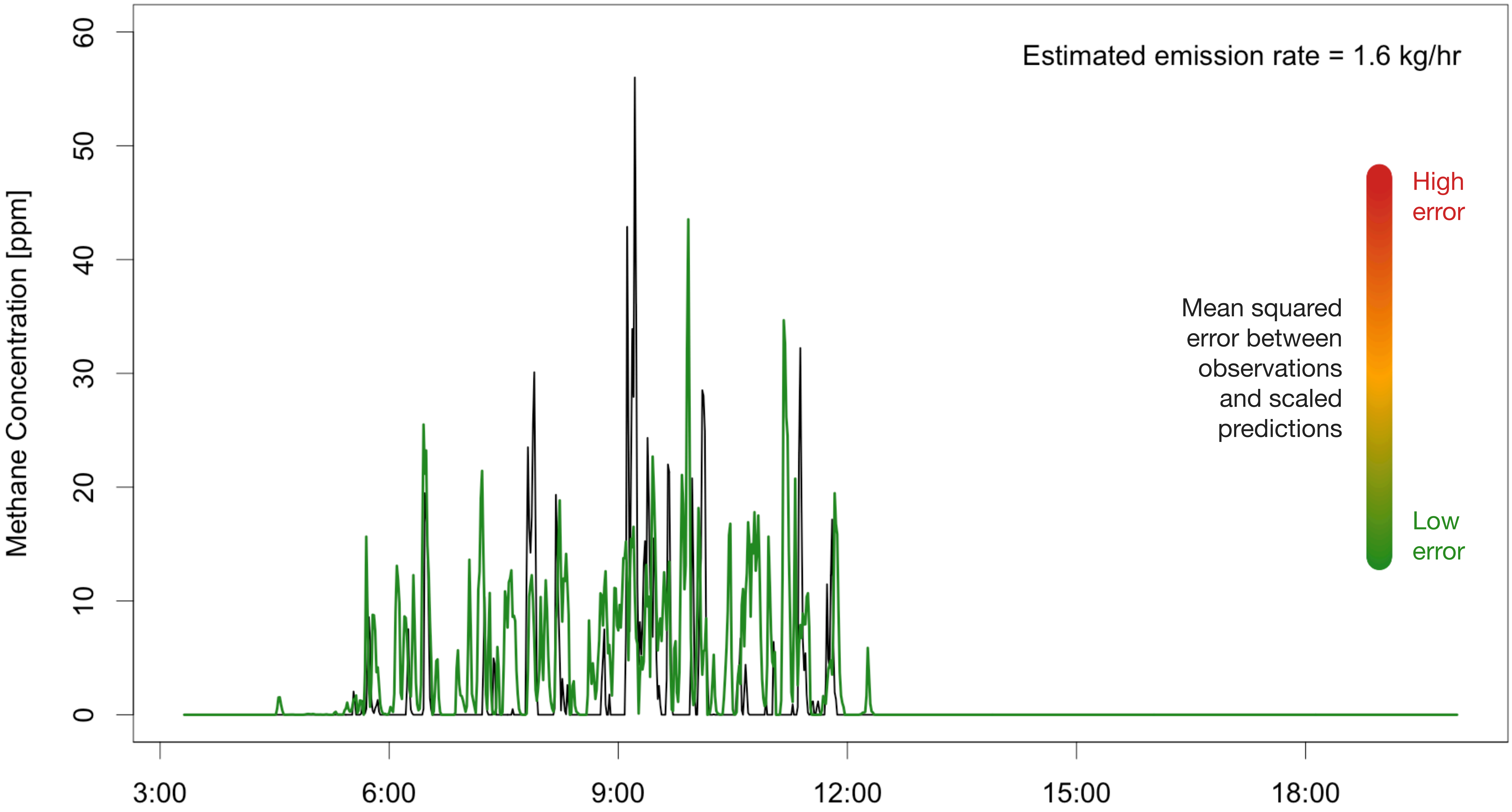
Emission rate estimate

Simulated concentrations



Estimated emission rate = 0.1 kg/hr
Methane Concentration [ppm]
0
10
20
30
40
50
60
3:00
6:00
9:00
12:00
15:00
18:00
Mean squared error between observations and scaled predictions
High error
Low error


Estimated emission rate = 8.7 kg/hr
Methane Concentration [ppm]
0
10
20
30
40
50
60
3:00
6:00
9:00
12:00
15:00
18:00
High error
Mean squared error between observations and scaled predictions
Low error


Estimated emission rate = 1.6 kg/hr
Methane Concentration [ppm]
0
10
20
30
40
50
60
3:00
6:00
9:00
12:00
15:00
18:00
Mean squared error between observations and scaled predictions
High error
Low error

Open source framework for solving inverse problem

STEP 1:
Background removal
and event detection

→

STEP 2:
Simulation

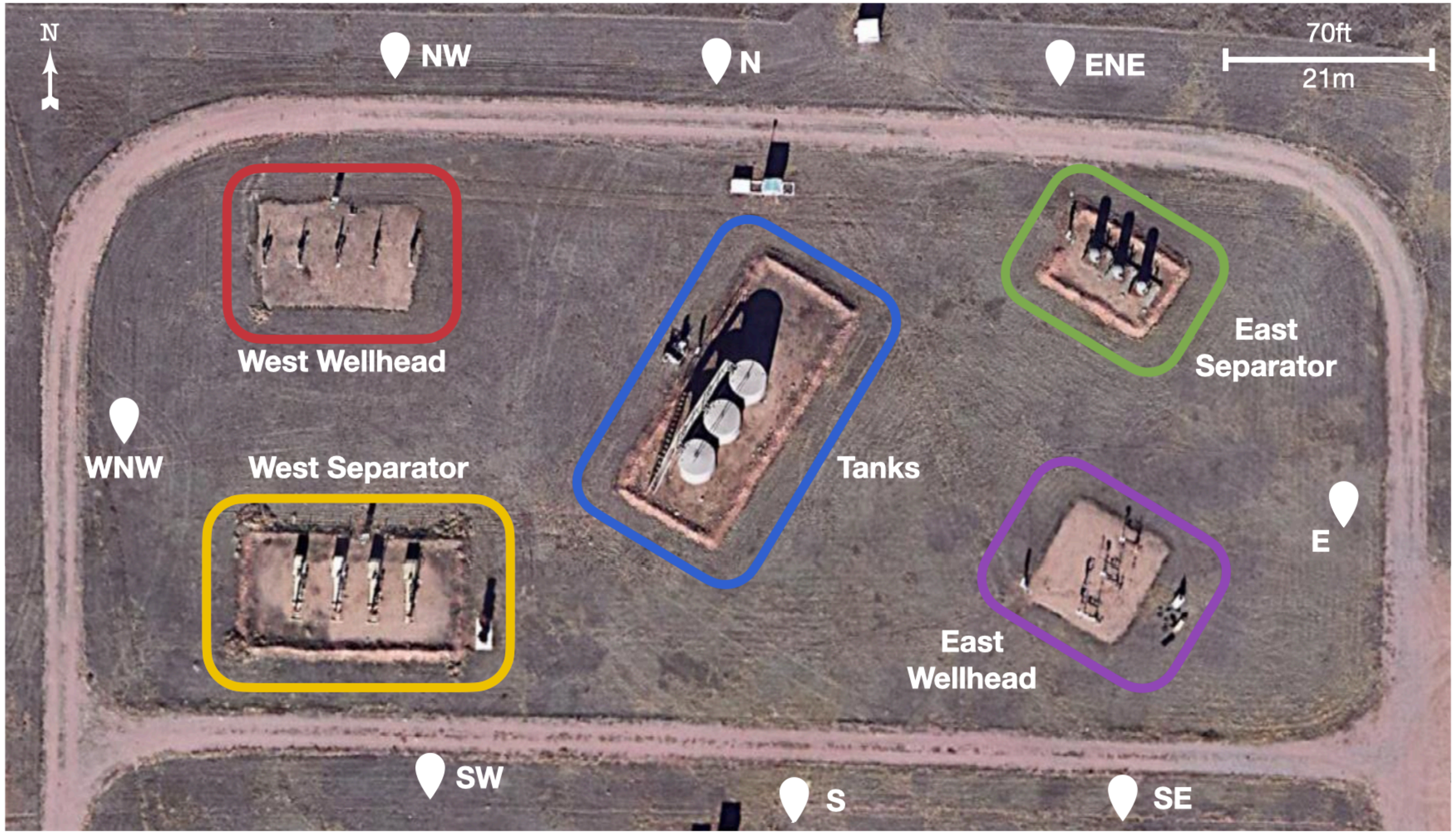
→

STEP 3:
Localization

→

STEP 4:
Quantification

Evaluation on single-source controlled releases



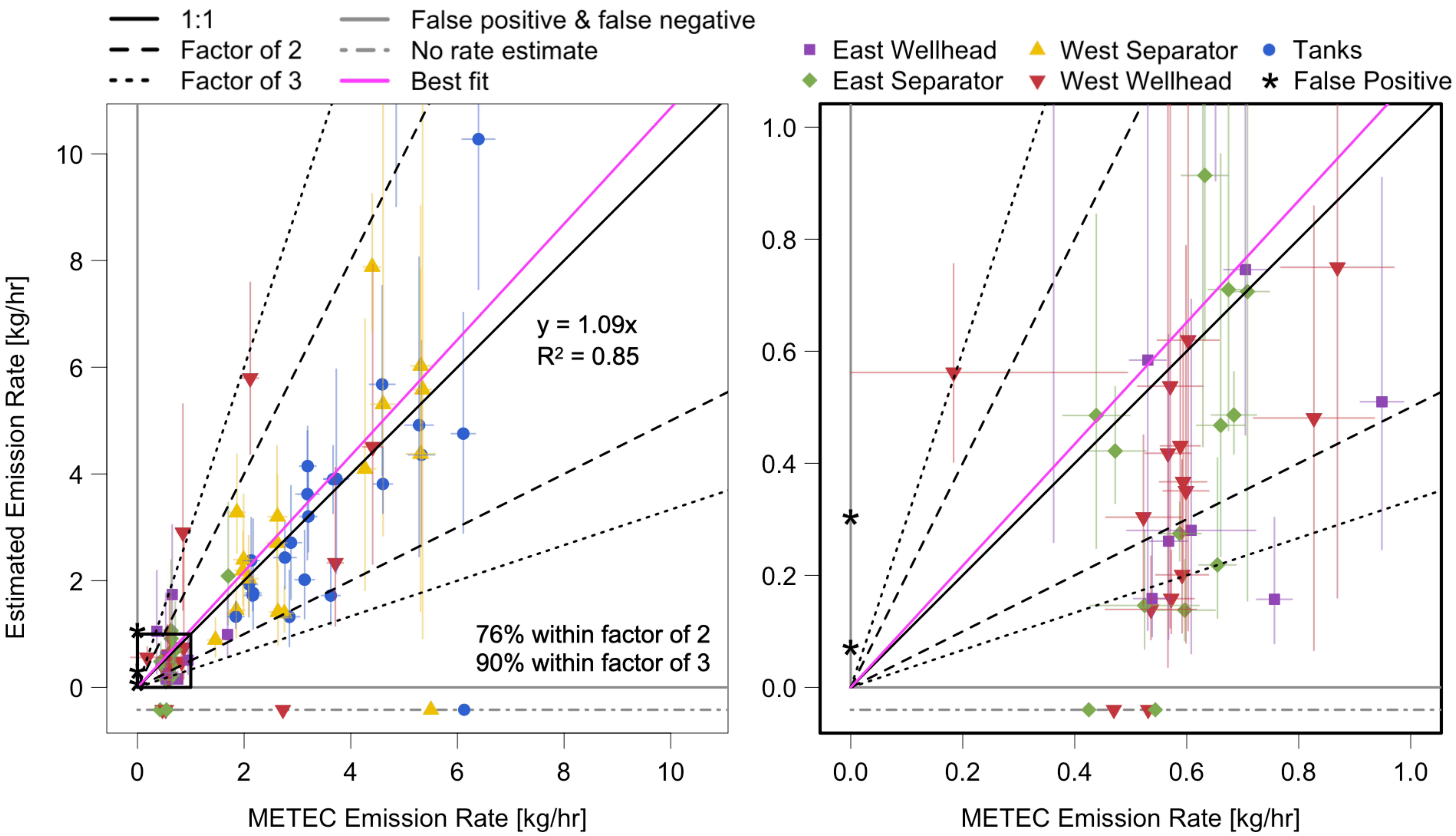
Methane Emissions Technology Evaluation Center (METEC)

85 single-source controlled releases

Emission rates range from **0.2** to **6.4** kg/hr

Emission durations range from **0.5** to **8.25** hours

Evaluation on single-source controlled releases



Part 1: Single-source emission detection, localization, and quantification

Detection, localization, and quantification of single-source methane emissions on oil and gas production sites using point-in-space continuous monitoring systems.

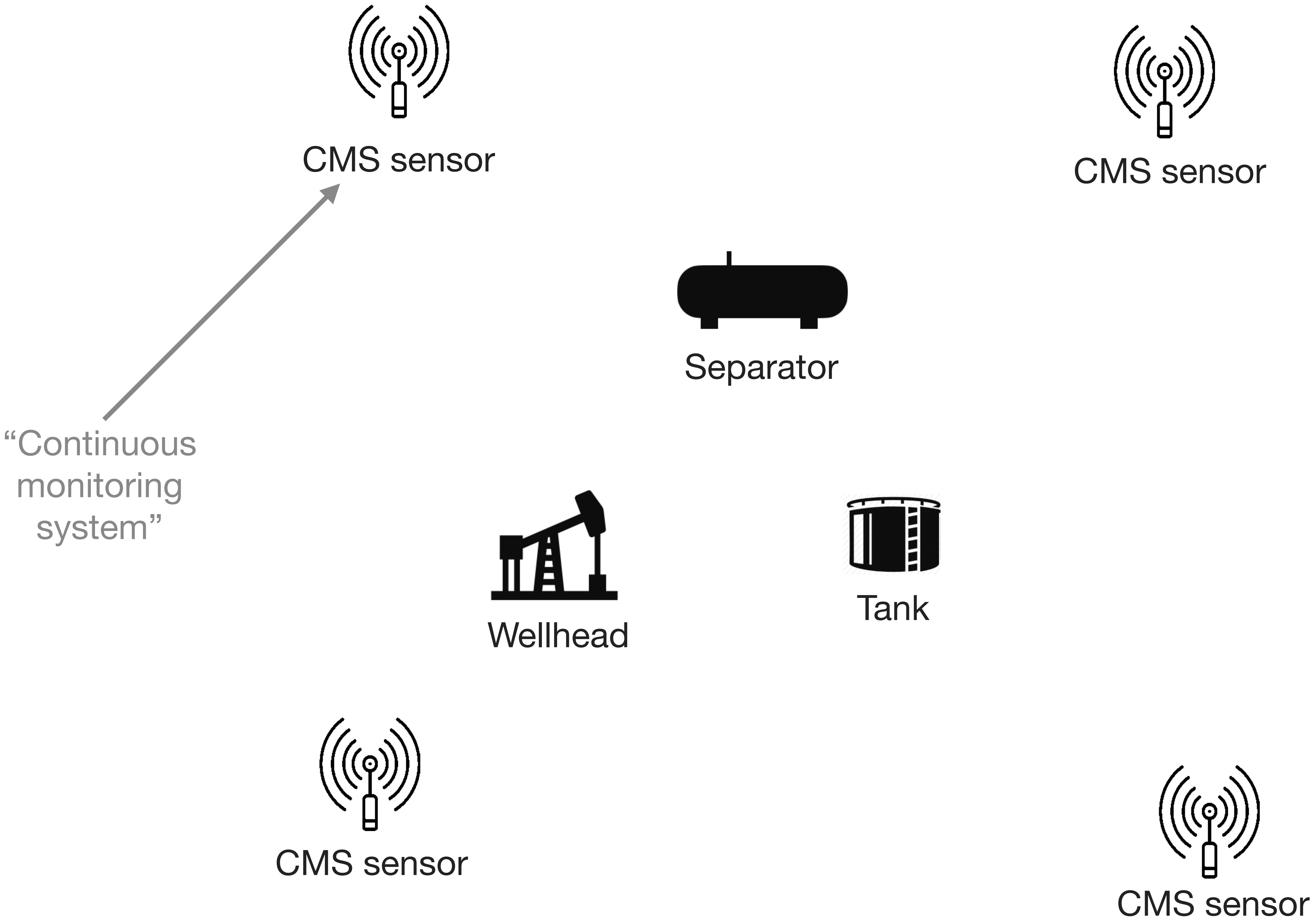
William Daniels, Meng Jia, Dorit Hammerling.

Elementa: Science of the Anthropocene, 12(1), 00110, (2024).

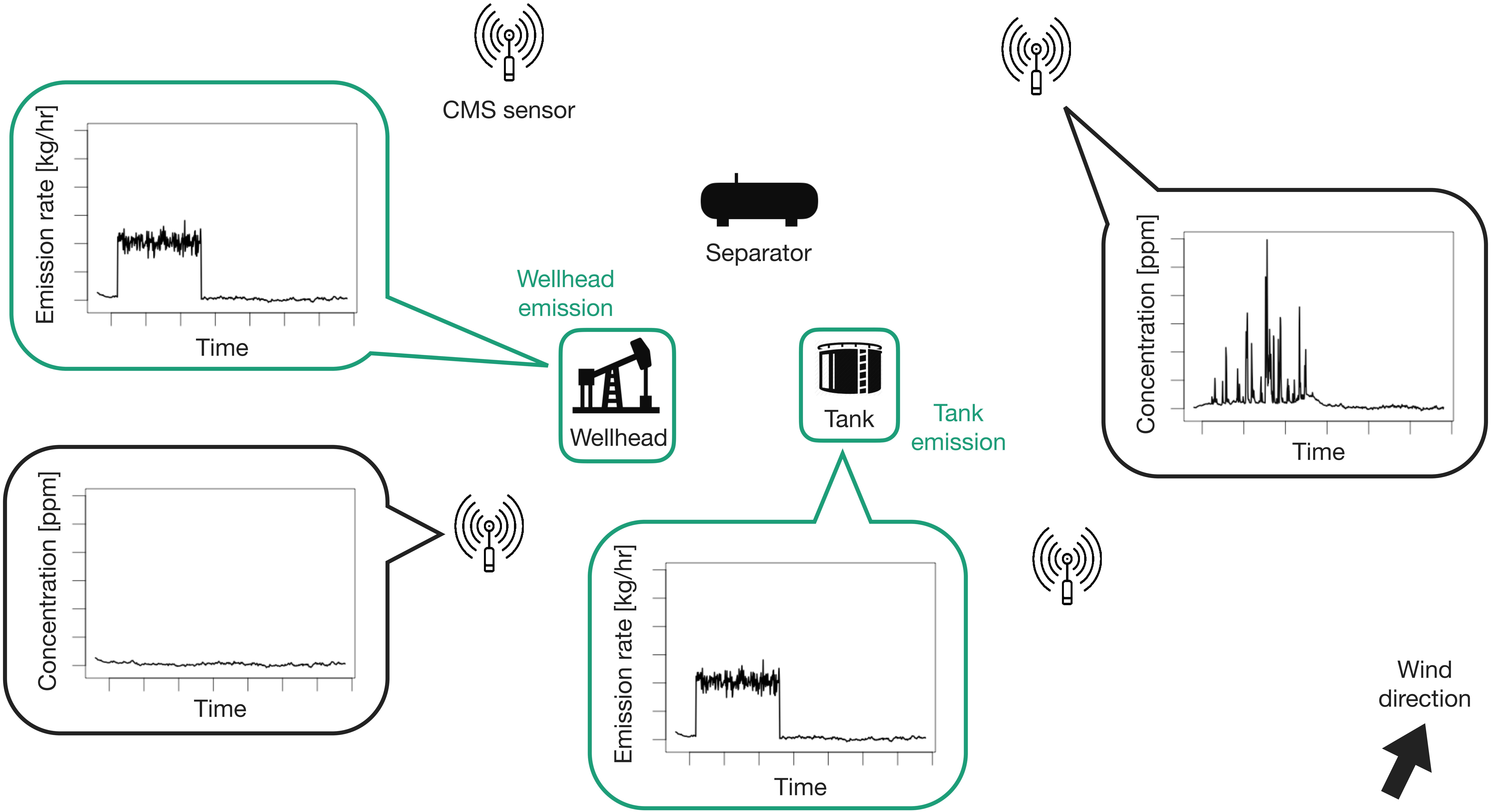
A fast and lightweight implementation of the Gaussian puff model for near-field atmospheric transport of trace gasses.

Meng Jia, Ryker Fish, William Daniels, Brennan Sprinkle, Dorit Hammerling.

Scientific Reports, 15, 18710 (2025).



The multi-source continuous monitoring inverse problem


CMS sensor
Emission rate [kg/hr]
Time
Separator
Wellhead emission
Wellhead
Tank
Tank emission
Concentration [ppm]
Time
Concentration [ppm]
Time
Emission rate [kg/hr]
Time
Wind direction

Model hierarchy

Assume a multiple linear regression model at the data level

n = number of observations
p = number of potential sources

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \ldots, y_n\}, \beta \equiv \{\beta_1, \ldots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Concentration observations from CMS sensors

Emission rates for each source

Simulated concentrations from forward model, with each column assuming a different source

Model hierarchy

n = number of observations
p = number of potential sources

Assume a multiple linear regression model at the data level

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \ldots, y_n\}, \beta \equiv \{\beta_1, \ldots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

$$\epsilon \sim N(0, \sigma^2 R)$$

Model hierarchy

n = number of observations
p = number of potential sources

Assume a multiple linear regression model at the data level

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \ldots, y_n\}, \beta \equiv \{\beta_1, \ldots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

$$\epsilon \sim N(0, \sigma^2 R)$$

Let the errors follow an AR(1) process such that

$$\epsilon_i = r\epsilon_{i-1} + w$$

Autocorrelation coefficient

Gaussian white noise

Model hierarchy

n = number of observations
p = number of potential sources

Assume a multiple linear regression model at the data level

$$y = X\beta + \epsilon$$

$$y \equiv \{y_1, \ldots, y_n\}, \beta \equiv \{\beta_1, \ldots, \beta_p\}, X \in \mathbb{R}^{n \times p}$$

Assume that the errors $\epsilon \equiv \{\epsilon_1, \ldots, \epsilon_n\}$ are are identically distributed, Gaussian, and autocorrelated such that

$$\epsilon \sim N(0,\sigma^2 R)$$

Let the errors follow an AR(1) process such that

$$\epsilon_i = r\epsilon_{i-1} + w$$

This gives us: $y \sim N(X\beta, \sigma^2 R)$

Model hierarchy

n = number of observations
p = number of potential sources

Given an AR(1) process for ϵ, the correlation matrix is

$$R = \begin{bmatrix} 1 & r & r^2 & \dots & r^{n-1} \\ r & 1 & r & \dots & \vdots \\ r^2 & r & 1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r^{n-1} & \dots & \dots & \dots & 1 \end{bmatrix}$$

Model hierarchy

n = number of observations
p = number of potential sources

Given an AR(1) process for ϵ, the correlation matrix is

$$R = \begin{bmatrix} 1 & r & r^2 & \dots & r^{n-1} \\ r & 1 & r & \dots & \vdots \\ r^2 & r & 1 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r^{n-1} & \dots & \dots & \dots & 1 \end{bmatrix}$$

which has closed form expressions for the inverse and determinant:

$$R^{-1} = \frac{1}{(1-r^2)} \begin{bmatrix} 1 & -r & 0 & \dots & 0 \\ -r & 1+r^2 & -r & \dots & \vdots \\ 0 & -r & 1+r^2 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 1 \end{bmatrix}$$

and

$$|R| = (1-r^2)^{n-1}$$

Model hierarchy

n = number of observations
p = number of potential sources

Data-level: $y = X\beta + \epsilon$

$\epsilon \sim N(0,\sigma^2 R)$

The remainder of the hierarchy takes the following form

Spike-and-slab prior allows samples to be identically zero

$$\beta_i \sim \begin{cases} 0, & z_i = 0 \\ \mathrm{Exp}(\tau_i^2\sigma^2), & z_i = 1 \end{cases}$$

"Slab" component is non-negative

$$z_i \sim \mathrm{Bernoulli}(\theta_i)$$

$$\theta_i \sim \mathrm{Beta}(a_i, b_i)$$

$$\tau_i^2 \sim \text{Inv-Gamma}(c_i, d_i)$$

$$\sigma^2 \sim \text{Inv-Gamma}(\nu/2,\ \nu/2)$$

$$\nu \sim \text{Inv-Gamma}(\alpha_1, \alpha_2)$$

$$r \sim \mathrm{Uniform}(0, 1)$$

Proportion of samples where $z_i = 1$ gives posterior probability that source i is emitting

a_i, b_i, c_i, d_i can contain operator insight

Model hierarchy

$$\beta_i \sim \begin{cases} 0, & z_i = 0 \\ \mathrm{Exp}(\tau_i^2 \sigma^2), & z_i = 1 \end{cases}$$

$$z_i \sim \mathrm{Bernoulli}(\theta_i)$$

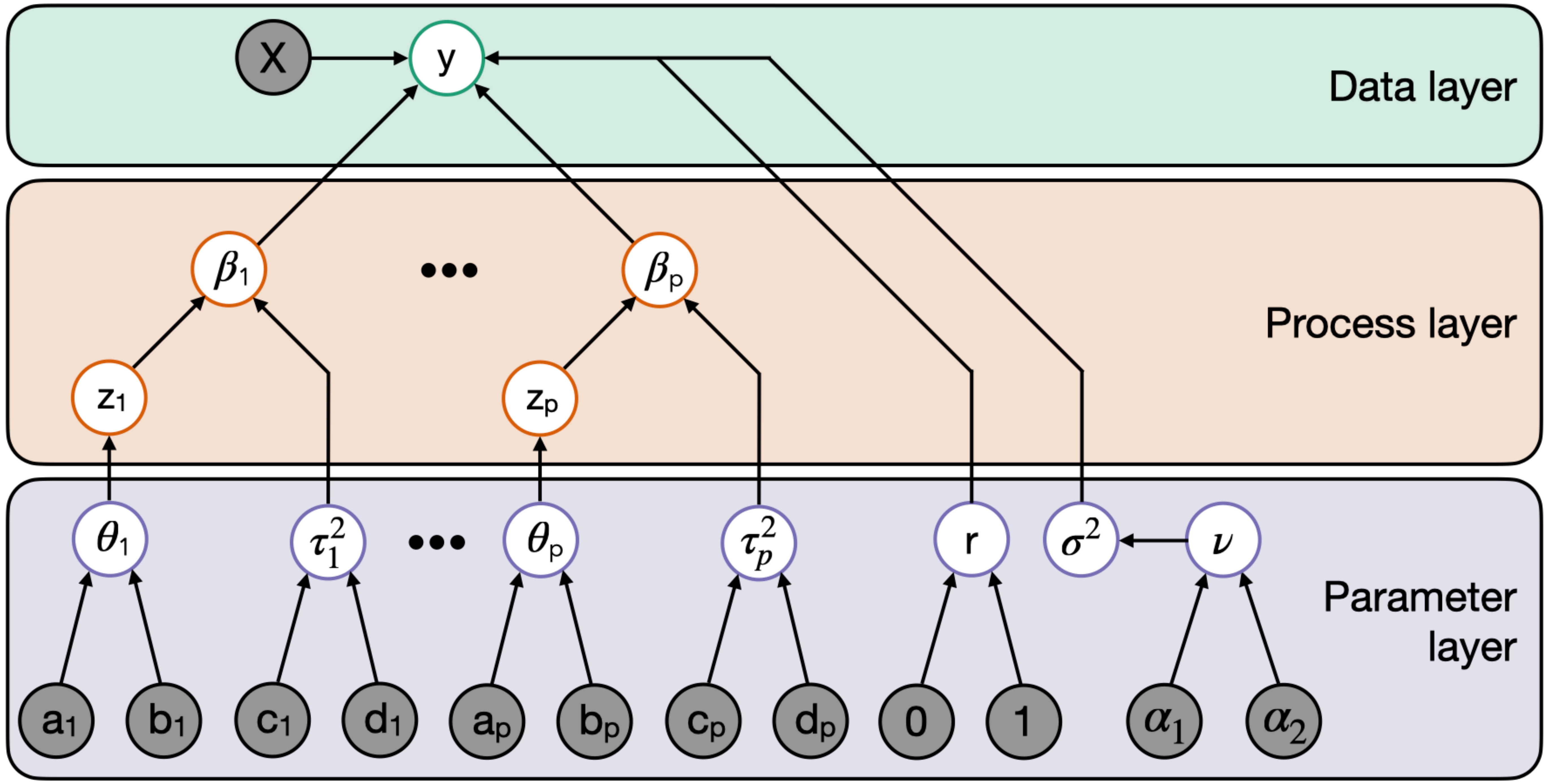
$$\theta_i \sim \mathrm{Beta}(a_i, b_i)$$

$$\tau_i^2 \sim \text{Inv-Gamma}(c_i, d_i)$$

$$\sigma^2 \sim \text{Inv-Gamma}(\nu/2,\ \nu/2)$$

$$\nu \sim \text{Inv-Gamma}(\alpha_1, \alpha_2)$$

$$r \sim \mathrm{Uniform}(0, 1)$$



Sampling from the posterior

We can derive Gibbs updates for all parameters except ν.

$$\theta_i|\xi \sim \text{Beta}(z_i + a_i, 1 - z_i + b_i)$$

$$\sigma^2|\xi \sim \text{Inv-Gamma}\left(\frac{\nu}{2} + \frac{n}{2}, \frac{\nu}{2} + \frac{1}{2}(y - X\beta)^T R^{-1}(y - X\beta)\right)$$

$$r|\xi \sim \begin{cases} \mathcal{N}(X\beta, \sigma^2 R) & 0 < r < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_i^2|\xi \sim \text{Inv-Gamma}\left(z_i + c_i, \frac{\beta_i}{\sigma^2} + d_i\right)$$

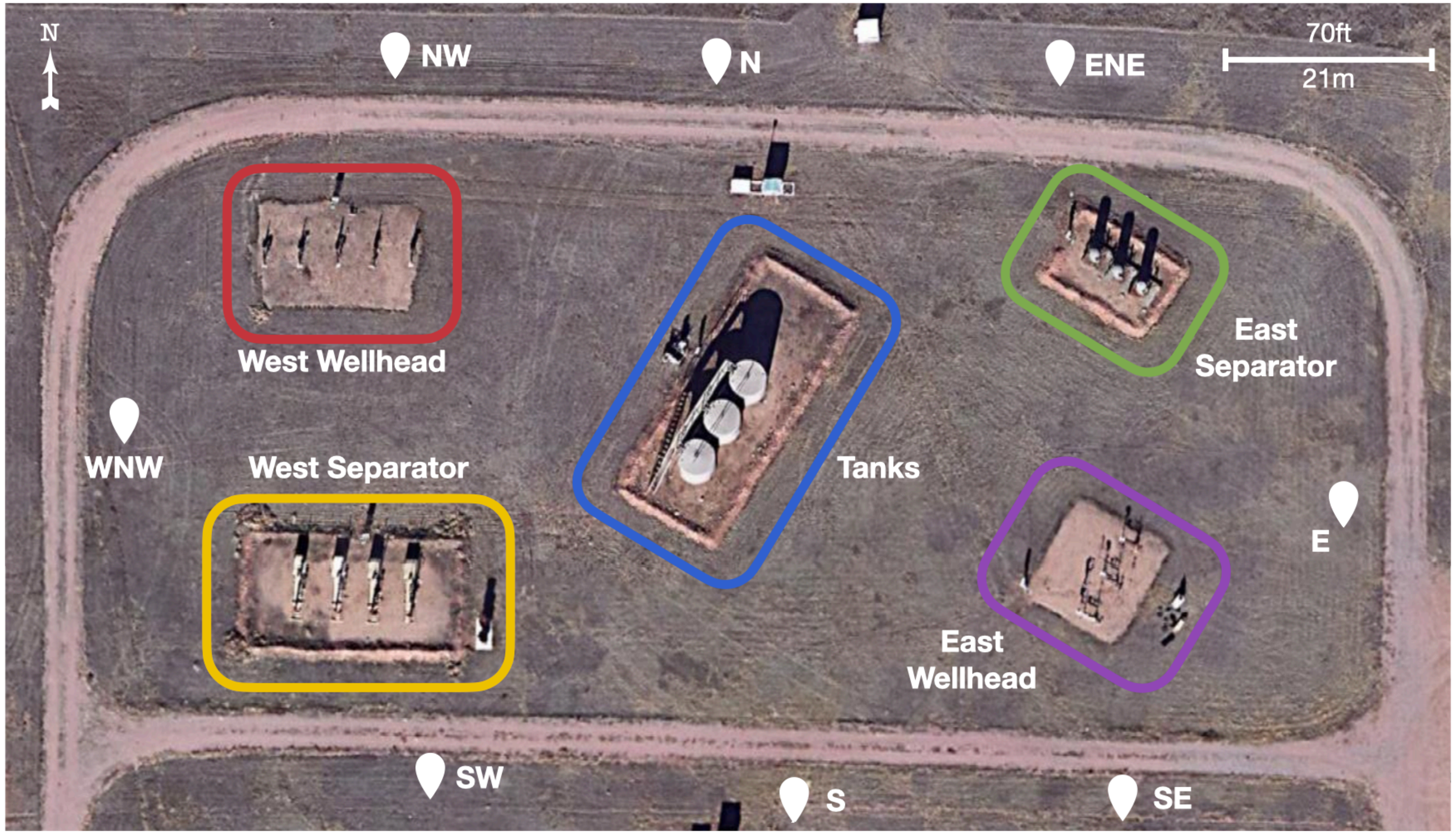
$$\beta_i|\xi \sim \begin{cases} 0 & z_i = 0 \\ \mathcal{N}\left(\left(\frac{X^T R^{-1} X}{\sigma^2}\right)^{-1}\left(\frac{X^T R^{-1} y}{\sigma^2} - \frac{e_i}{\tau_i^2 \sigma^2}\right), \left(\frac{X^T R^{-1} X}{\sigma^2}\right)^{-1}\right) & z_i = 1 \end{cases}$$

$$z_i|\xi \sim \text{Bernoulli}\left(1 - \frac{1 - \theta_i}{(1 - \theta_i) + \theta_i\left(\frac{1}{\tau_i^2 \sigma^2}\right)\exp\left(\frac{\left(\sum_{j=1}^n (w_j X_{j,i}^* + w_j^* X_{j,i}) - \frac{2}{\tau_i^2}\right)^2}{4\sigma^2 \sum_{j=1}^n X_{j,i} X_{j,i}^*}\right)\left(\frac{2\sigma^2\pi}{\sum_{j=1}^n X_{j,i} X_{j,i}^*}\right)^{1/2}\left(\frac{1}{2}\right)}\right)$$

$$\nu|\xi \sim ? \quad \text{(Use a Metropolis–Hastings step)}$$

Iterative samples from each full conditional gives you samples from the joint posterior!

Model evaluation on multi-source controlled release data



Methane Emissions Technology Evaluation Center (METEC)

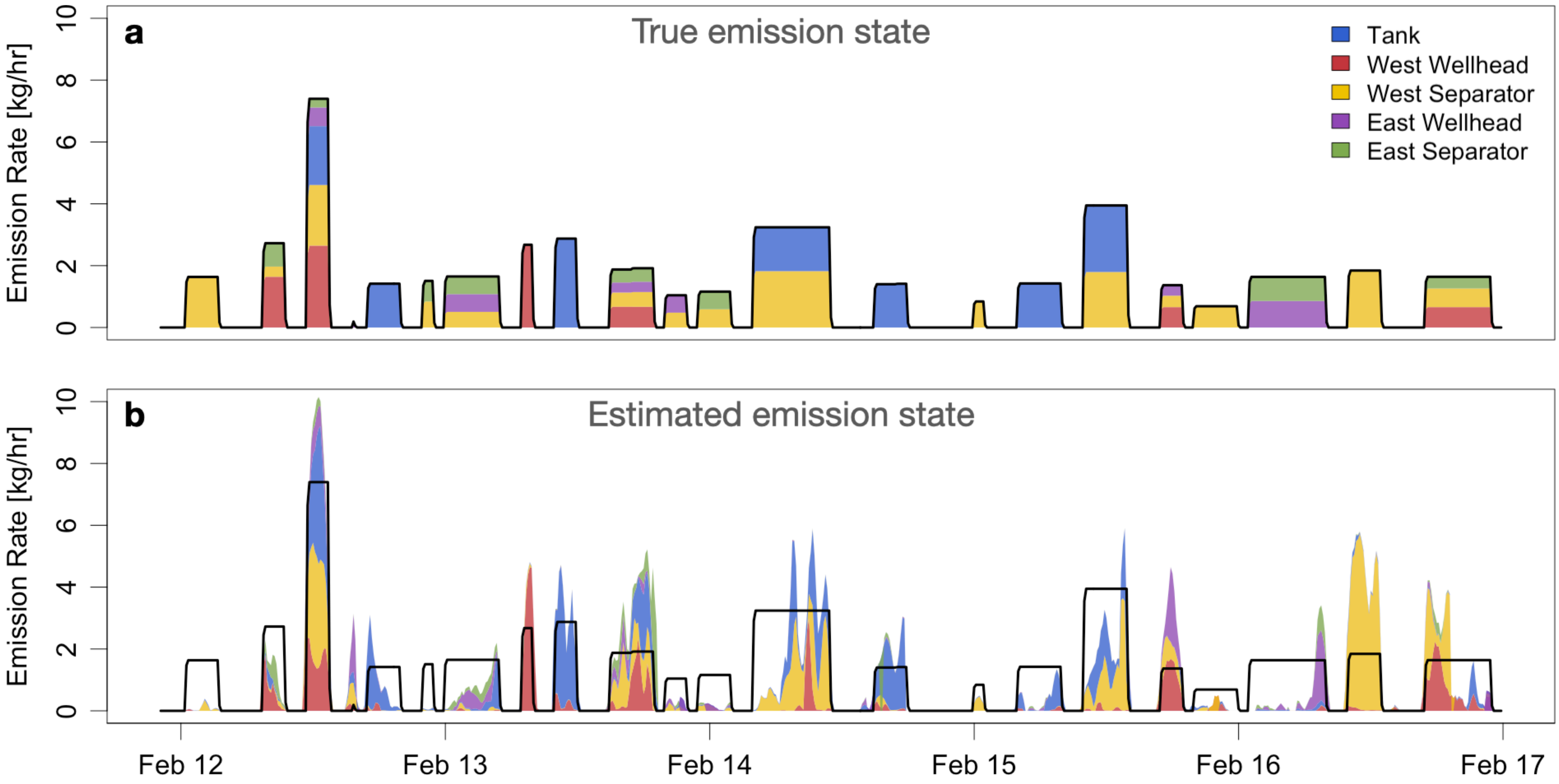
337 controlled releases:

- **99** (29%) single-source
- **238** (71%) multi-source

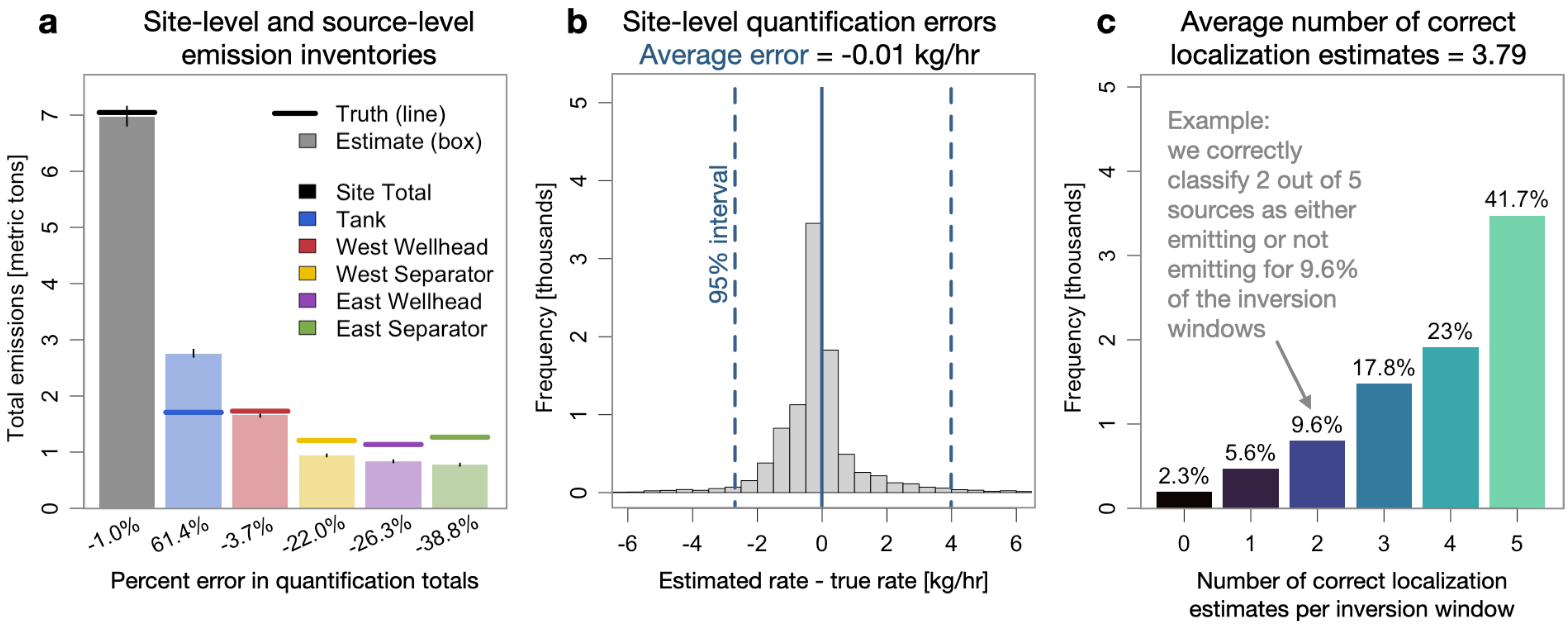
Emission rates range from **0.08** to **7.2** kg/hr

Emission durations range from **0.5** to **8** hours

Model evaluation on multi-source controlled release data



Model evaluation on multi-source controlled release data



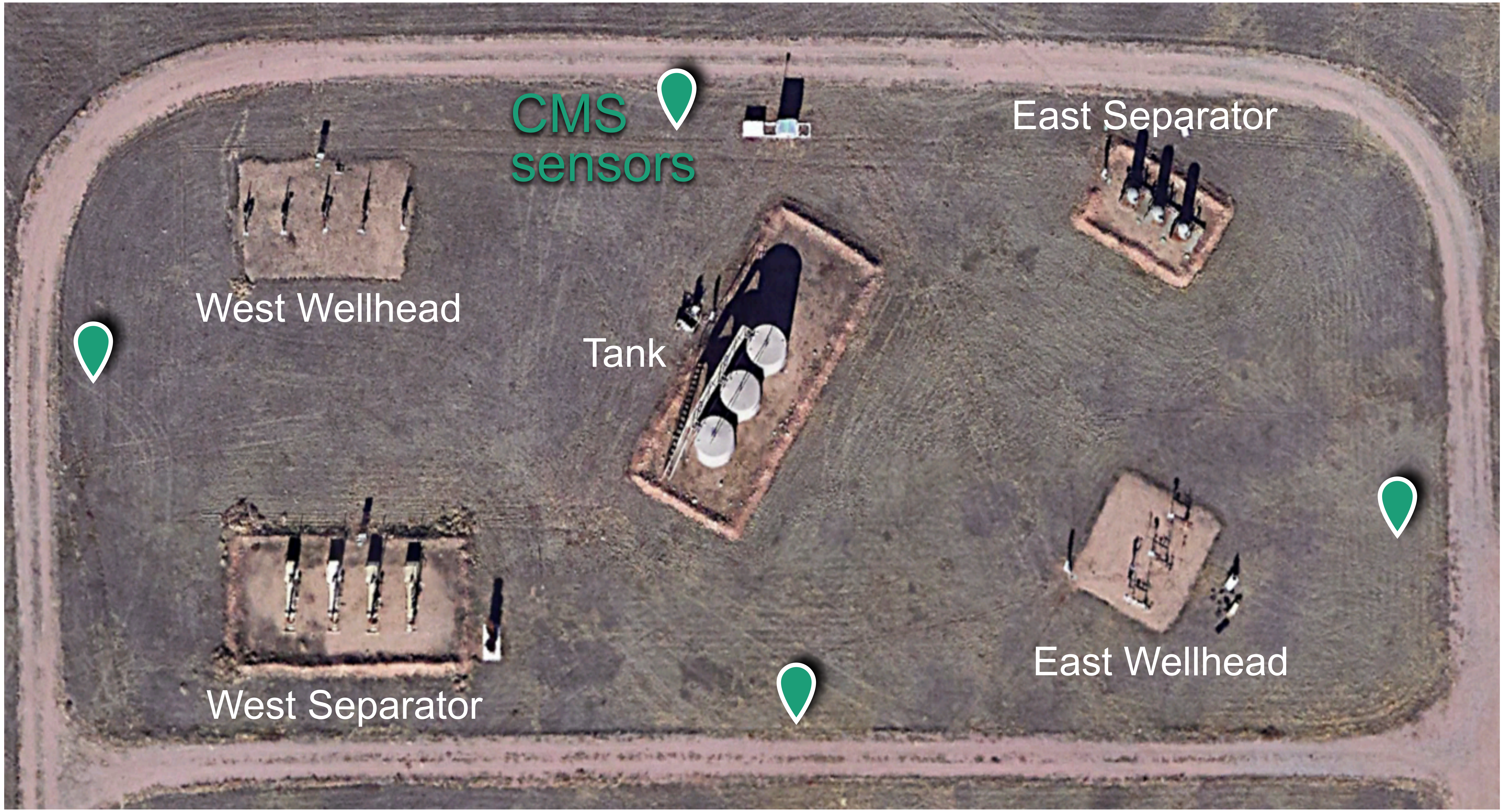
Part 2: Multi-source emission detection, localization, and quantification

A Bayesian hierarchical model for methane emission source apportionment.

William Daniels, Douglas Nychka, Dorit Hammerling.

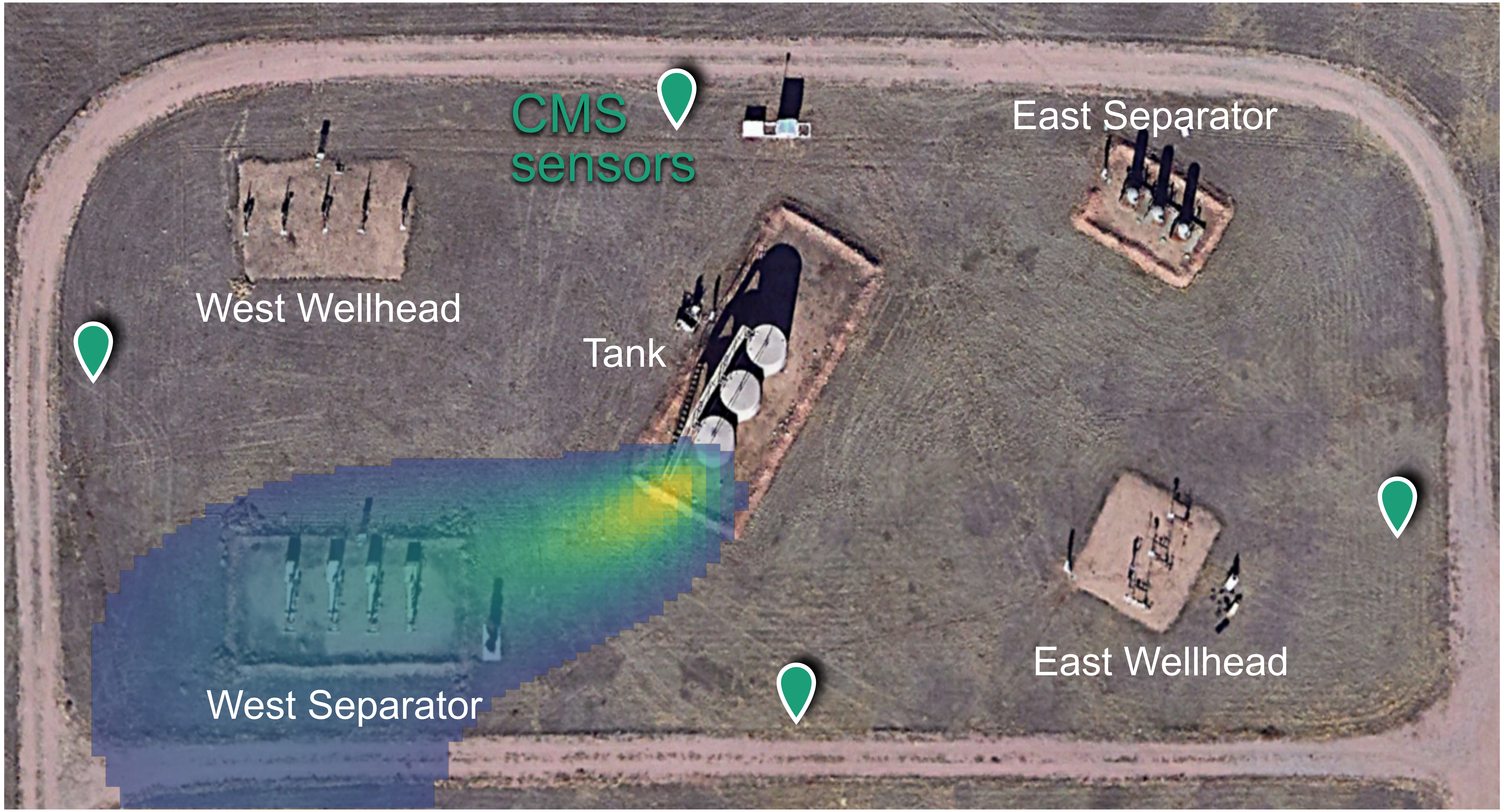
Annals of Applied Statistics, submitted, (2025).

One problem… incomplete sensor coverage



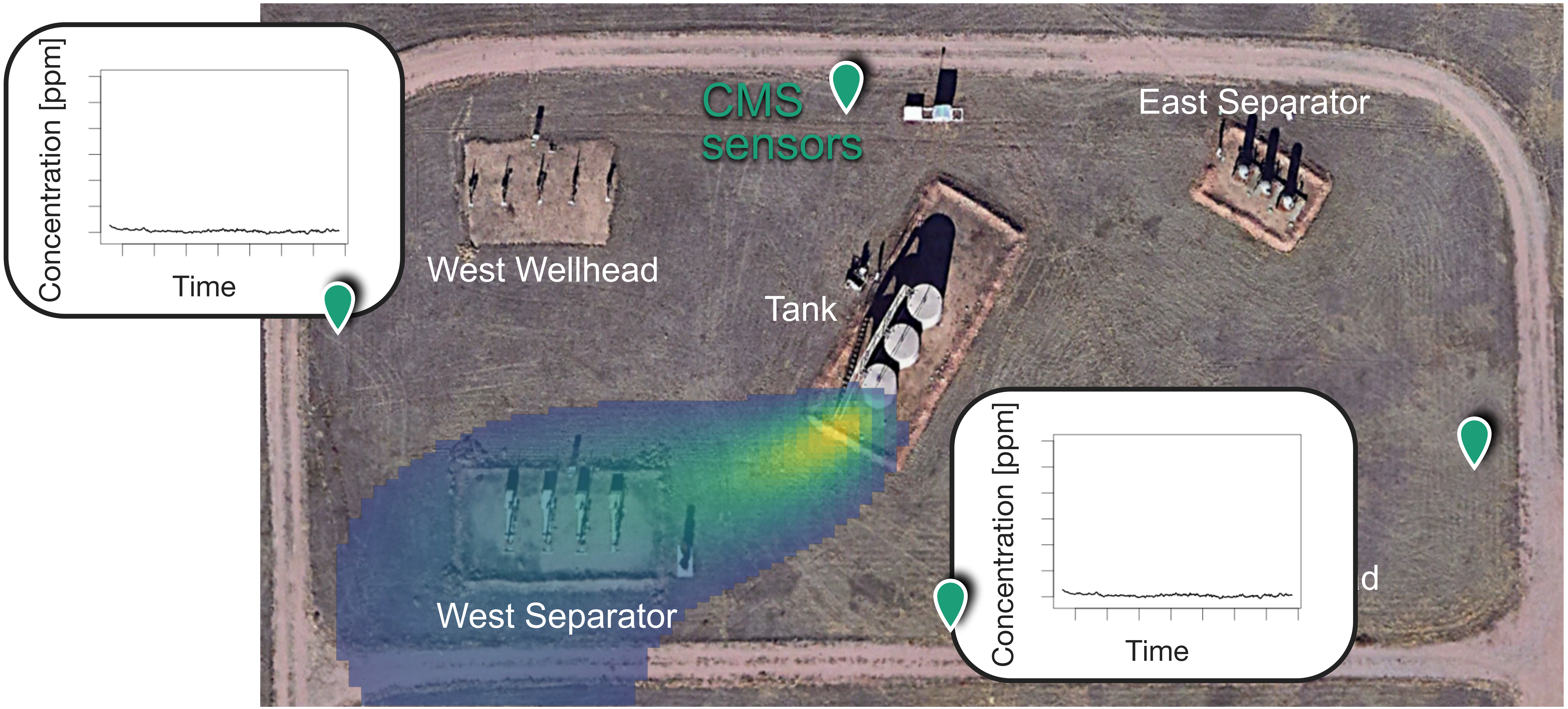
One problem… incomplete sensor coverage

Wind direction



One problem… incomplete sensor coverage

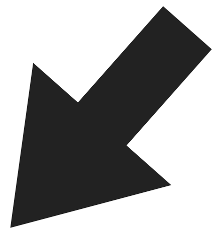
Wind direction

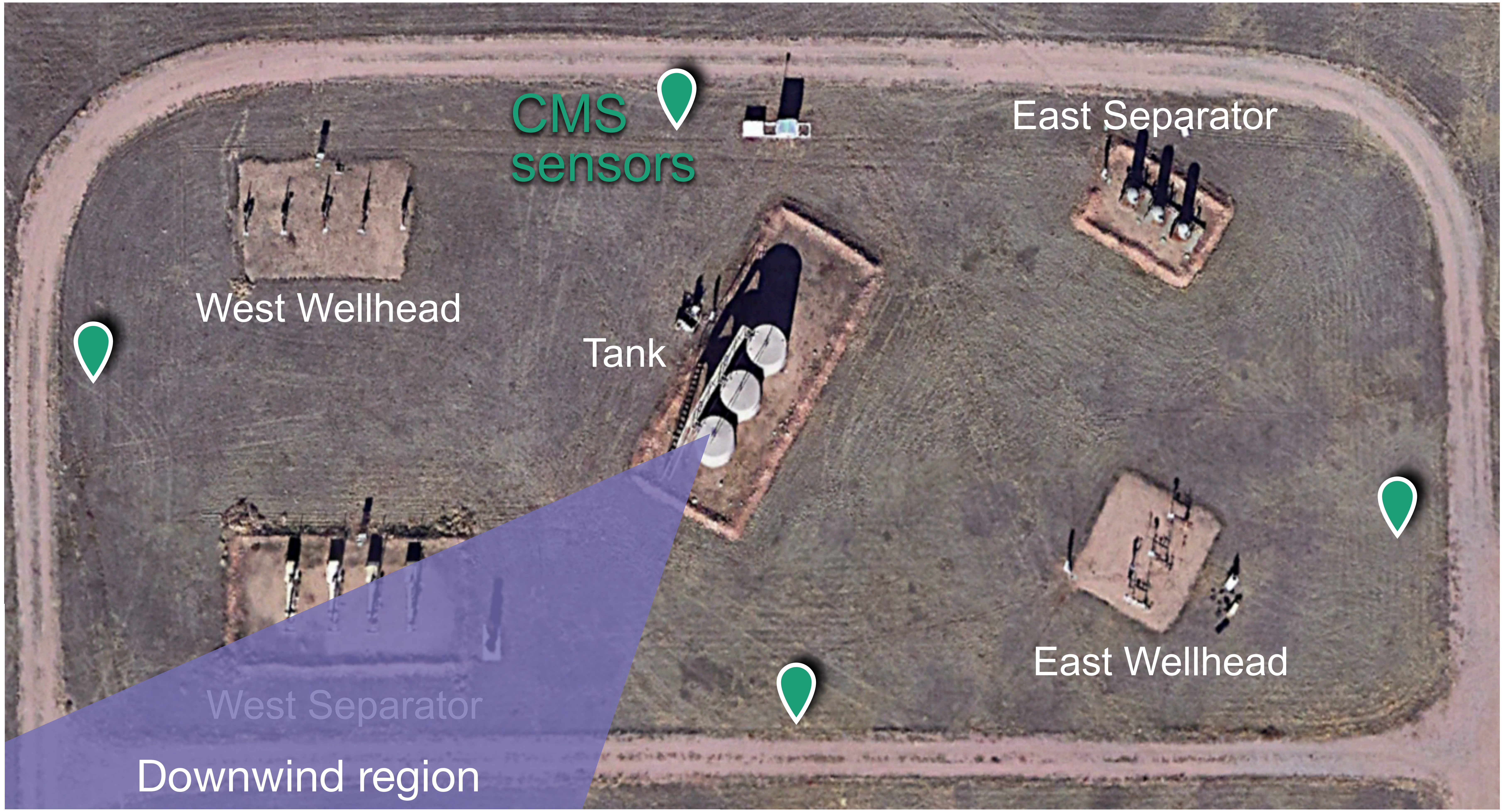


CMS do not provide emission information when the wind blows between sensors

However, we can estimate when this happens!

Wind direction

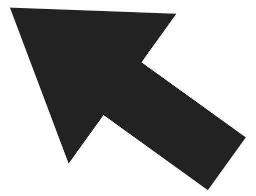


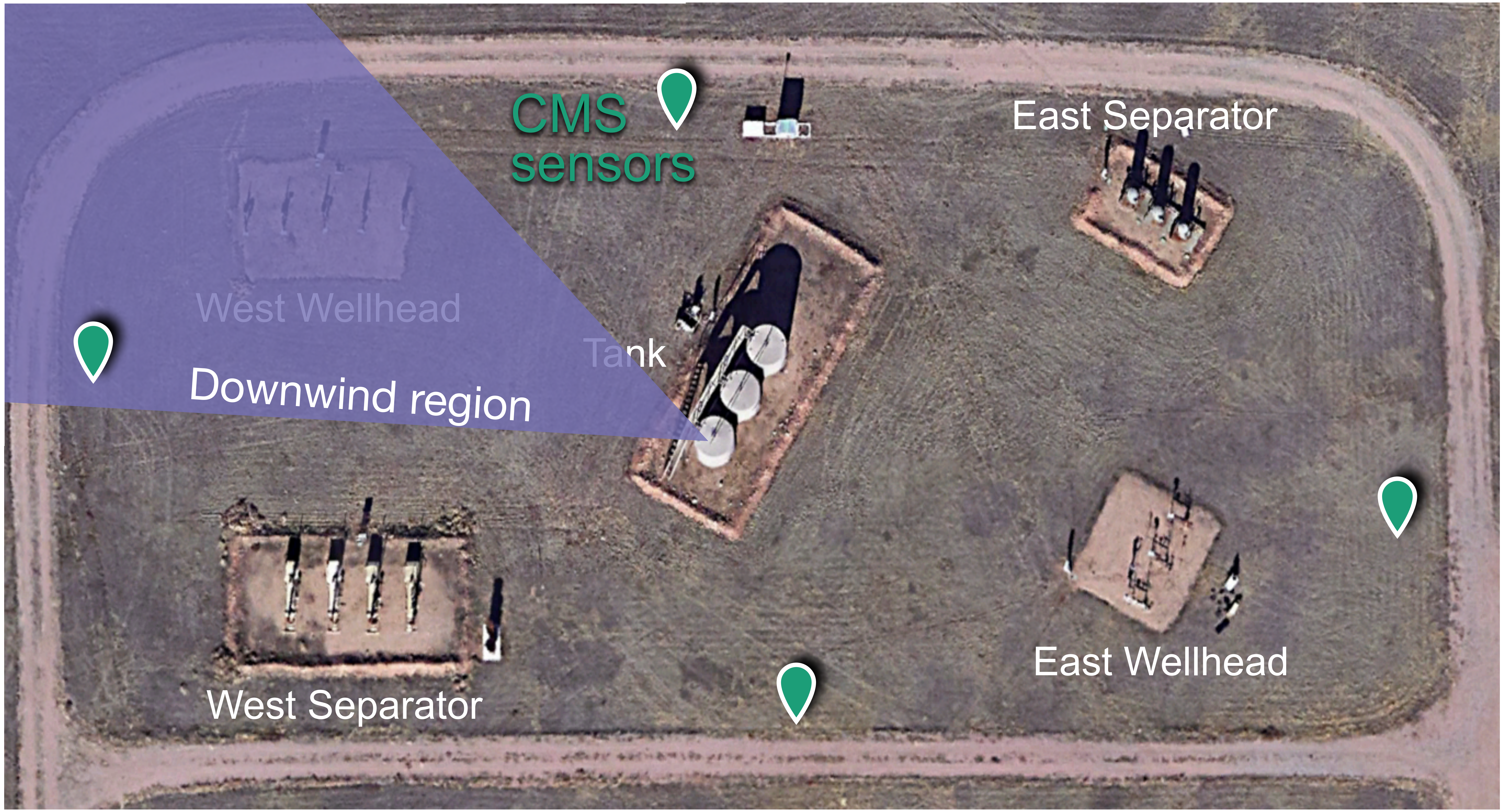


Downwind region **does not** overlap with CMS sensors = period of “**no information**”

However, we can estimate when this happens!

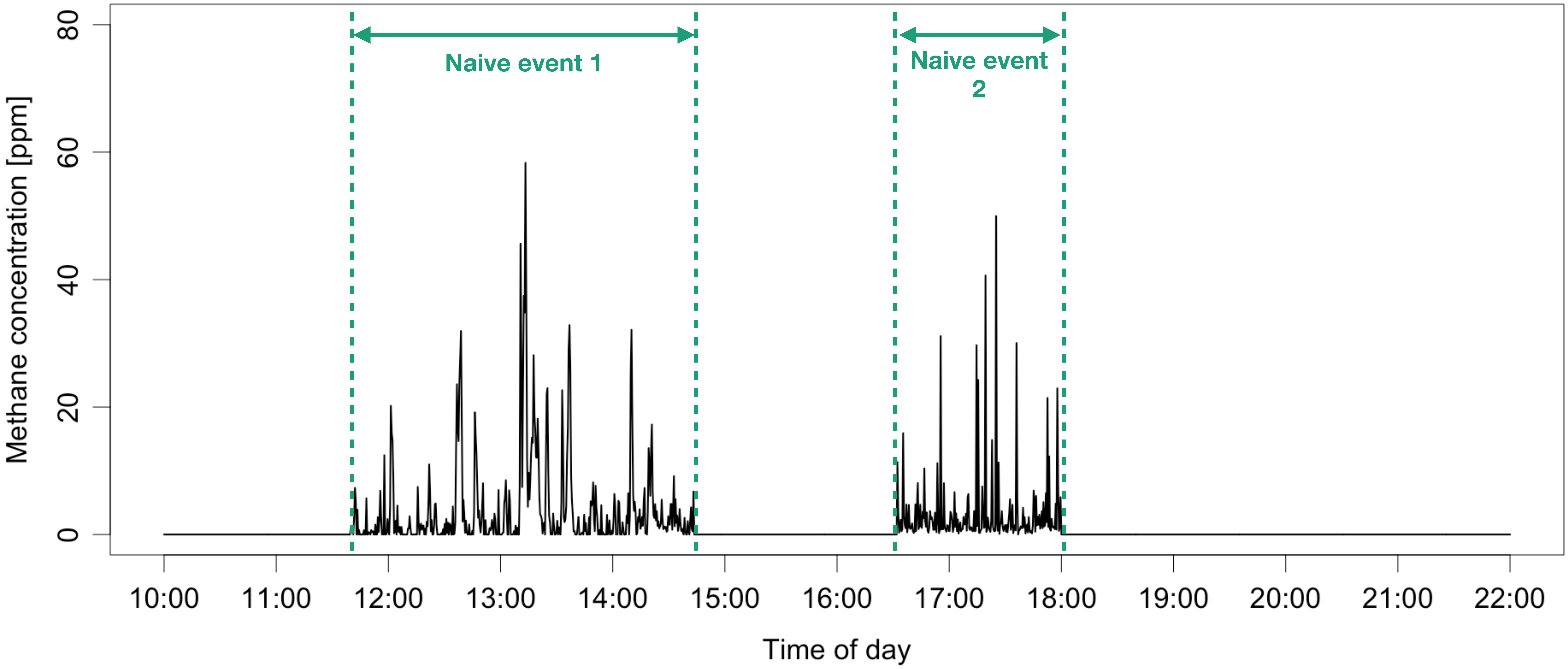
Wind direction



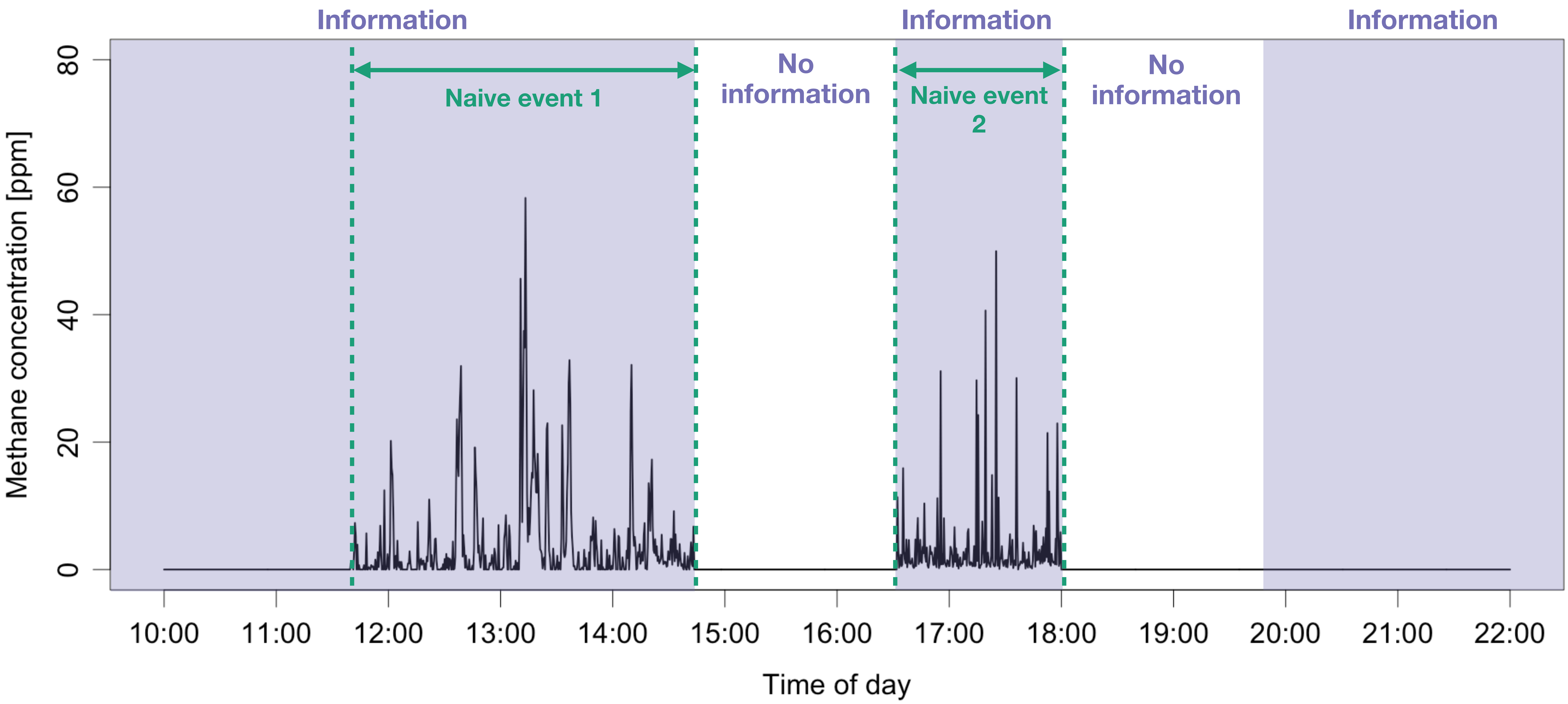


Downwind region **does** overlap with CMS sensors = period of “**information**”

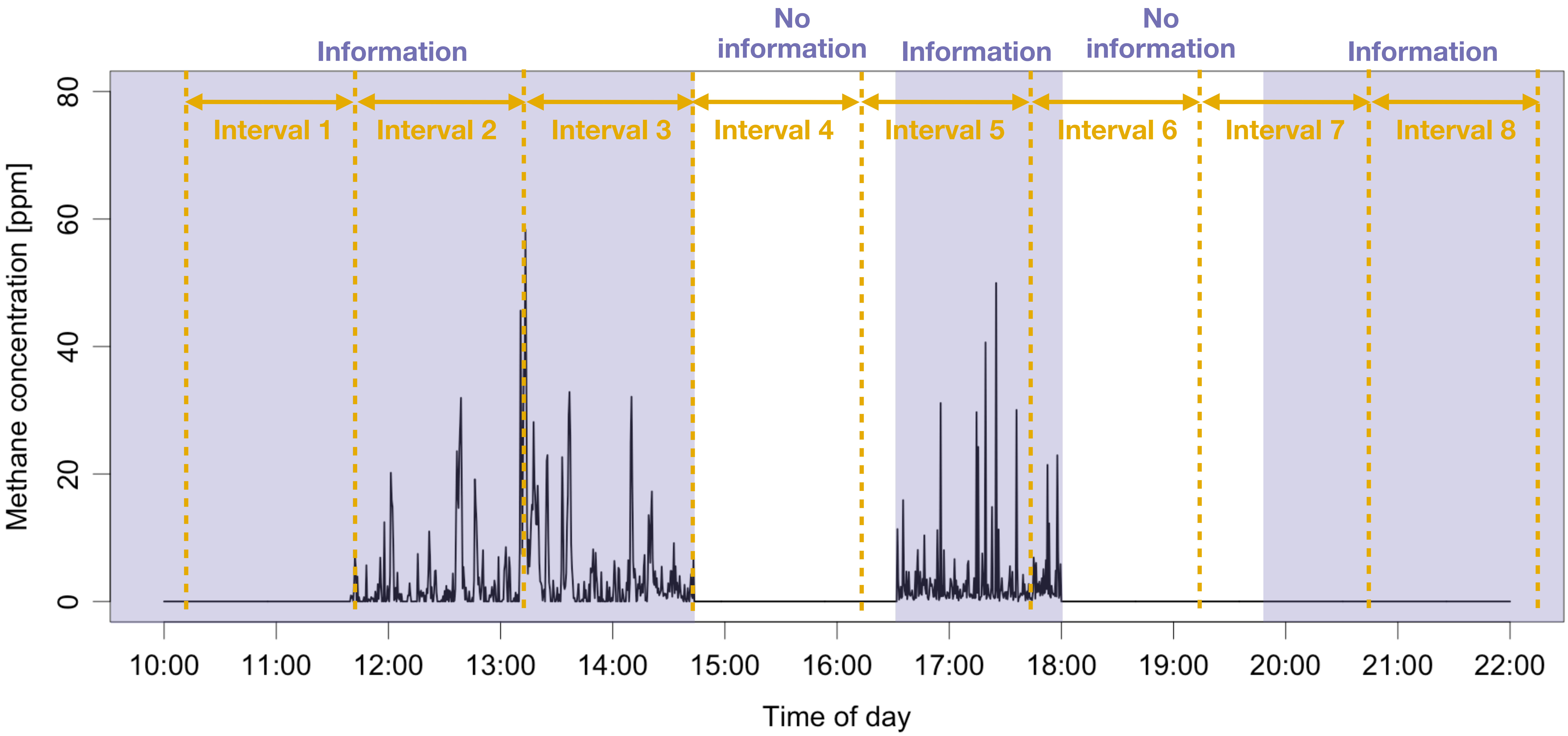
How do periods of information and no information present themselves in the data?



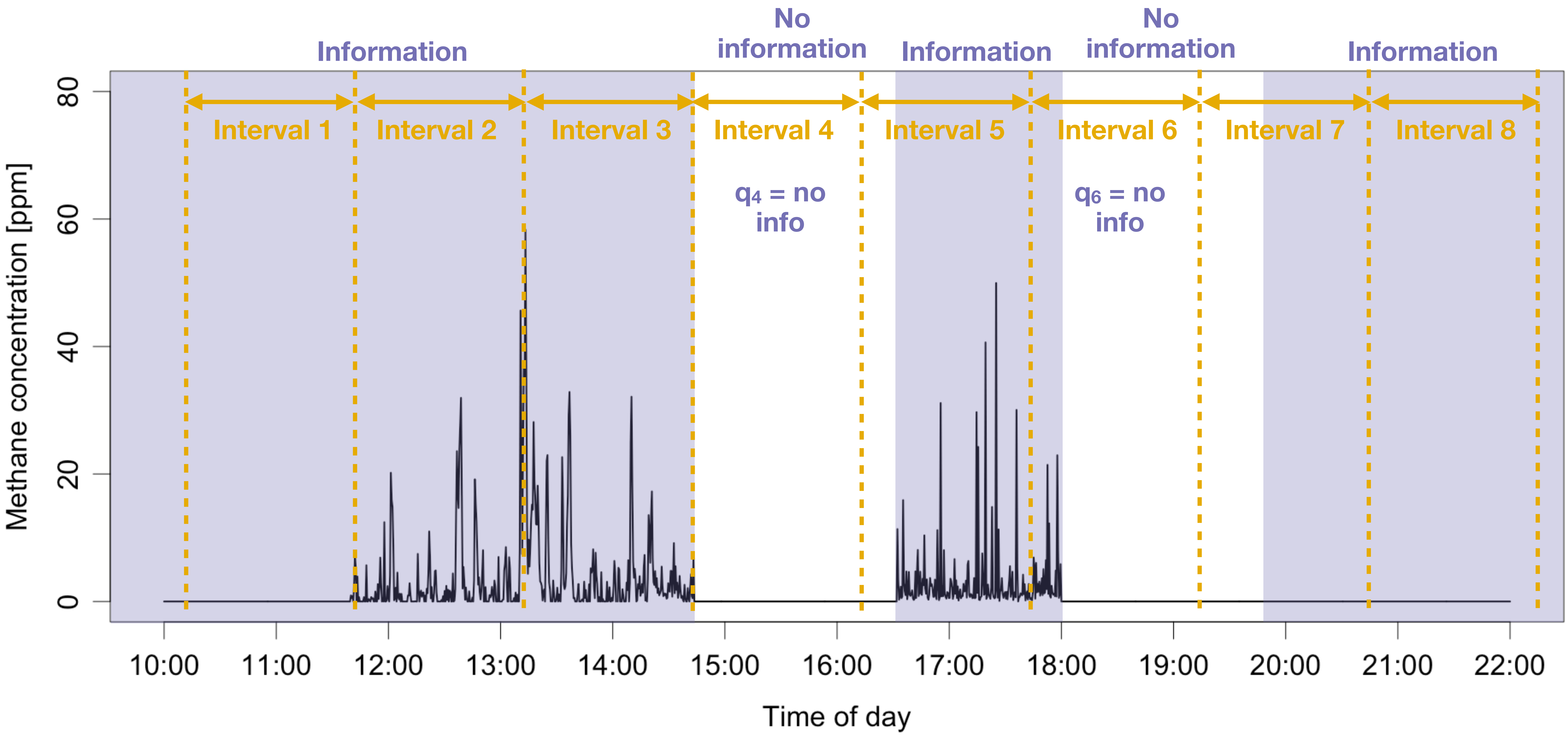
How do periods of information and no information present themselves in the data?



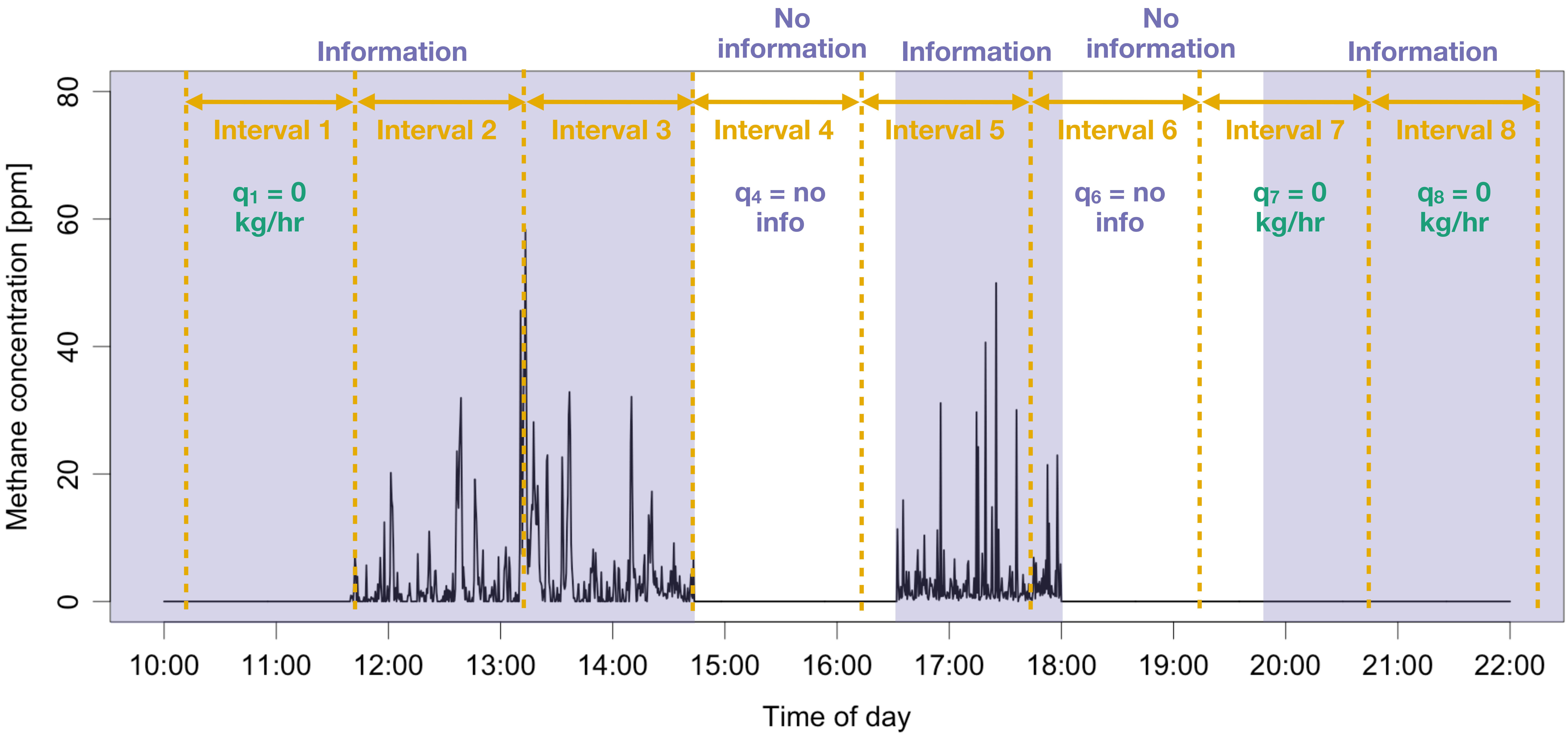
In practice, run the MDLQ (or DLQ) model on fixed intervals



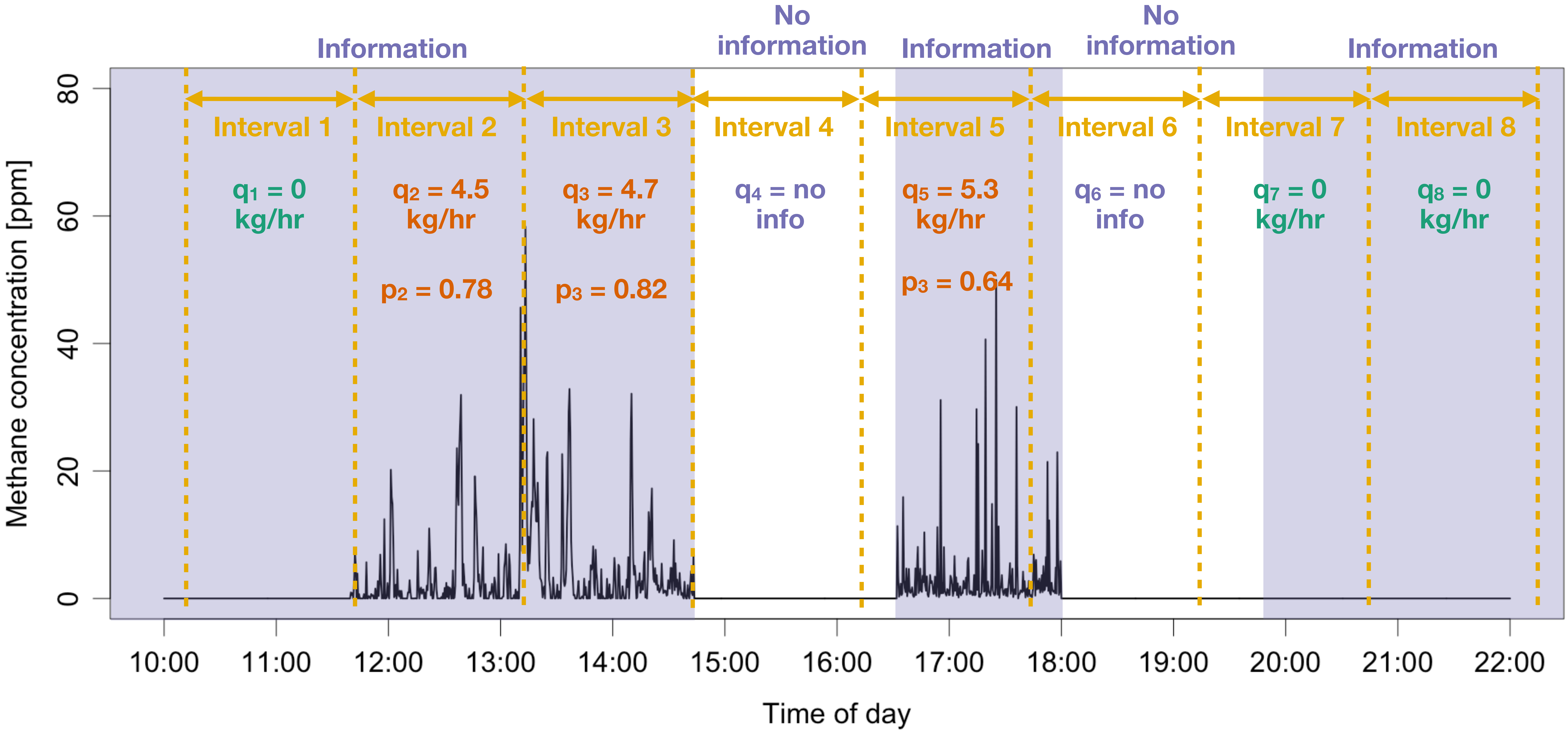
Whether an interval is no information,



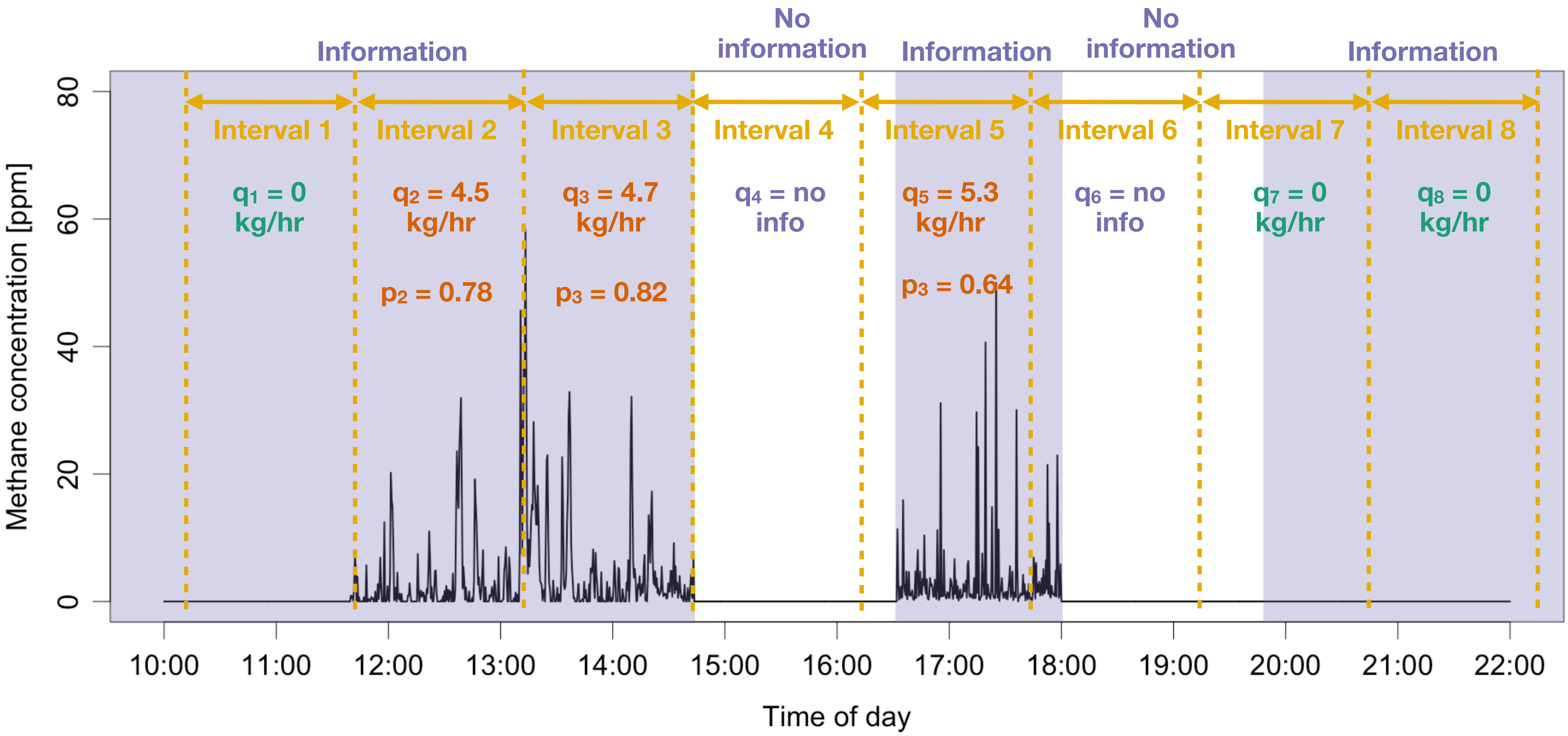
Whether an interval is no information, zero emission rate,



Whether an interval is no information, zero emission rate, or non-zero emission rate depends on the data.

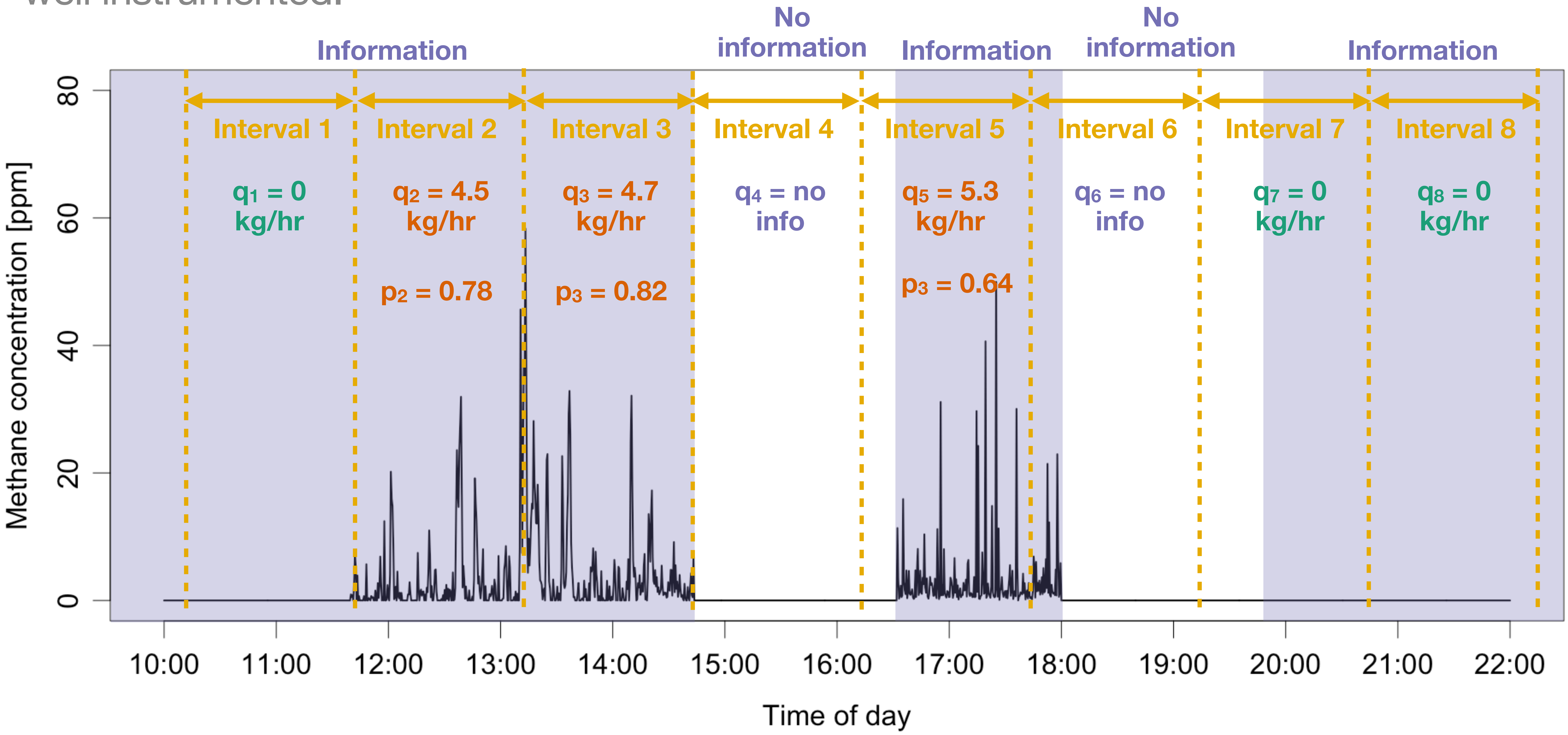


How do you turn these estimates into a measurement-derived inventory?



How do you turn these estimates into a measurement-derived inventory?

One option: run it long enough to build stable distributions. How long? Depends on how well instrumented.



Thank you!

COLORADO SCHOOL OF MINES

EEMDL
Energy Emissions Modeling and Data Lab

U.S. DEPARTMENT OF ENERGY