# Gaussian Puff Atmospheric Dispersion Model: An Analysis at Low Wind Speeds

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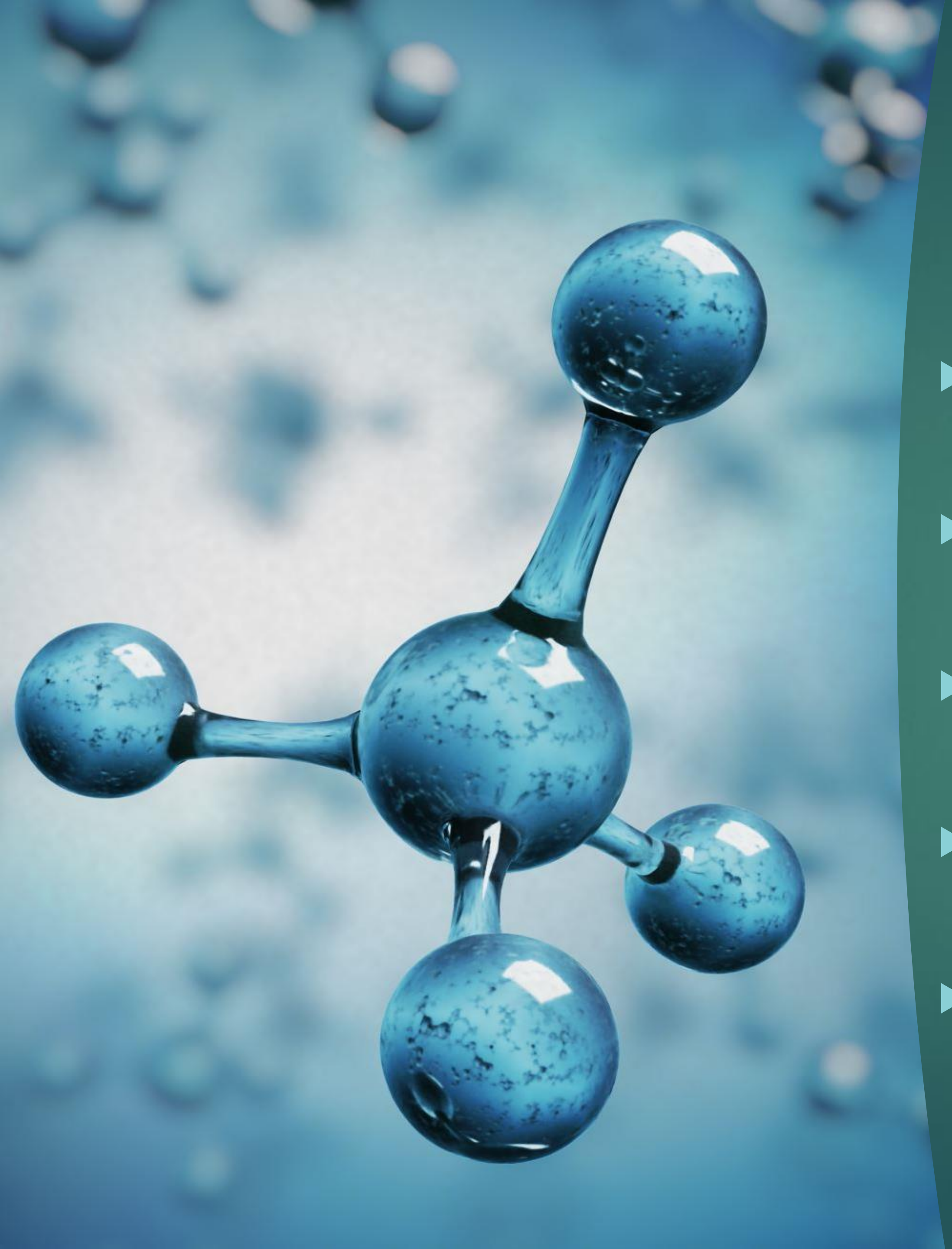


# Background

- The Gaussian Puff model is important when **monitoring methane emissions.**
- Methane has a **28x** greater global warming potential than CO2 over a 100-year period
- Methane accounts for **16%** of global greenhouse gas emissions
- Methane has a lifespan of only **7-12 years** in the atmosphere
- Natural gas and petroleum accounts for **29%** of all U.S. methane emissions.

# Background

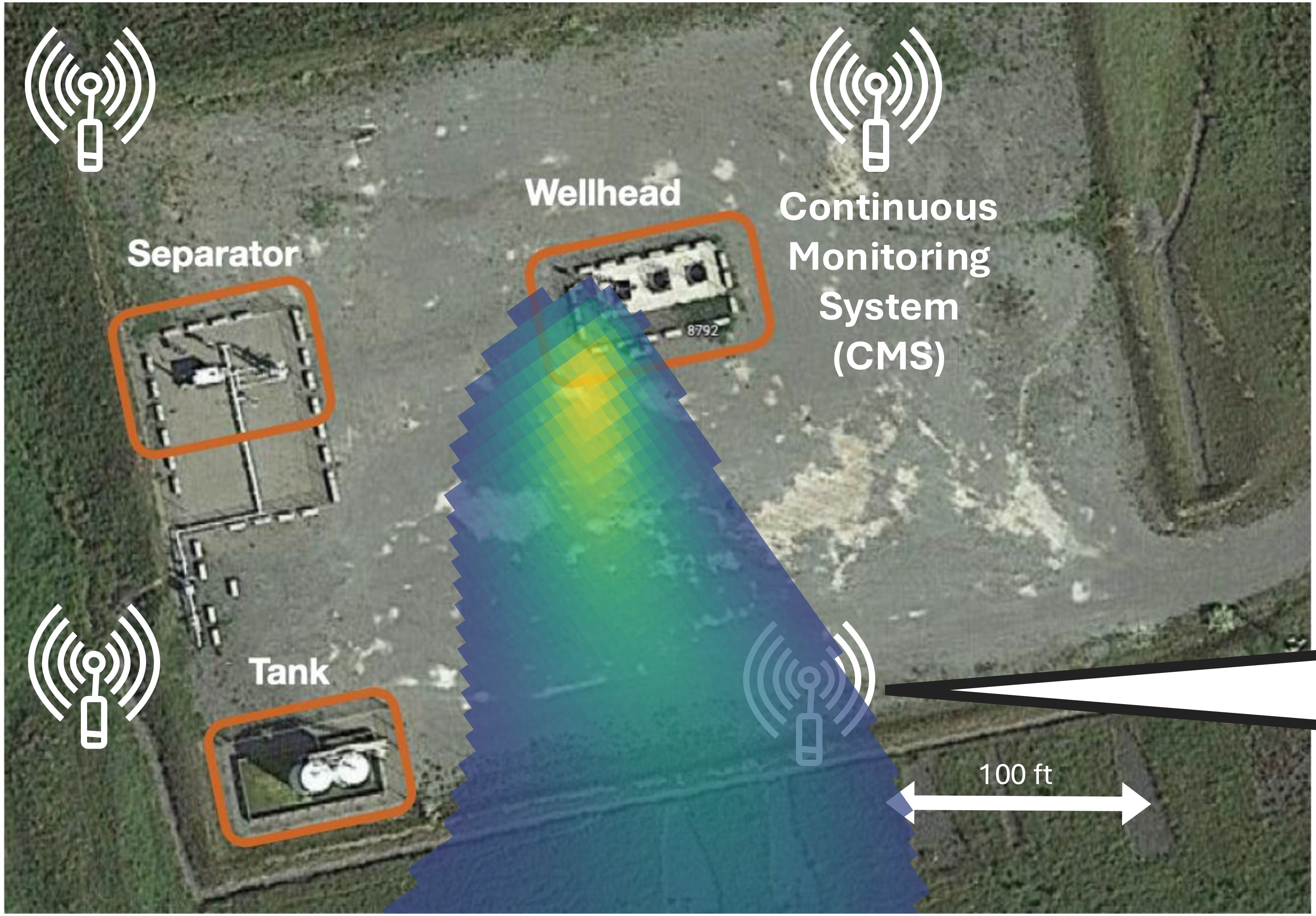
Methane matters, and we can do something about it!

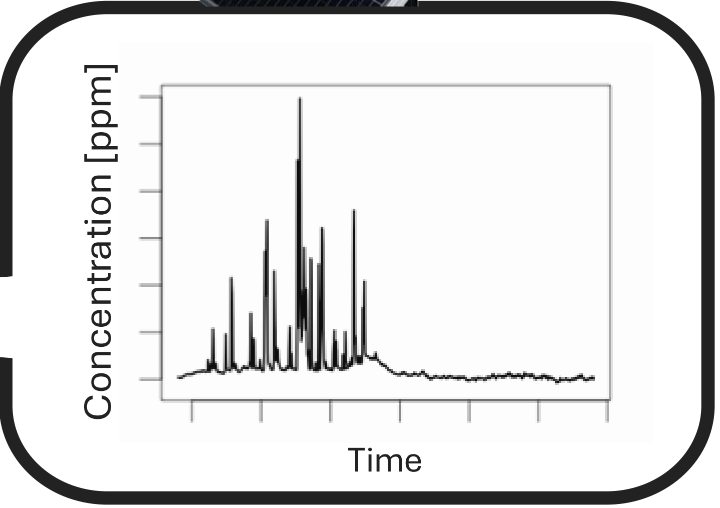
…arming …riod…

…**years** in…

…petroleum accounts for **29%** of …methane emissions.

# Monitoring methane emissions

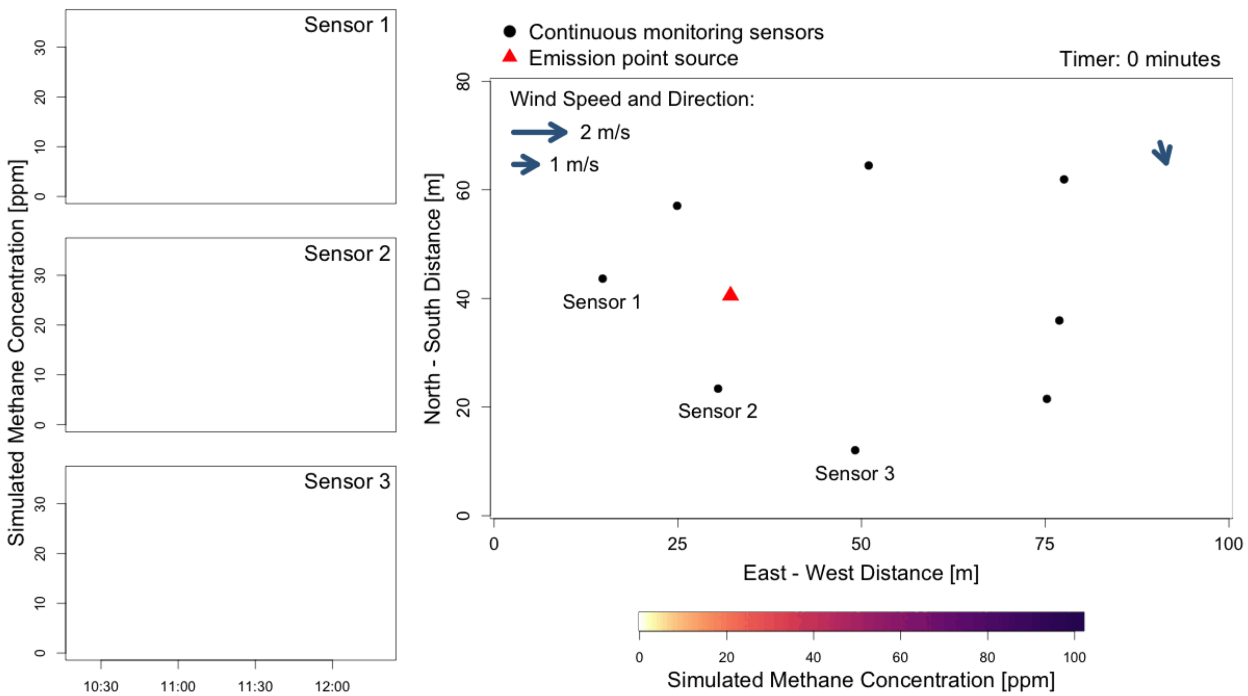


# Gaussian Puff Model

$$c(x, y, z, t) = \sum_{p \in S_t} c_p(x, y, z, t)$$

$$c_p(x, y, z, t) = \frac{q}{(2\pi)^{3/2}\sigma_y^2\sigma_z} \exp\left(-\frac{(x - ut)^2 + y^2}{2\sigma_y^2}\right)\left[\exp\left(-\frac{(z - z_0)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z + z_0)^2}{2\sigma_z^2}\right)\right]$$



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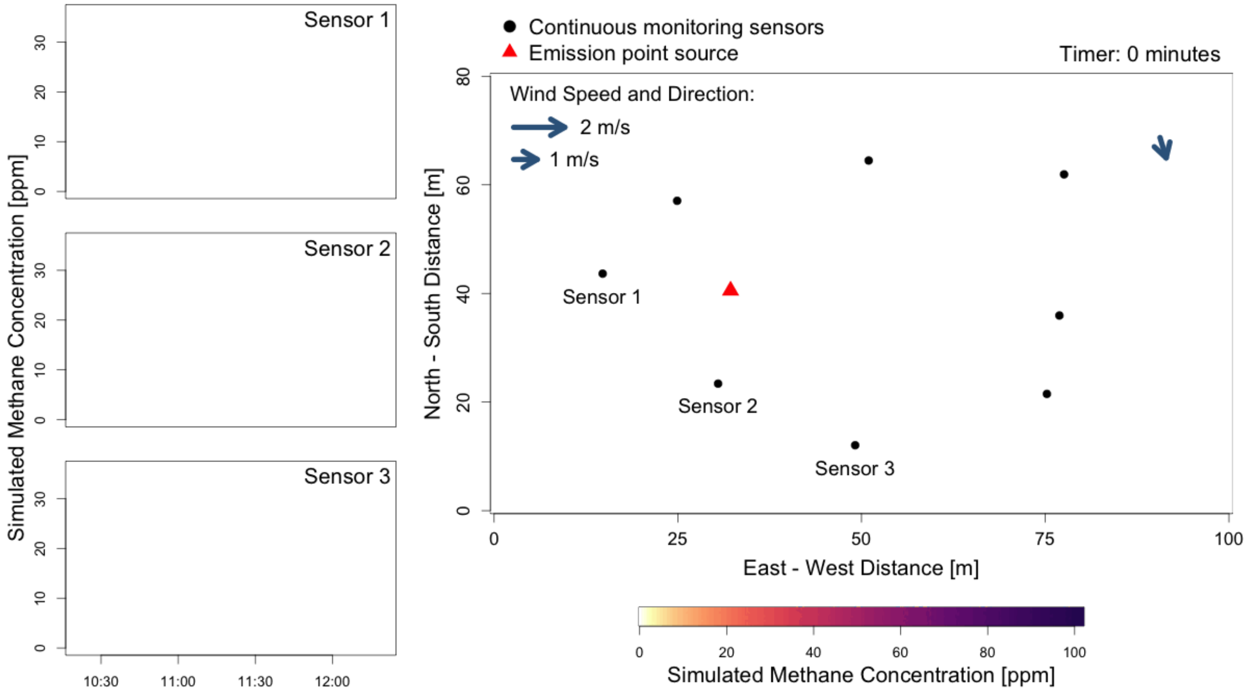


Output: methane concentration estimates across time and space. This is the sum of contributions from each puff.

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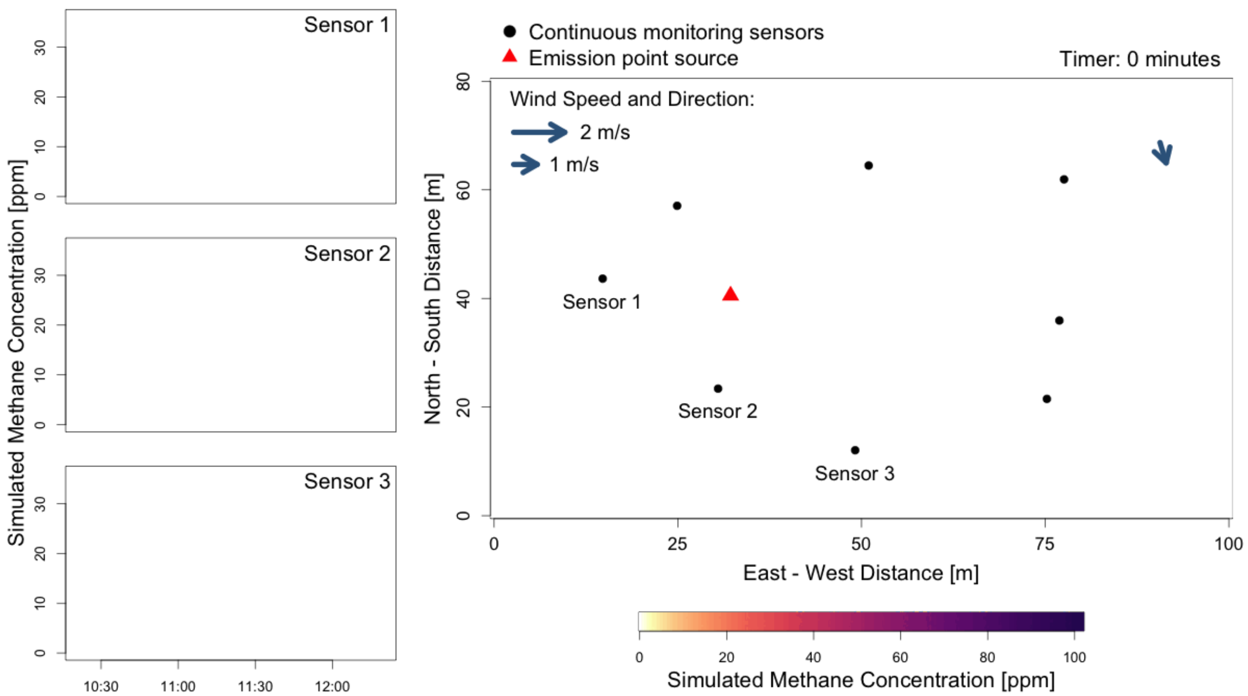


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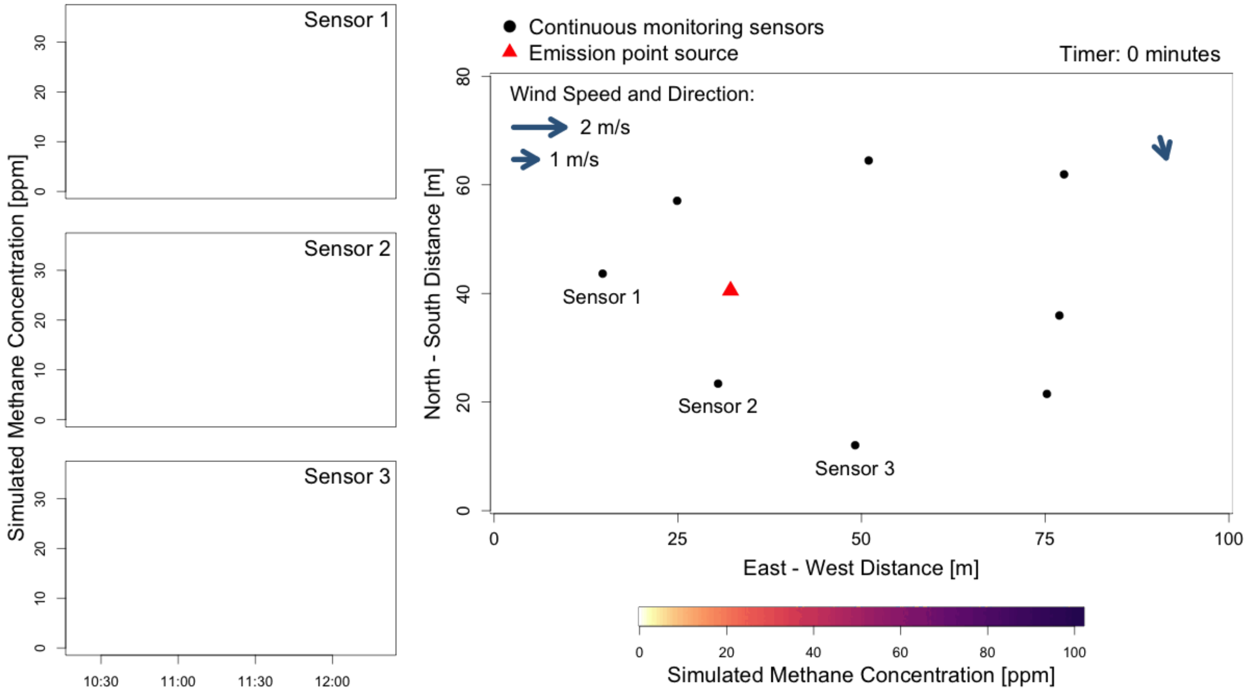


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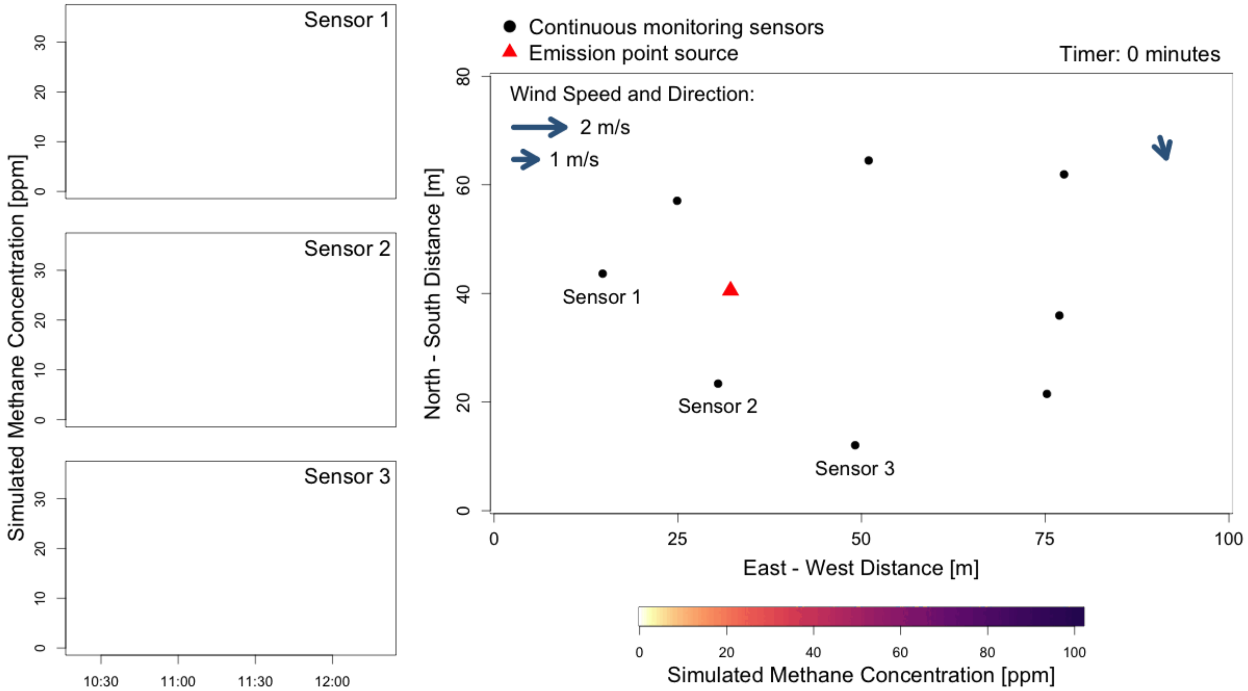


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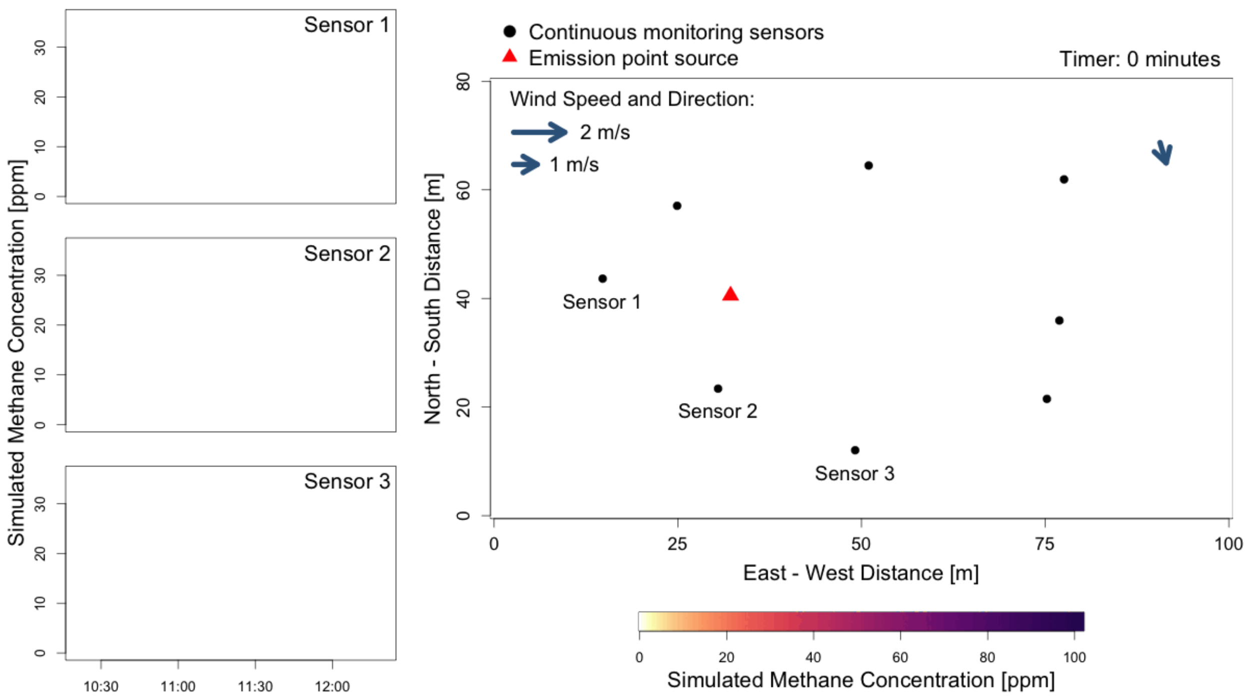


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Model Inputs:

- Wind direction
- Emission source location
- Sensor locations and heights

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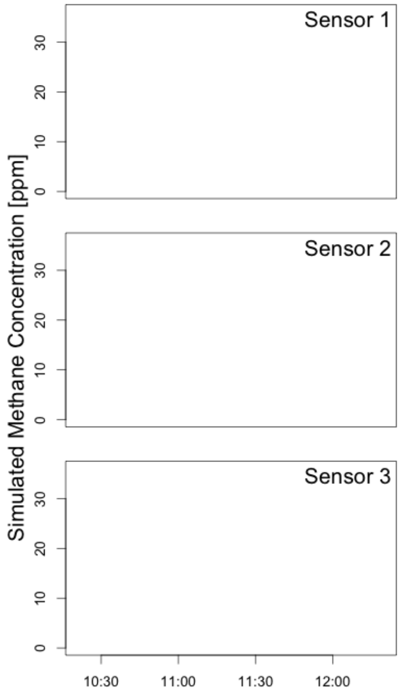


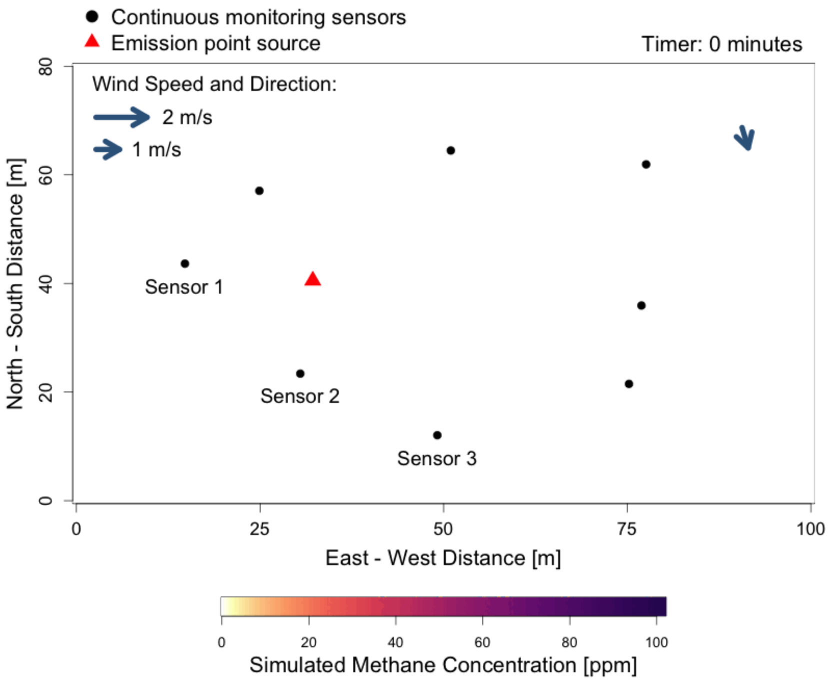


<u>Output:</u> methane concentration estimates across time and space. This is the sum of contributions from each puff.

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- Wind direction
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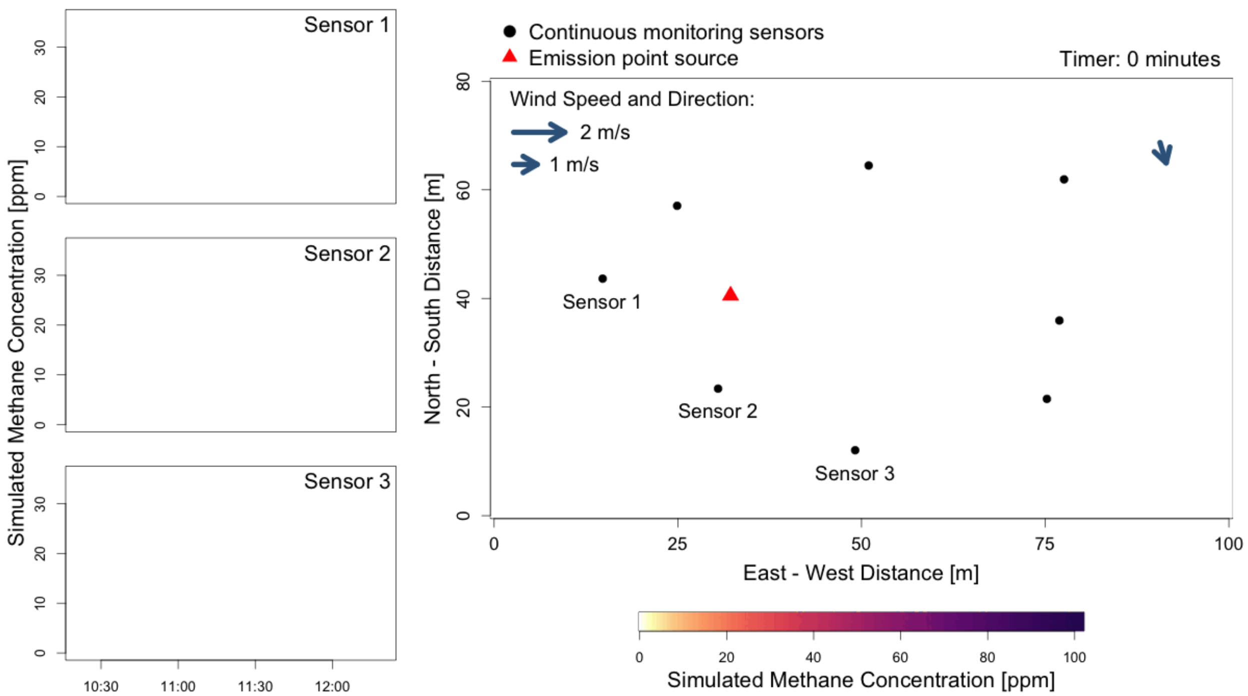


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Model Inputs:

- Wind direction
- Emission source location
- Sensor locations and heights
- Emission source height
- Wind speed

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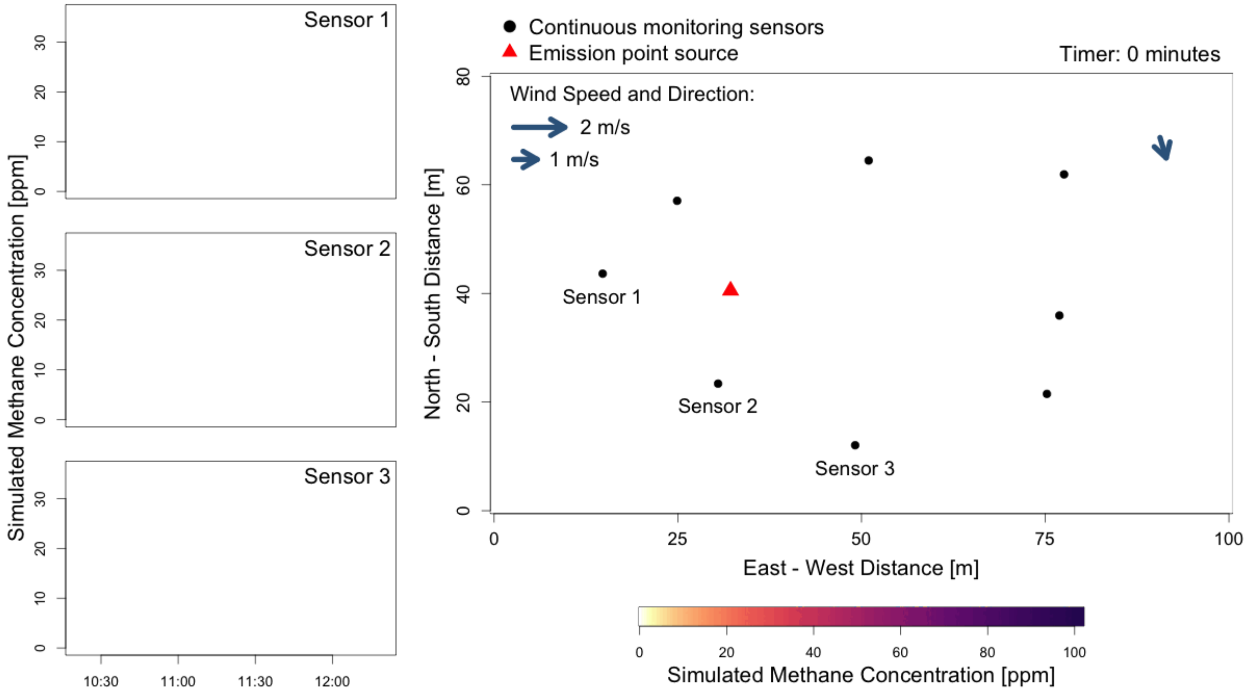


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Model Inputs:

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- Emission source location
- Sensor locations and heights
- Emission source height
- Wind speed
- Emission rate

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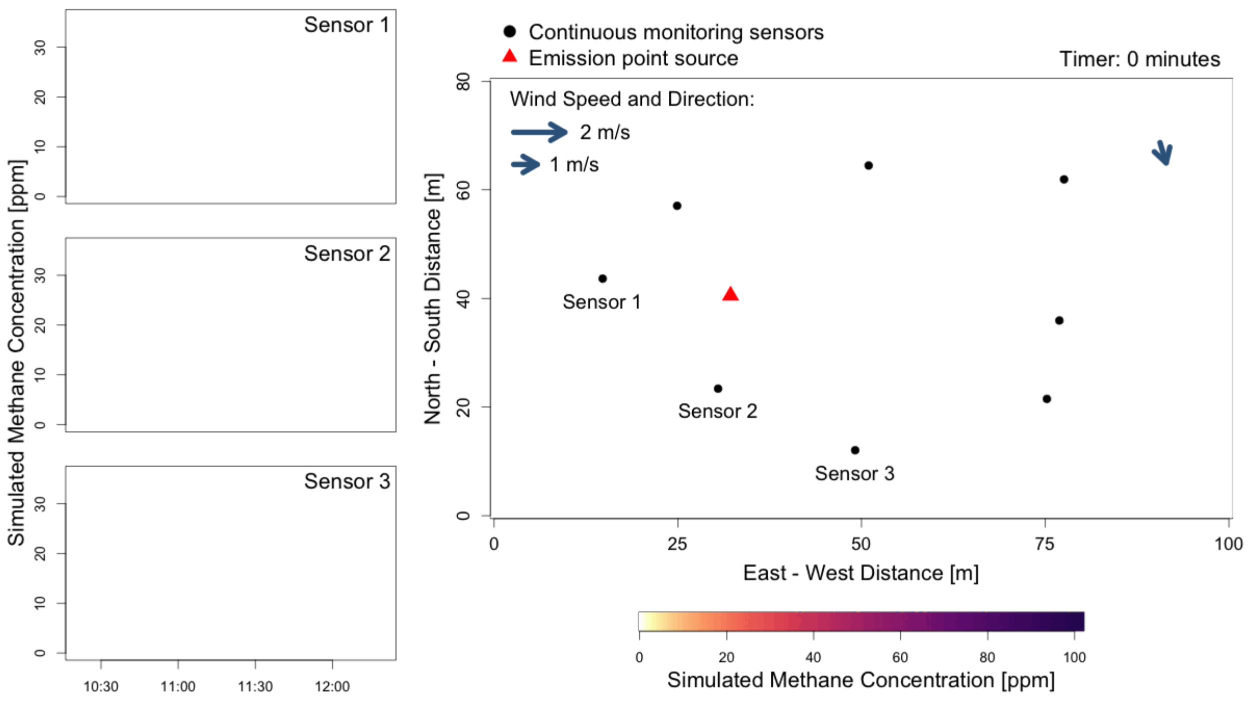


Output: methane concentration estimates across time and space. This is the sum of contributions from each puff.

Model Inputs:
- Wind direction
- Emission source location
- Sensor locations and heights
- Emission source height
- Wind speed
- Emission rate
- Dispersion parameters
- Time-related parameters

Problem: The Gaussian Puff model performs poorly at low wind speeds.

## What we don’t know:

- The wind speed threshold at which model performance begins to decline
- Underlying phenomena leading to underperformance

# Data

- Controlled methane releases at METEC, run by Colorado State University
- Time period of almost 3 months
- Data on observed and simulated concentrations at each sensor

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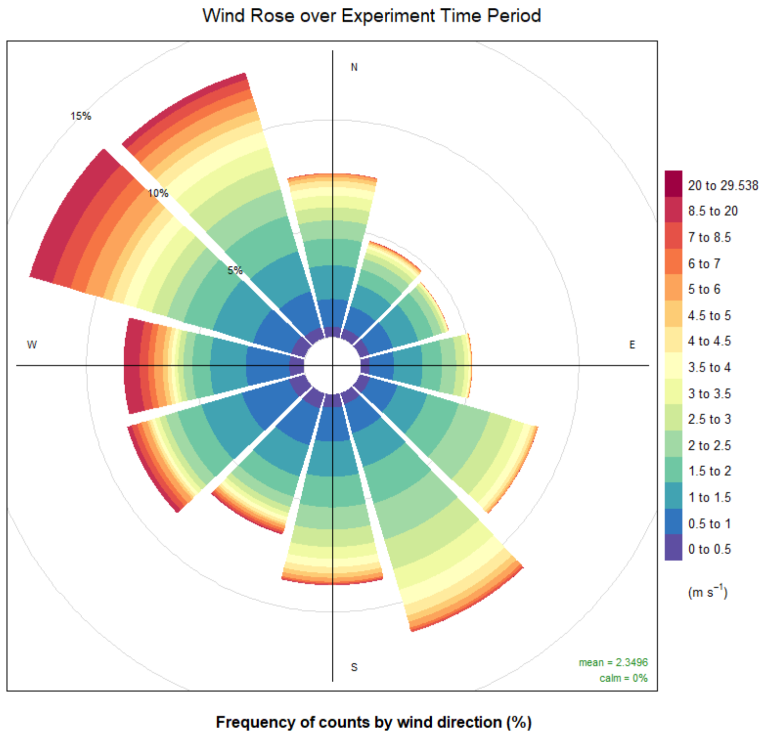

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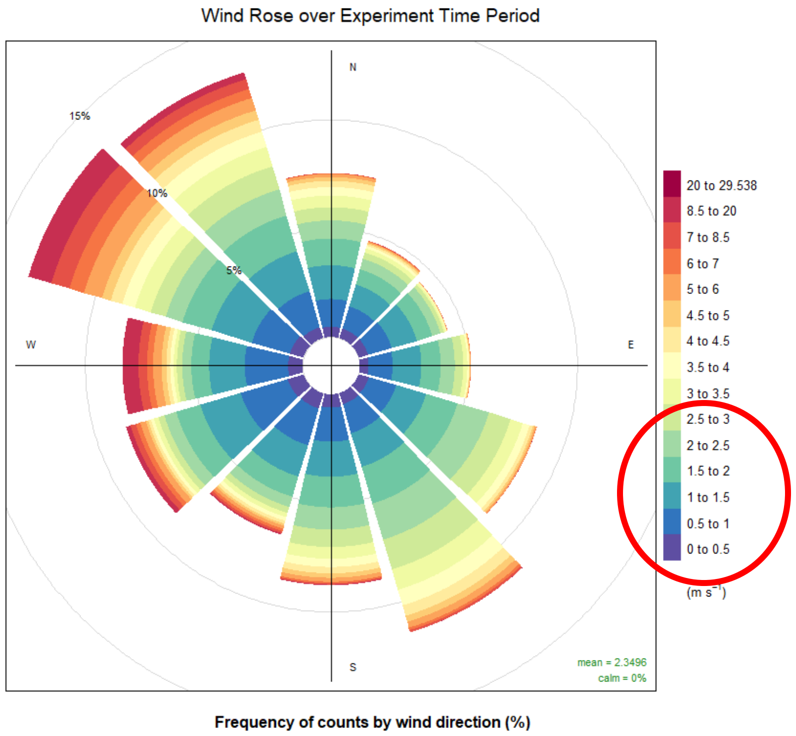
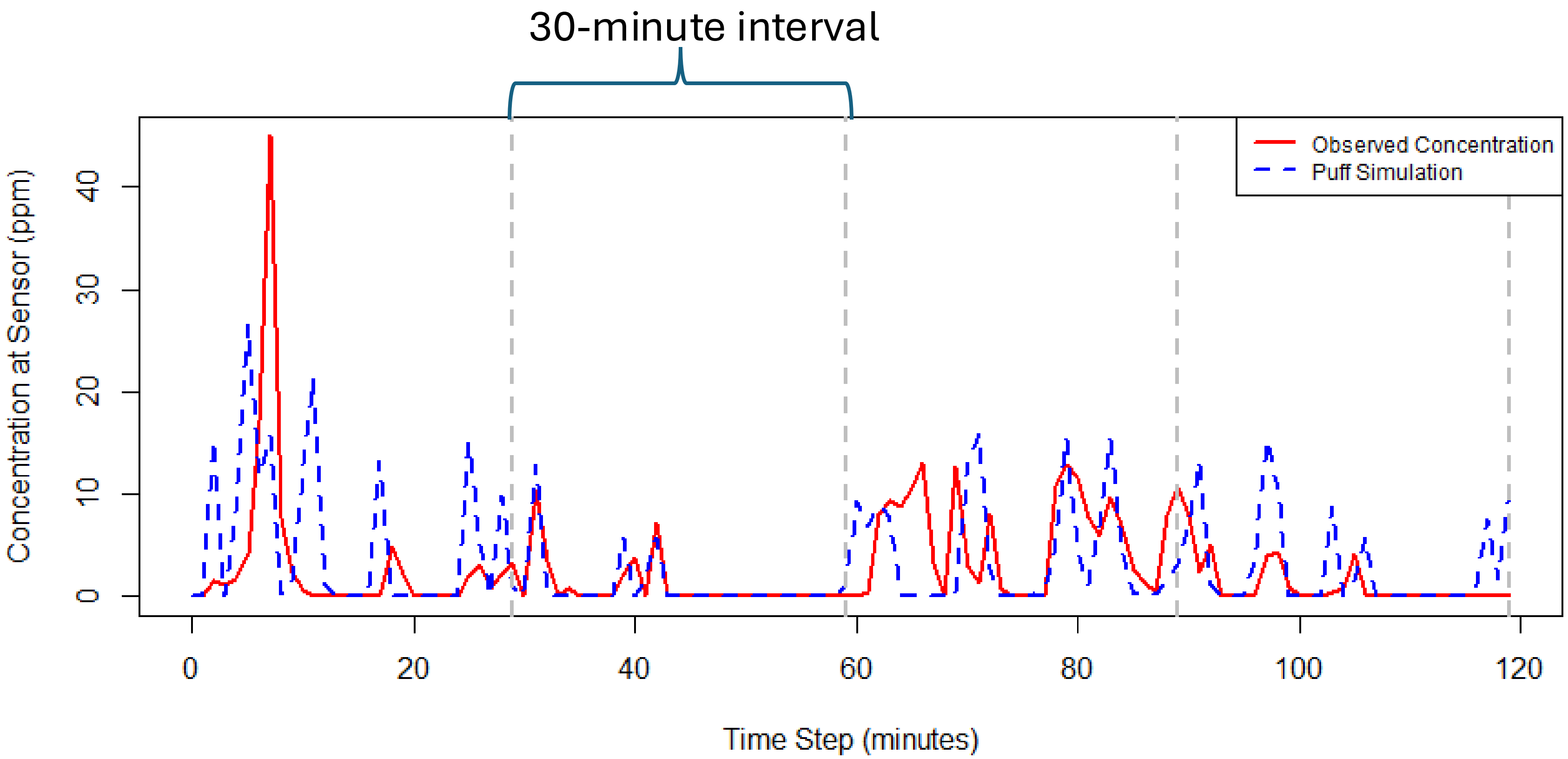


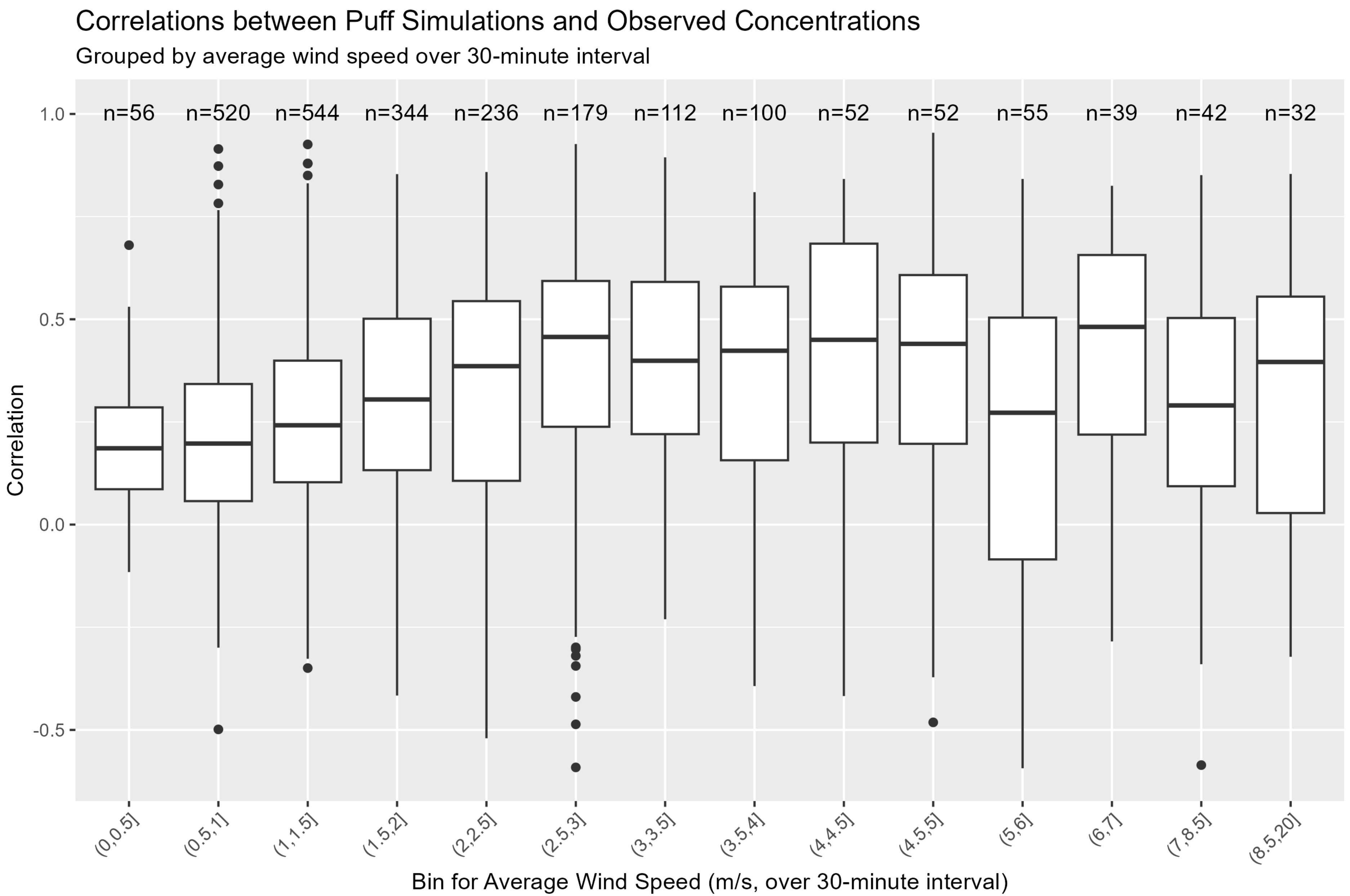
# Methods

Metrics used:

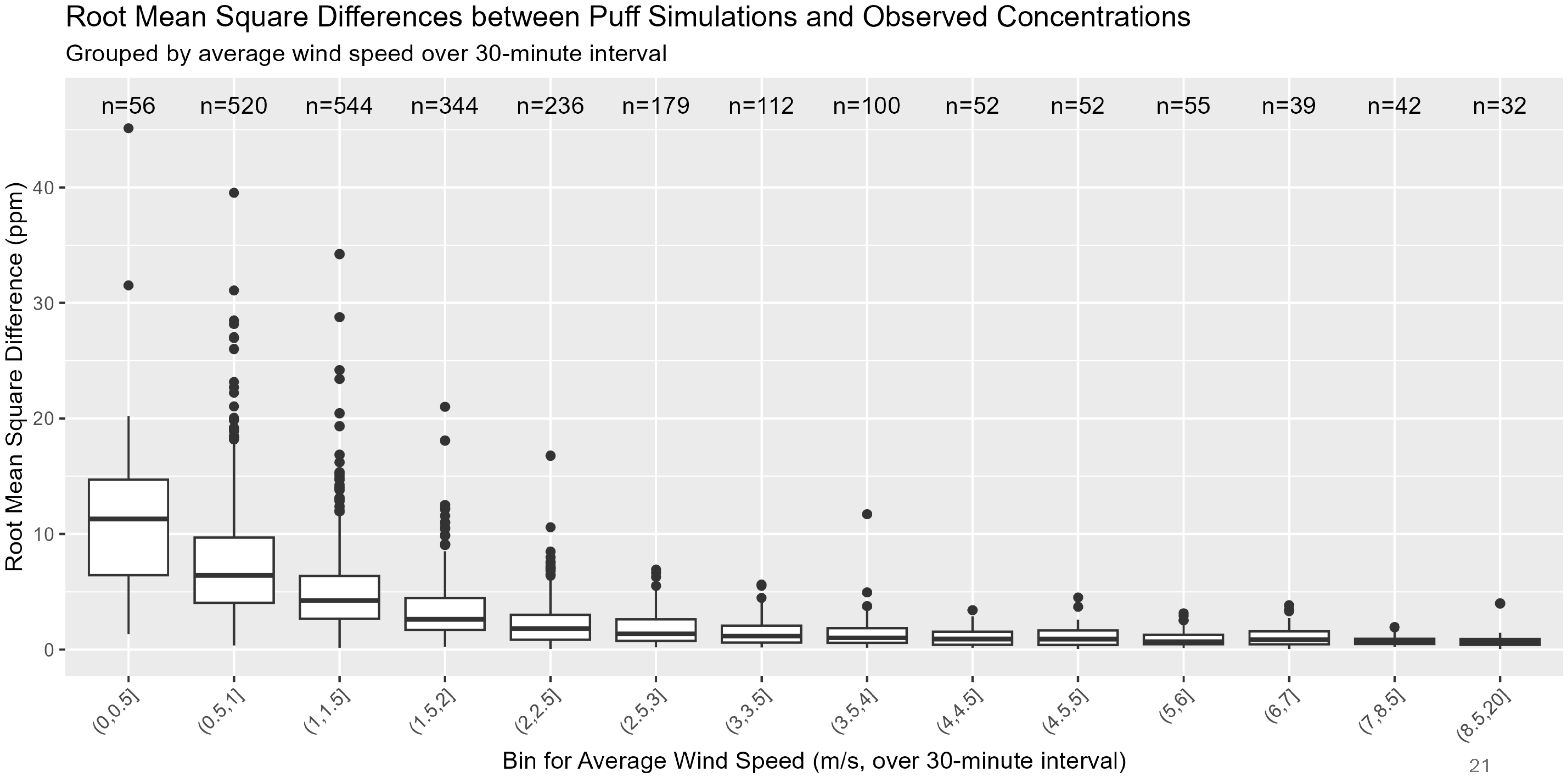
- Correlations
- Root-mean-square differences
- Percent differences



# Results


Correlations between Puff Simulations and Observed Concentrations
Grouped by average wind speed over 30-minute interval
n=56
n=520
n=544
n=344
n=236
n=179
n=112
n=100
n=52
n=52
n=55
n=39
n=42
n=32
1.0
0.5
0.0
-0.5
Correlation
(0,0.5]
(0.5,1]
(1,1.5]
(1.5,2]
(2,2.5]
(2.5,3]
(3,3.5]
(3.5,4]
(4,4.5]
(4.5,5]
(5,6]
(6,7]
(7,8.5]
(8.5,20]
Bin for Average Wind Speed (m/s, over 30-minute interval)

# Results


Root Mean Square Differences between Puff Simulations and Observed Concentrations
Grouped by average wind speed over 30-minute interval
n=56
n=520
n=544
n=344
n=236
n=179
n=112
n=100
n=52
n=52
n=55
n=39
n=42
n=32
Root Mean Square Difference (ppm)
40
30
20
10
0
(0,0.5]
(0.5,1]
(1,1.5]
(1.5,2]
(2,2.5]
(2.5,3]
(3,3.5]
(3.5,4]
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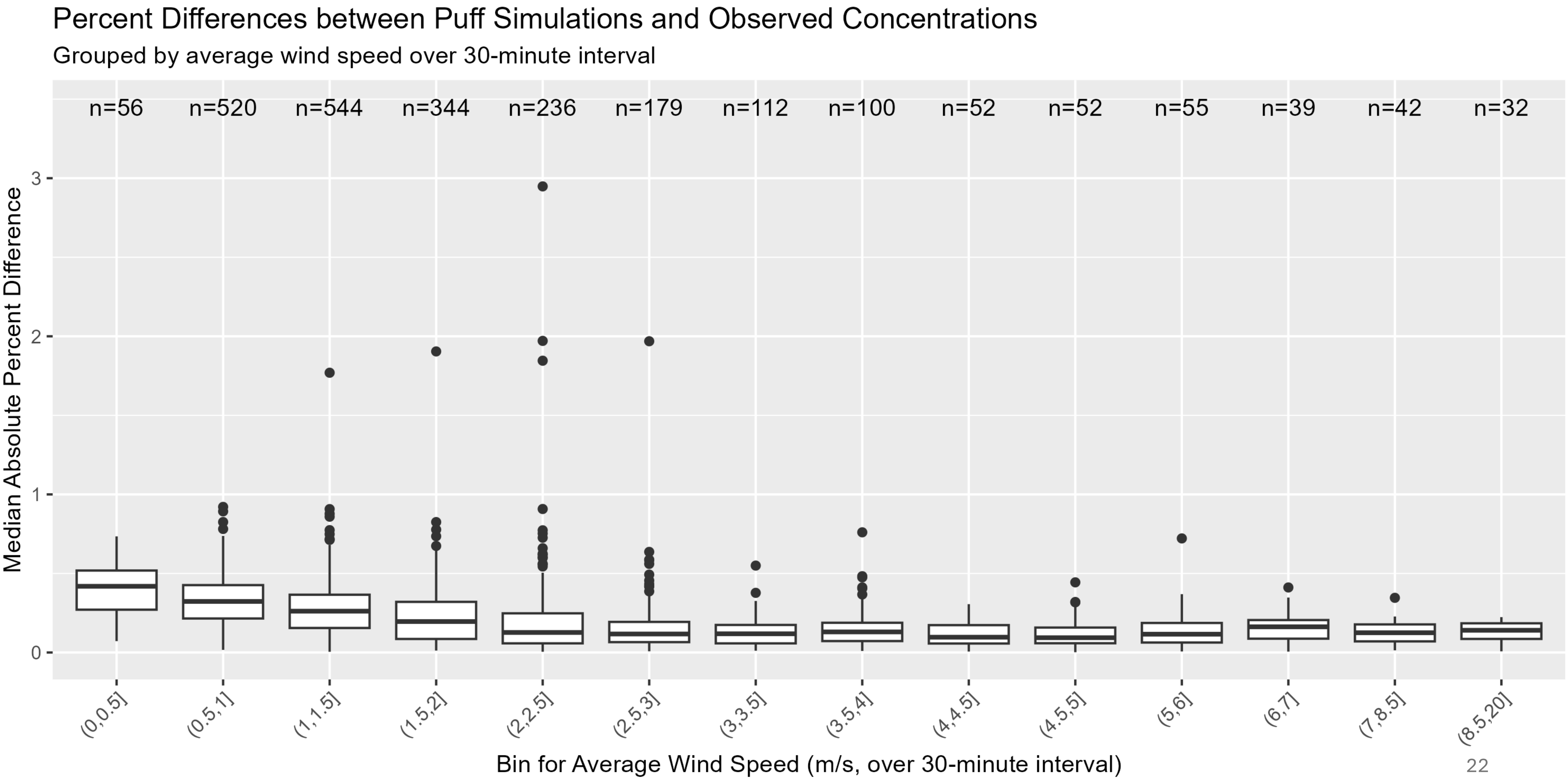

Percent Differences between Puff Simulations and Observed Concentrations
Grouped by average wind speed over 30-minute interval
n=56
n=520
n=544
n=344
n=236
n=179
n=112
n=100
n=52
n=52
n=55
n=39
n=42
n=32
Median Absolute Percent Difference
0
1
2
3
(0,0.5]
(0.5,1]
(1,1.5]
(1.5,2]
(2,2.5]
(2.5,3]
(3,3.5]
(3.5,4]
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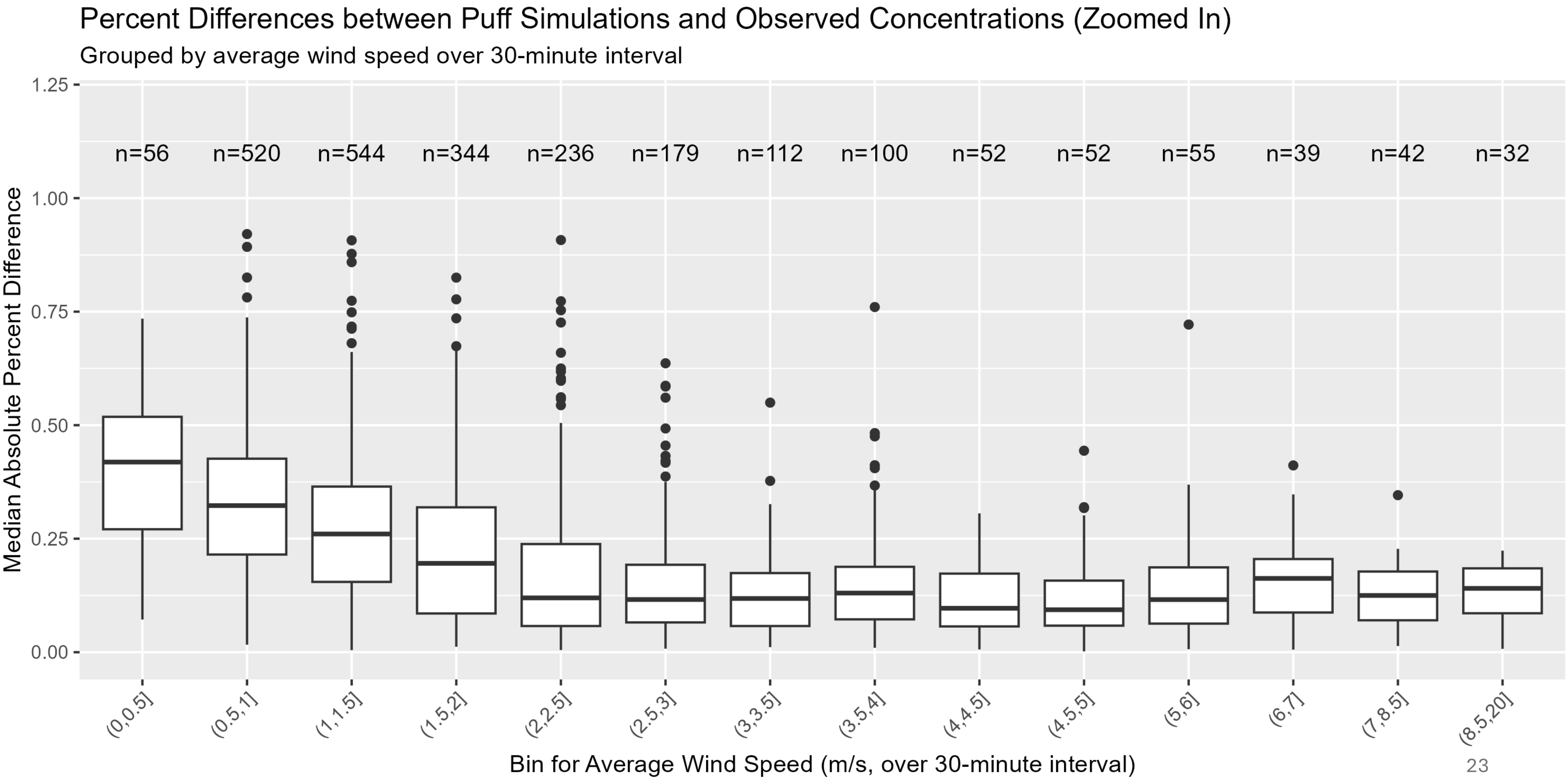
(8.5,20]
Bin for Average Wind Speed (m/s, over 30-minute interval)

# Results



Percent Differences between Puff Simulations and Observed Concentrations (Zoomed In)
Grouped by average wind speed over 30-minute interval
Median Absolute Percent Difference
1.25
1.00
0.75
0.50
0.25
0.00
n=56
n=520
n=544
n=344
n=236
n=179
n=112
n=100
n=52
n=52
n=55
n=39
n=42
n=32
(0,0.5]
(0.5,1]
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(4,4.5]
(4.5,5]
(5,6]
(6,7]
(7,8.5]
(8.5,20]
Bin for Average Wind Speed (m/s, over 30-minute interval)

# Initial Takeaways

As expected, the model performs worse at low wind speeds.

The model starts to underperform around 2.5 m/s. Performance continues to decline as wind speed approaches zero.

# What does poor model performance at low wind speeds look like?

To answer this question, we simulate over specific time periods.

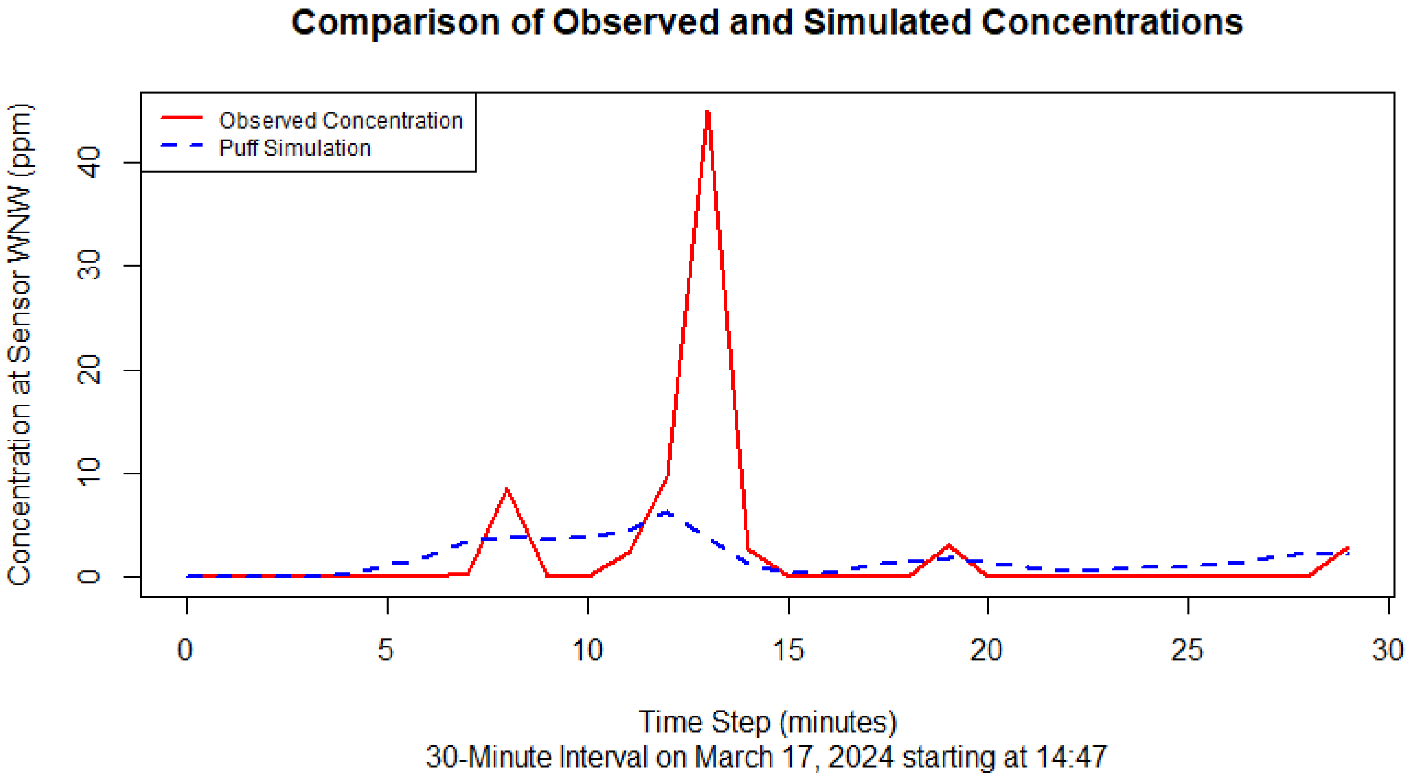
# Low Wind Speed Time Period

## Average Wind Speed: 0.652 m/s

Wind Direction Variance: 0.081

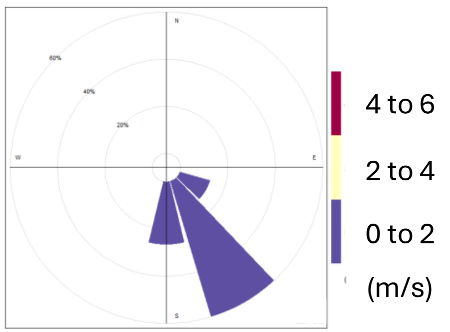
Emission Source: West Wellhead

Emission Rate: 0.746 kg/hr




Denotes emission source

Denotes sensor of interest

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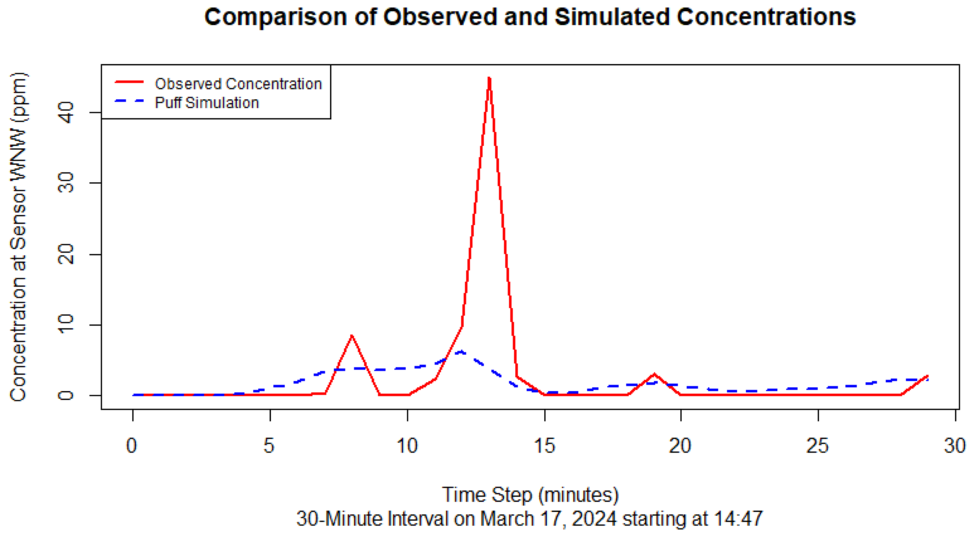
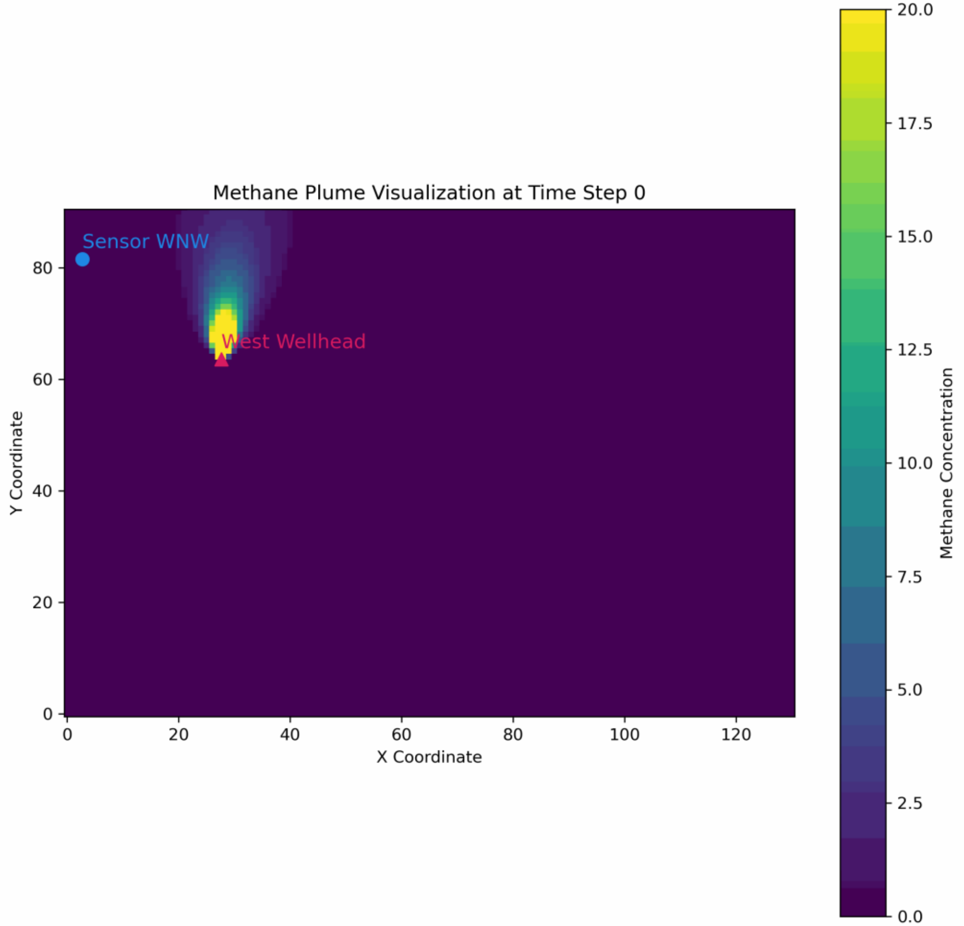


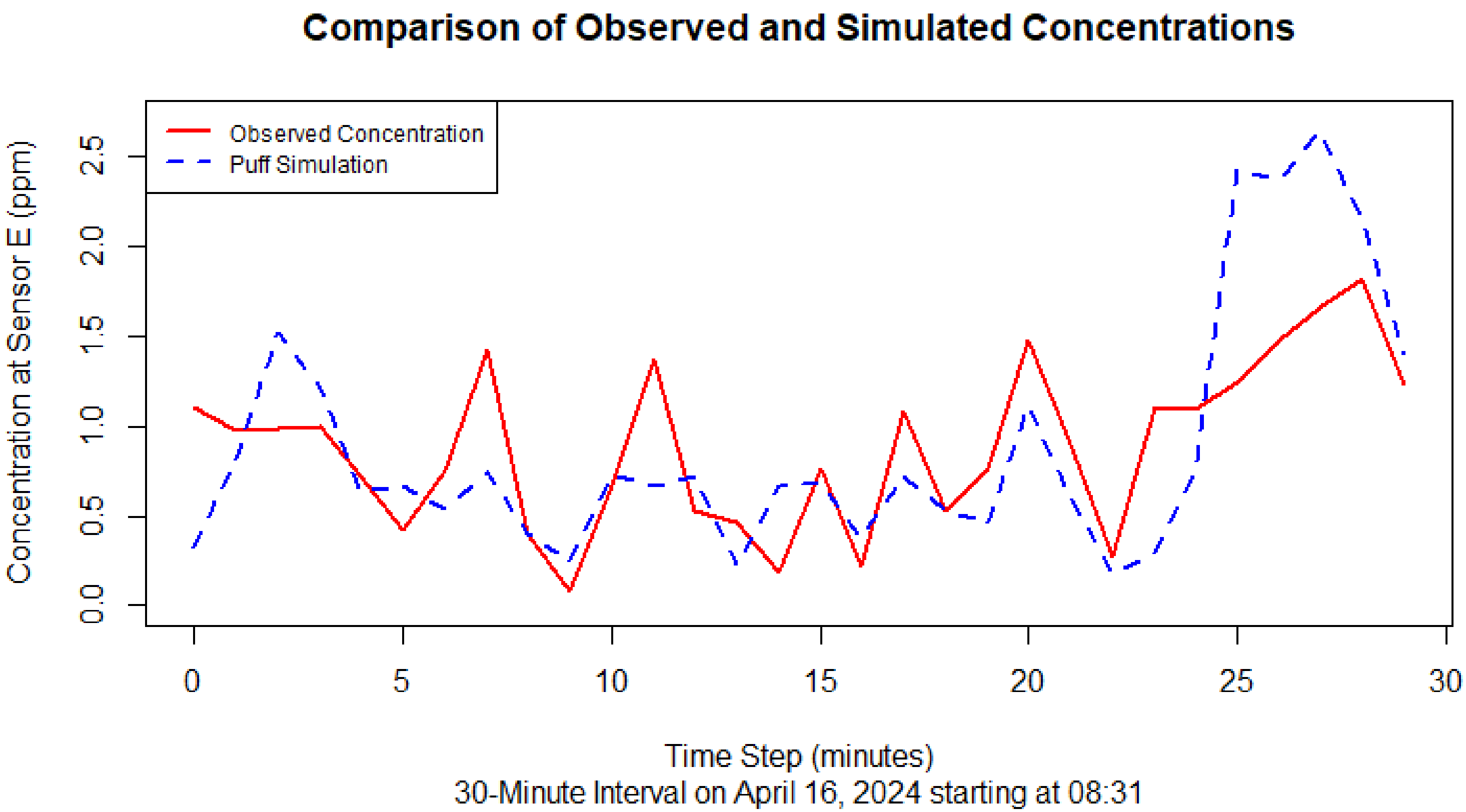
# High Wind Speed Time Period

**Average Wind Speed: 11.082 m/s**

Wind Direction Variance: 0.004

Emission Source: East Separator

Emission Rate: 1.247 kg/hr

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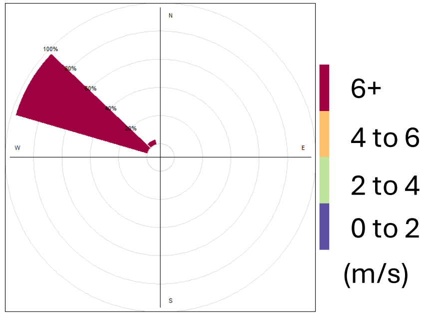

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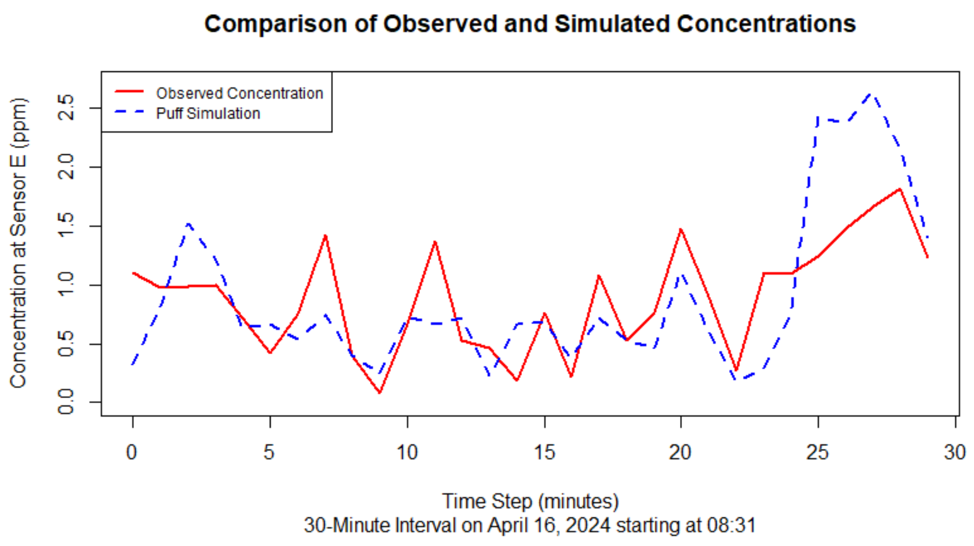


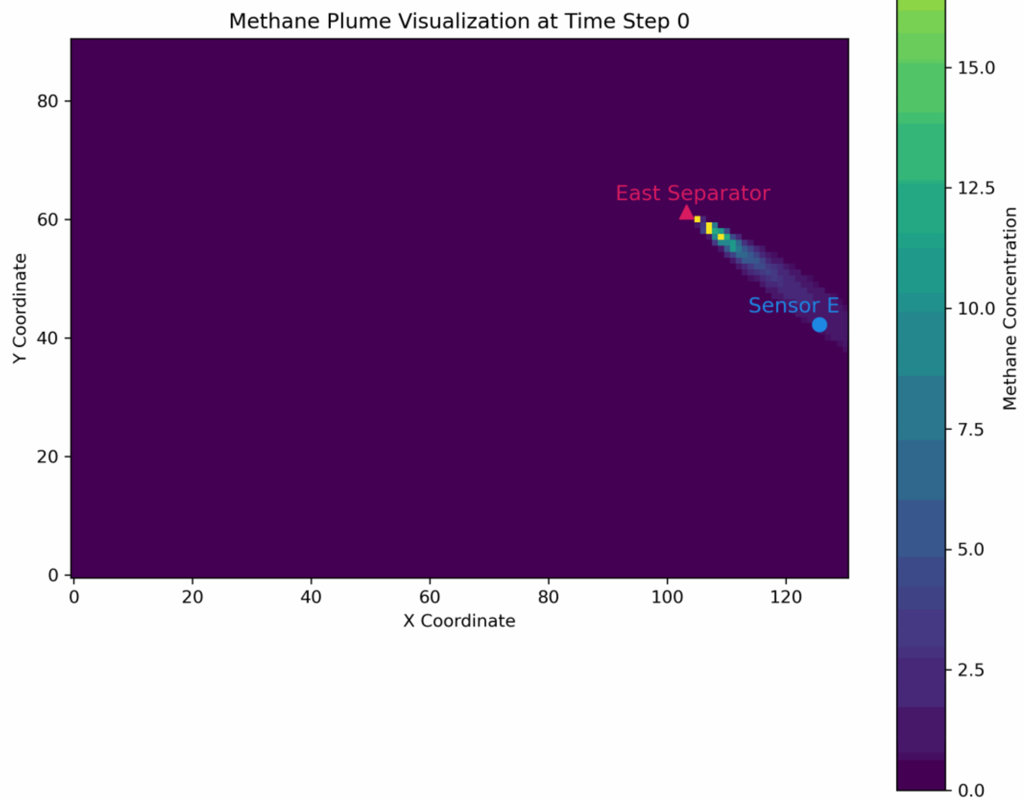

# Conclusions

At low wind speeds, model simulations are too smooth compared to observed concentrations.

At low wind speeds, noisy wind data may cause the plume's path to fluctuate too rapidly, causing lower concentration enhancements.

# Future Research Directions

Obtain **more data** to analyze the model under low wind speeds and a relatively unchanging wind direction.

**More robustly** determine the threshold at which the model starts to underperform.

Apply **different degrees of smoothing** to the wind data in low wind speed regimes.

Investigate other theoretical sources of error: **no diffusion in downwind direction**, and **dispersion parameters** set via lookup tables which could be out of date.

Dorit Hammerling

Spencer Kidd

Philip Waggoner

Meng Jia

Ryker Fish

Cal Okenberg

Troy Sorensen

Teagan Ward

William Daniels

Dishita Sharma

Thanks for listening!

Questions?

# Simulations vs. Observed Concentrations